Boundary value problems for Schrödinger equations with Hardy type potentials.

Abstract. We consider equations of the form $-L_V\, u = \tau$ where $L_V = \Delta + V$ and τ is a Radon measure in a Lipschitz bounded domain $D \subset R^N$. The assumptions on V include the condition $|V(x)| \leq a\delta(x)^{-2}$ - where $a > 0$ and $\delta(x) = \mathrm{dist}(x, \partial D)$. An additional condition guarantees the existence of a ground state Φ_V. The model example is $V = \gamma\delta(x)^{-2}$ where $\gamma < c_H$ ($c_H =$ Hardy constant).

For positive solutions of the equation we define the L_V *boundary trace*. If $\int_D \Phi_V\, d\tau < \infty$, the boundary trace is well defined as a positive, bounded measure ν on ∂D. We consider the corresponding b.v.p., namely, $-L_V\, u = \tau$ in D, $u = \nu$ on ∂D. We show that, for τ and ν as above, the b.v.p. has a unique solution. Further, under some conditions on the ground state - satisfied for a large family of potentials - we obtain sharp two sided estimates of positive solutions of the b.v.p. Finally we discuss some applications to semilinear problems involving the operator L_V.