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SECTION I

Original Research Papers

NOTE FROM THE EDITOR-IN-CHIEF

In the first few volumes of this journal, the aim is to publish papers in mathematical sciences from authors of IIT Kanpur Research group in Mathematical Modeling and the Members of the Indian Academy of Mathematical Modeling and Simulation having Head Quarters at IIT Kanpur.

Researchers from elsewhere will be invited later to publish their research work in this journal. Self submission is not encouraged. But letters to the Editors are invited from researchers on ideas relevant to Nature and Society for publication in the journal.

In the present special volume of Environmental Systems, two problems related to removal of carbon dioxide from the atmosphere by using external species and greenbelts plantation have been included. The third problem deals with the effects of emissions of hot pollutants from thermal power stations on the suppression of rainfall. The fourth problem studies the removal of dust particles emitted from cement industries in the atmosphere by using water sprays and greenbelts plantation. These papers will provide young researchers a good idea as how to conduct research in these areas.

I also take this opportunity to thank the guest editor and all the authors for their cooperation in this venture.

J.B Shukla

NOTE FROM THE GUEST EDITOR

This special volume consists of four papers related to Environmental Systems. The following problems have been solved:

1. **Reduction of the concentration carbon-di-oxide by discharging particulate matters and liquid droplets around the sources of emission in the atmosphere to control global warming.**
2. **Reduction of the concentration of carbon-di-oxide by discharging particulate matters above the sources of emission in the atmosphere and greenbelts plantation around the sources of emissions to control global warming.**
3. **Effects of hot gases and particulate matters emitted from thermal power plants to reduce rainfall.**
4. **Removal of dust particles using water sprays and greenbelts plantation.**

The papers are basic in nature and very useful to young researchers working in these areas.

Ram Naresh

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Modeling the removal of carbon dioxide from the atmosphere by spraying external species above the sources of emissions: A mechanism to reduce global warming

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Abstract

Today our planet is facing a serious problem of climate change caused by rise in its average temperature due to discharge of global warming gases from various industrial and other sources leading to undesirable consequences such as melting of glaciers, see level rise, etc. The carbon dioxide is one of the most important greenhouse gases responsible for climate change. Therefore, it is necessary to find a mechanism by which the global warming gases such as carbon dioxide from the atmosphere can be removed. In view of this, in this paper, a nonlinear mathematical model is proposed and analyzed to suggest a mechanism for the removal of carbon dioxide by spraying external species such as liquid species and particulate matters in the atmosphere above the source of emissions. When these externally sprayed species, interact with the gas, the secondary phases are formed which then get removed from the atmosphere by gravity. The dynamics of the system in such a case is assumed to be governed by five nonlinearly interacting dependent variables, namely, the concentration of carbon dioxide, the concentrations of externally sprayed liquid species and solid particulate matters, the concentration of carbon dioxide absorbed in the liquid phase and the concentration of carbon dioxide transformed by the solid particulate matters forming secondary species. The model analysis shows that the concentration of global warming gas, CO_2, decreases as the rates of spray of liquid phase and solid particulate matters increase in the atmosphere. It is noted that this gas can be removed almost completely from the atmosphere, if the rates of spray of these external species are very large. The numerical simulation confirms these analytical results.

Key words: Global warming; carbon dioxide; liquid species; particulate matters; equilibrium; stability

1. Introduction

The main global warming contributors in the atmosphere are carbon dioxide (CO_2), methane (CH_4) and hydro fluorocarbons ($HFCs$). Due to continuous increase in the concentration of these global warming gases the average temperature of the earth's atmosphere increases leading to various climate change consequences such as melting of ice at north and south poles, rise in see levels, etc. An increase in global temperature can also change the pattern of precipitation causing floods in some parts of the earth and droughts in other leading to expansion of deserts ,WHO (2003), UNFC (2006) Robinson et al (2007), Millstein and Harley (2009), Zhang and Wang(2011)..

Some studies have been conducted in the past few years for the removal of gaseous pollutants and particulate matters from the atmosphere by precipitation Naresh et al. (2007), Shukla et al (2008), Sundar and Naresh (2011). In this regard, a nonlinear mathematical model for the removal of a gaseous pollutant and two particulate matters by rain in the atmosphere of a city has been studied Shukla et al (2008). They have also studied the effect of acid rain on plant growth Shukla et al (2013). The role of cloud droplets on the removal of gaseous pollutants from the atmosphere has been studied by Sundar and Naresh (2011).

