

Fundamentals of Computing: Lecture 11

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Summary of the previous lecture

- ▶ Variables declared outside all functions are global.
- ▶ Variables declared inside functions are local.
- ▶ The scope of the variables inside a block is from the start of the declaration to the end of the corresponding brace.
- ▶ Local variables hide global variables.

Designing Loops

Loop invariant

A loop invariant is a condition that is true always at the beginning of the loop.

The key idea in designing a loop is finding a good loop invariant.

Finding the smallest in a sequence of n numbers

$$s = \min\{a[0], \dots, a[n-1]\}.$$

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The invariant

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```
s = a[0];
i = 0
while( i < n )
{
    /* Do something to preserve the invariant */
    i++;
}
```

Invariant $\wedge \neg(i \leq n) \Rightarrow s = \min\{a[0], \dots, a[n-1]\}.$

Outline of the method

- ▶ First logically specify the desired condition φ

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- ▶ Write the loop with condition C such that $I \wedge \neg C = \varphi$.

Sorting

Let us define what is sorted array

$$\text{SortedArray}(a) \equiv \forall i \ 0 \leq i < \text{length}(a) - 1 \Rightarrow a[i] \leq a[i + 1].$$

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$$\text{SortedArray}(a) \equiv \forall i \ 0 \leq i < \text{length}(a) - 1 \Rightarrow a[i] \leq a[i + 1].$$

$$\text{Sorted}(a, r, s) \equiv \forall i \ r \leq i < s \Rightarrow a[i] \leq a[i + 1]$$

Observations

- ▶ For all integers r $\text{Sorted}(a, r, r + 1)$.
- ▶ $\text{Sorted}(a, r, s) \wedge \text{Sorted}(a, s, t) \Rightarrow \text{Sorted}(a, r, t)$.

Choose the invariant $\text{Sorted}(a, 0, i)$ for a parameter i .

```
i = 0;
while( i < n)
{
    /* Do something to restore invariant */
    i++;
}
```