Some other modeling studies have been conducted to study the dynamics of carbon dioxide in the atmosphere using nonlinear mathematical models,Misra and Verma (2013), Sundar et al (2014), Shukla et al (2015). A nonlinear mathematical model is proposed and analyzed to study the dynamics of CO_2 in the atmosphere considering three dependent variables; density of human population, CO_2 concentration and density of forest biomass Misra and Verma (2013). They have shown that the concentration of CO_2 can be stabilized by preventing deforestation and promoting reforestation. Sundar et al (2014) have modeled a phenomenon to study the control of CO_2 using a suitable absorbent (such as aqueous ammonia solution, amines, sodium hydroxide, etc.) near the source of emission. They have shown that the concentration of CO_2 decreases in the atmosphere if suitable absorbent is used.

It is noted here that this concept has not been used for removal of global warming gas from the atmosphere. Therefore, in this paper, to apply this concept, we propose and analyze a nonlinear model for the removal of a global warming gas (such as CO_2) by infusing liquid drops (such as water) and suitable particulate matters (such as calcium oxide) in the atmosphere. When this global warming gas interacts with these externally introduced species, secondary phases are formed which are then removed from the atmosphere by gravity reducing the concentration of global warming gas in the atmosphere. The following reactions illustrate this idea Gao et al (2012).

$$CO_2 + H_2O \rightarrow H_2CO_3$$

$$CO_2 + CaO \rightarrow CaCO_3$$

In the following section, we propose a model using this concept.

2. Mathematical model

We consider a regional atmosphere where the phenomenon of removal of CO_2 by introducing external liquid species and particulate matters takes place. To model the problem, we have made the following assumptions,

(i) The rate of emission of carbon dioxide is a constant.
(ii) Externally sprayed species are introduced at the rate proportional to the rate of emission of CO_2.
(iii) When carbon dioxide comes in contact with external species, it would get removed from the regional atmosphere due to gravity.
(iv) Due to evaporation or any other process, a fraction of removed carbon dioxide may re-enter into the regional atmosphere increasing the growth of carbon dioxide.

Let C be the concentration of CO_2 gas in the atmosphere emitted with a constant rate Q, C_i be the concentration of externally sprayed liquid species in the atmosphere above the source of emission with a rate λQ, C_p be the concentration of particulate matters sprayed above the source of emission with a rate rQ and C_a, C_{pa} be the concentrations of secondary species formed due to interaction of CO_2 with externally sprayed liquid species and particulate matters respectively. Here λ and r are the proportionality constants. The natural depletion of CO_2 is assumed to be proportional to its concentration i.e. $\lambda_0 C$. It is assumed further that the rate by which CO_2 is removed from the atmosphere is directly proportional to its concentration as well as the concentration of externally sprayed liquid species and the density of particulate matters, (i.e. $\lambda_1 CC_i$ and $r_1 CC_p$ respectively). The constants λ_1 and r_1 are the depletion rate coefficients of CO_2 due to interaction with externally sprayed liquid species and particulate matters respectively. It is considered that the concentrations (densities) of both the secondary species formed after reaction are proportional to the concentration of CO_2 and the concentrations of externally sprayed species in the atmosphere, (i.e. $\theta_1 CC_i$ and $\theta_2 CC_p$ respectively) with θ_{10} and θ_{20} as their natural depletion rates respectively. Here, $\theta_1 \le \lambda_1$ and $\theta_2 \le r_1$ are the rates by which secondary species are formed due to interaction of CO_2 with externally sprayed liquid species and particulate matters. A fraction of these removed amounts of CO_2 (i.e. $\pi_{10}\theta_{10}C_a$ and $\pi_{20}\theta_{20}C_{pa}$) may re-enter into the regional atmosphere increasing the growth of CO_2 where $0 \le \pi_{10} \le 1$ and $0 \le \pi_{20} \le 1$ are reversible rate coefficients.

Keeping in view of these considerations, the problem is governed by the following nonlinear ordinary differential equations,

$$\frac{dC}{dt} = Q - \delta_0 C - \lambda_1 CC_i - r_1 CC_p + \pi_{10}\theta_{10}C_a + \pi_{20}\theta_{20}C_{pa} \tag{1}$$

$$\frac{dC_i}{dt} = \lambda Q - \lambda_0 C_i - \lambda_1 CC_i \tag{2}$$

$$\frac{dC_p}{dt} = rQ - r_0 C_p - r_1 CC_p \tag{3}$$

$$\frac{dC_a}{dt} = \theta_1 CC_i - \theta_{10}C_a \tag{4}$$

$$\frac{dC_{pa}}{dt} = \theta_2 CC_p - \theta_{20}C_{pa} \tag{5}$$

with $C(0) > 0,\ C_i(0) > 0,\ C_p(0) > 0,\ C_a(0) > 0,\ C_{pa} > 0$

Remark It is noted here that if λ_1 and r_1 are very large then $\frac{dC}{dt}$ may become negative which implies that the concentration of CO_2 can be stabilized significantly in the atmosphere by spraying external species in large quantities.

To analyze the model (1-5), we state the region of attraction Ω of the solutions in the following lemma (without proof),

Lemma

The region of attraction for all solutions initiating in the positive octant is given by the set $\Omega = \left\{(C, C_i, C_p, C_a, C_{pa}) : 0 \le C \le C_m,\ 0 \le C_i + C_a \le \frac{\lambda Q}{\lambda_m},\ 0 \le C_p + C_{pa} \le \frac{rQ}{r_m}\right\}$

where $C_m = \frac{Q}{\delta_0}\left\{1 + \frac{\pi_{10}\theta_{10}\lambda}{\lambda_m} + \frac{\pi_{20}\theta_{20}r}{r_m}\right\}$, $\lambda_m = \min\{\lambda_0, \theta_{10}\}$ and $r_m = \min\{r_0, \theta_{20}\}$

3. Equilibrium analysis

The model (1-5) has only one non-negative equilibrium namely $E^*(C^*, C_i^*, C_p^*, C_a^*, C_{pa}^*)$ which is obtained by solving the following algebraic equations:

$$Q - \delta_0 C - \lambda_1 CC_i - r_1 CC_p + \pi_{10}\theta_{10}C_a + \pi_{20}\theta_{20}C_{pa} = 0 \tag{6}$$

$$C_i = \frac{\lambda Q}{\lambda_0 + \lambda_1 C} = f_1(C) \tag{7}$$

$$C_p = \frac{rQ}{r_0 + r_1 C} = f_2(C) \tag{8}$$

$$C_a = \frac{\theta_1}{\theta_{10}} C C_i = \frac{\theta_1}{\theta_{10}} C f_1(C) \tag{9}$$

$$C_{pa} = \frac{\theta_2}{\theta_{20}} C C_p = \frac{\theta_2}{\theta_{20}} C f_2(C) \tag{10}$$

We can write equation (6) after using (7) and (8) as follows,

$$F(C) = Q - \delta_0 C - (\lambda_1 - \pi_{10}\theta_1) C f_1(C) - (r_1 - \pi_{20}\theta_2) C f_2(C) = 0 \tag{11}$$

From (11), we note the following,

(i) $F(0) = Q > 0$

(ii) $F(C_m) = -Q\left\{\dfrac{\pi_{10}\theta_{10}\lambda}{\lambda_m} + \dfrac{\pi_{20}\theta_{20} r}{r_m}\right\} - (\lambda_1 - \pi_{10}\theta_1) C_m f_1(C_m)$

$-(r_1 - \pi_{20}\theta_2) C_m f_2(C_m) < 0$

(iii) $F'(C) = -\delta_0 - (\lambda_1 - \pi_{10}\theta_1)\{C f_1'(C) + f_1(C)\} - (r_1 - \pi_{20}\theta_2)\{C f_2'(C) + f_2(C)\} < 0$

This implies that there exist a unique root of $F(C) = 0$ (say C^*) in $0 \le C \le C_m$. By knowing the value of C^*, we can find the values of C_i^*, C_p^*, C_a^* and C_{pa}^* from equations (7) – (10) respectively.

Variations of dependent variables with different parameters

In the following, we check the variations of dependent variables with respect to relevant parameters.

From equation (11), we have

$$Q - \delta_0 C - (\lambda_1 - \pi_{10}\theta_1) C f_1(C) - (r_1 - \pi_{20}\theta_2) C f_2(C) = 0$$

Differentiating the above equation with respect to λ_1, we get,

$$\frac{dC}{d\lambda_1} = -\frac{C f_1(C)}{\delta_0 + (\lambda_1 - \pi_{10}\theta_1)\{C f_1'(C) + f_1(C)\} + (r_1 - \pi_{20}\theta_2)\{C f_2'(C) + f_2(C)\}} < 0$$

Similarly, we can show that $\dfrac{dC}{dr_1} < 0$.

Thus, the concentration of CO_2 gas decreases as the rate of interaction of CO_2 with external species increases.

4. Stability Analysis

In the following, we state the theorems for local and nonlinear stability of the equilibrium of model (1-5).

Theorem 4.1 The equilibrium E^* is locally asymptotically stable provided the following conditions are satisfied,

$$\lambda_1^{\,2}(C^* + C_i^{\,*})^2 < \frac{1}{3}(\delta_0 + \lambda_1 C_i^{\,*} + r_1 C_p^{\,*})(\lambda_0 + \lambda_1 C^*) \qquad (12)$$

$$r_1^{\,2}(C^* + C_p^{\,*})^2 < \frac{1}{3}(\delta_0 + \lambda_1 C_i^{\,*} + r_1 C_p^{\,*})(r_0 + r_1 C^*) \qquad (13)$$

$$\frac{\pi_{10}^{\;2}}{(\delta_0 + \lambda_1 C_i^{\,*} + r_1 C_p^{\,*})^2} < \frac{4}{27\theta_1^{\,2}} \min\left\{\frac{1}{3}\frac{(\delta_0 + \lambda_1 C_i^{\,*} + r_1 C_p^{\,*})}{C_i^{\,*2}}, \frac{\lambda_0 + \lambda_1 C^*}{C^{*2}}\right\} \qquad (14)$$

$$\frac{\pi_{20}^{\;2}}{(\delta_0 + \lambda_1 C_i^{\,*} + r_1 C_p^{\,*})^2} < \frac{4}{27\theta_2^{\,2}} \min\left\{\frac{1}{3}\frac{(\delta_0 + \lambda_1 C_i^{\,*} + r_1 C_p^{\,*})}{C_p^{\,*2}}, \frac{r_0 + r_1 C^*}{C^{*2}}\right\} \qquad (15)$$

(see Appendix A for proof).

Theorem 4.2 The equilibrium E^* is globally stable provided the following conditions hold inside the region of attraction Ω,

$$\lambda_1^{\,2}(C^* + C_i^{\,*})^2 < \frac{1}{3}\delta_0 \lambda_0 \qquad (16)$$

$$r_1^{\,2}(C^* + C_p^{\,*})^2 < \frac{1}{3}\delta_0 r_0 \qquad (17)$$

$$\frac{\pi_{10}^{\;2}}{\delta_0} < \frac{4}{27\theta_1^{\,2}} \min\left\{\frac{1}{3}\frac{\delta_0 \lambda_m^{\;2}}{Q^2 \lambda^2}, \frac{\lambda_0}{C^{*2}}\right\} \qquad (18)$$

$$\frac{\pi_{20}^{\;2}}{\delta_0} < \frac{4}{27\theta_2^{\,2}} \min\left\{\frac{1}{3}\frac{\delta_0 r_m^{\;2}}{Q^2 r^2}, \frac{r_0}{C^{*2}}\right\} \qquad (19)$$

(see Appendix B for proof).

The above discussion about equilibrium analysis and the stability conditions (12) – (19) imply that the system attains equilibrium under some conditions and the concentration of CO_2 decreases as the rates of spray of external species increase.

Numerical simulations

In this section, numerical simulation of the model (1-5) with respect to E^* is conducted using Maple 7, to see the effect of externally sprayed species and particulate matters on the removal of CO_2 from the regional atmosphere and feasibility of stability conditions by choosing the following set of parameter values,

$$Q = 2,\ \delta_0 = 1,\ \lambda_1 = 0.3, r_1 = 0.2,\ \lambda = 1, r = 1,\ \lambda_0 = 1.8,\ r_0 = 1.6, \theta_1 = 0.2, \theta_2 = 0.15,$$

$$\theta_{10} = 0.18,\ \theta_{20} = 0.12,\ \pi_{10} = 0.02,\ \pi_{20} = 0.02$$

The equilibrium values of components of $E^*(C^*, C_i^*, C_p^*, C_a^*, C_{pa}^*)$ for the model system (1-5) are obtained as,

$$C^* = 1.352224, C_i^* = 0.906755,\ C_p^* = 1.069264,\ C_a^* = 1.362373,\ C_{pa}^* = 1.807356$$

The eigenvalues corresponding to $E^*(C^*, C_i^*, C_p^*, C_a^*, C_{pa}^*)$ for model system (1-5) are $-2.353494,\ -1.937516,\ -1.271662,\ -0.179573,\ -0.119744$. Since all the eigenvalues are negative, the interior equilibrium E^* is locally asymptotically stable. Stability conditions stated in theorems 4.1 and 4.2 are also satisfied for the foregoing set of parameter values.

The nonlinear stability behavior of E^* in $C - C_i$ and $C - C_p$ plane is shown in figures 1 and 2 respectively.

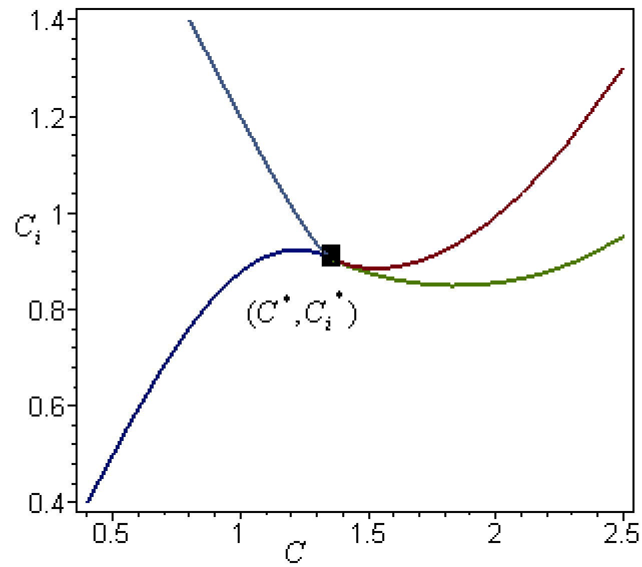

Figure 1. Nonlinear stability in $C - C_i$ plane

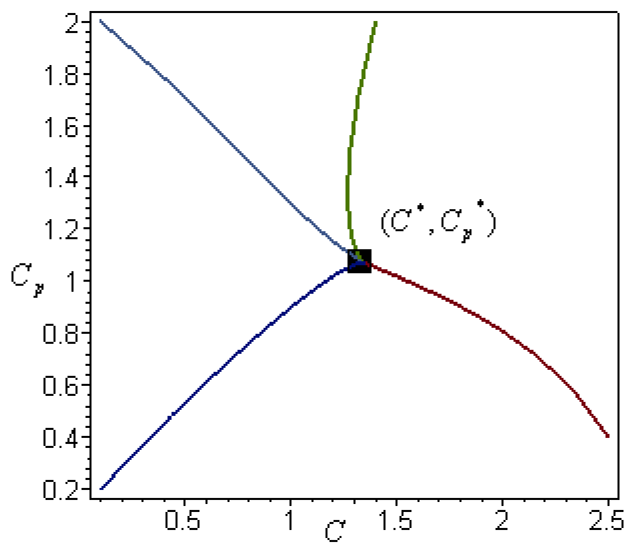

Figure 2. Nonlinear stability in $C - C_p$ plane

In figures 3 and 4, the variation of concentration of carbon dioxide (C) with time $'t'$ is shown for different values of λ (the rate of spray of external liquid species) and r (the rate of spray of particulate matters) respectively. It is evident from these figures that the concentration of CO_2 decreases as λ or r increases. In figures 5 and 6, the variation of concentration of carbon dioxide (C) with time $'t'$ is shown for different values of λ_1 and r_1 respectively. From these figures, we note that the concentration of CO_2 decreases as λ_1 and r_1 increase.

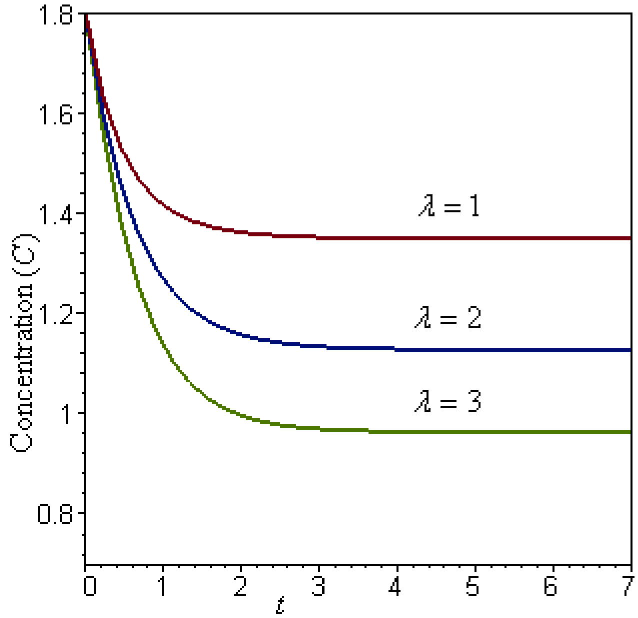

Figure 3. Variation of C with time 't' for different values of λ at $Q = 2$

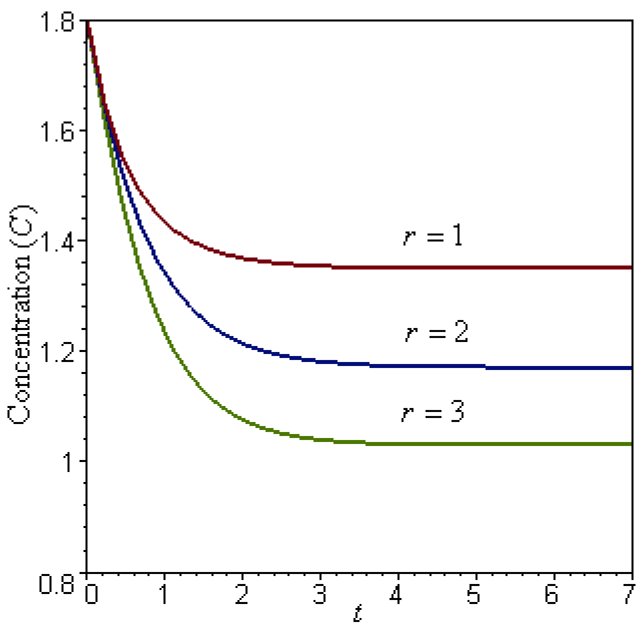

Figure 4. Variation of C with time 't' for different values of r

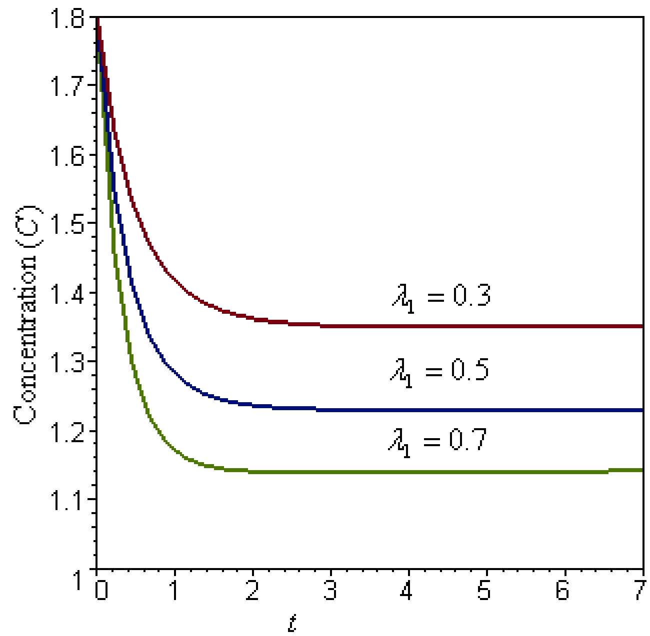

Figure 5. Variation of C with time 't' for different values of λ_1

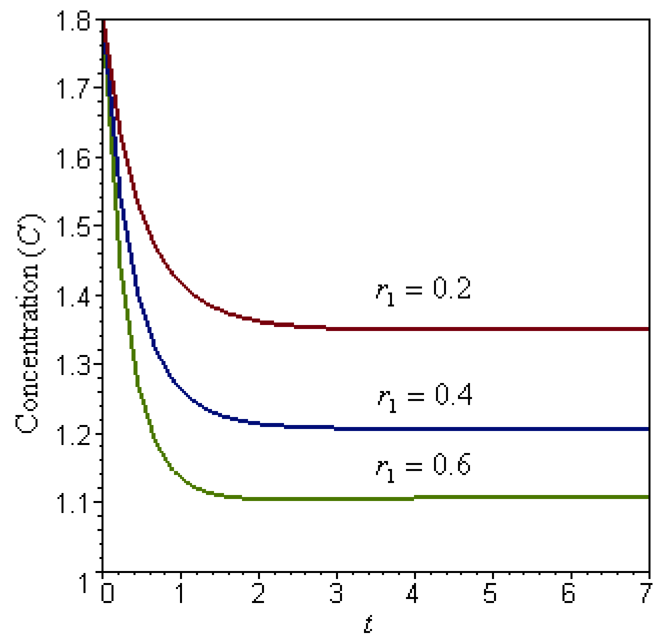

Figure 6. Variation of C with time 't' for different values of r_1

Conclusions

The presence of global warming gases in the atmosphere causes climate changes which are undesirable to our living. In this paper, through modeling study, it has been suggested that the global warming gases from the atmosphere can be removed by spraying liquid droplets and appropriate particulate matters which can react with CO_2 and remove it by gravity. The model analysis has shown that the concentration of global warming gas can be decreased considerably from the atmosphere by spraying suitable liquid droplets and appropriate particulate matters above the source of emission. The numerical simulation of the model confirms these analytical results. This study suggests a mechanism for reducing global warming in the atmosphere

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APPENDIX A

Proof of the Theorem 4.1: Using the following positive definite function in the linearized system of model (1-5),

$$V = \frac{1}{2}(k_1 C_1^{\,2} + k_2 C_{i1}^{\,2} + k_3 C_{p1}^{\,2} + k_4 C_{a1}^{\,2} + k_5 C_{pa1}^{\,2}) \qquad \text{(A.1)}$$

where k_i (i = 1 to 5) are constants, to be chosen appropriately and $C_1, C_{i1}, C_{p1}, C_1, C_{a1}, C_{pa1}$ are small perturbations from E^* as follows,

$$C = C^* + C_1,\ C_i = C_i^{\,*} + C_{i1},\ C_p = C_p^{\,*} + C_{p1},\ C_a = C_a^{\,*} + C_{a1},\ C_{pa} = C_{pa}^{\,*} + C_{pa1}$$

Differentiating (A.1) with respect to $'t'$ we get, in the linearized system corresponding to E^*,

$$\begin{aligned}\dot{V} = &-k_1(\delta_0 + \lambda_1 C_i^{\,*} + \lambda_2 C_p^{\,*})C_1^{\,2} - k_2(\lambda_0 + \lambda_1 C^*)C_{i1}^{\,2} - k_3(r_0 + r_1 C^*)C_{p1}^{\,2} \\ &- k_4\theta_{10} C_{a1}^{\,2} - k_5\theta_{20} C_{pa1}^{\,2} - \lambda_1(k_1 C^* + k_2 C_i^{\,*})C_1 C_{i1} - r_1(k_1 C^* + k_3 C_p^{\,*})C_1 C_{p1} \\ &+ k_1\pi_{10}\theta_{10} C_1 C_{a1} + k_4\theta_1 C_i^{\,*} C_1 C_{a1} + k_1\pi_{20}\theta_{20} C_1 C_{pa1} + k_5\theta_2 C_p^{\,*} C_1 C_{pa1} \\ &+ k_4\theta_1 C^* C_{i1} C_{a1} + k_5\theta_2 C^* C_{p1} C_{pa1}\end{aligned}$$

Now $\dot{V}$ will be negative definite under the following conditions,

$$\lambda_1^{\,2}[k_1 C^* + k_2 C_i^{\,*}]^2 < \frac{1}{3}k_1 k_2(\delta_0 + \lambda_1 C_i^{\,*} + r_1 C_p^{\,*})(\lambda_0 + \lambda_1 C^*) \qquad \text{(A.2)}$$

$$r_1^{\,2}[k_1 C^* + k_3 C_p^{\,*}]^2 < \frac{1}{3}k_1 k_3(\delta_0 + \lambda_1 C_i^{\,*} + r_1 C_p^{\,*})(r_0 + r_1 C^*) \qquad \text{(A.3)}$$

$$k_1(\pi_{10}\theta_{10})^2 < \frac{2}{9}k_4(\delta_0 + \lambda_1 C_i^{\,*} + r_1 C_p^{\,*})\theta_{10} \qquad \text{(A.4)}$$

$$k_4(\theta_1\, C_i^{\,*})^2 < \frac{2}{9}k_1(\delta_0 + \lambda_1 C_i^{\,*} + r_1 C_p^{\,*})\theta_{10} \qquad \text{(A.5)}$$

$$k_1(\pi_{20}\theta_{20})^2 < \frac{2}{9}k_5(\delta_0 + \lambda_1 C_i^{\,*} + r_1 C_p^{\,*})\theta_{20} \qquad \text{(A.6)}$$

$$k_5(\theta_2\, C_p^{\,*})^2 < \frac{2}{9}k_1(\delta_0 + \lambda_1 C_i^{\,*} + r_1 C_p^{\,*})\theta_{20} \qquad \text{(A.7)}$$

$$k_4(\theta_1\, C^*)^2 < \frac{2}{3}k_2(\lambda_0 + \lambda_1 C^*)\theta_{10} \qquad \text{(A.8)}$$

$$k_5(\theta_2\, C^*)^2 < \frac{2}{3}k_3(r_0 + r_1 C^*)\theta_{20} \qquad \text{(A.9)}$$

Choosing $k_1 = k_2 = k_3 = 1$, $\dot{V}$ will be negative definite provided the conditions (12) – (15) are satisfied. Hence, E^* is locally asymptotically stable.

APPENDIX B

Proof of the Theorem 4.2 Consider a positive definite function

$$U = \frac{1}{2}m_1(C - C^*)^2 + \frac{1}{2}m_2(C_i - C_i^*)^2 + \frac{1}{2}m_3(C_p - C_p^*)^2$$
$$+\frac{1}{2}m_4(C_a - C_a^*)^2 + \frac{1}{2}m_5(C_{pa} - C_{pa}^*)^2 \qquad \text{(B.1)}$$

Differentiating with respect to $'t'$ we get

$$\dot{U} = m_1(C - C^*)\dot{C} + m_2(C_i - C_i^*)\dot{C}_i + m_3(C_p - C_p^*)\dot{C}_p$$
$$+ m_4(C_a - C_a^*)\dot{C}_a + m_5(C_{pa} - C_{pa}^*)\dot{C}_{pa}$$

Putting the values of derivatives and simplifying, we get,

$$\dot{U} = -m_1(\delta_0 + \lambda_1 C_i + r_1 C_p)(C - C^*)^2 - m_2(\lambda_0 + \lambda_1 C)(C_i - C_i^*)^2$$
$$- m_3(r_0 + r_1 C)(C_p - C_p^*)^2 - m_4\theta_{10}(C_a - C_a^*)^2 - m_5\theta_{20}(C_{pa} - C_{pa}^*)^2$$
$$- \lambda_1(m_1 C^* + m_2 C_i^*)(C - C^*)(C_i - C_i^*) - r_1(m_1 C^* + m_3 C_p^*)(C - C^*)(C_p - C_p^*)$$
$$+ m_1\pi_{10}\theta_{10}(C - C^*)(C_a - C_a^*) + m_4\theta_1 C_i(C - C^*)(C_a - C_a^*)$$
$$+ m_1\pi_{20}\theta_{20}(C - C^*)(C_{pa} - C_{pa}^*) + m_5\theta_2 C_p^*(C - C^*)(C_{pa} - C_{pa}^*)$$
$$+ m_4\theta_1 C^*(C_i - C_i^*)(C_a - C_a^*) + m_5\theta_2 C^*(C_p - C_p^*)(C_{pa} - C_{pa}^*)$$

Now $\dot{U}$ will be negative definite under the following conditions,

$$\lambda_1^2[m_1 C^* + m_2 C_i^*]^2 < \frac{1}{3}m_1 m_2 \delta_0 \lambda_0 \qquad \text{(B.2)}$$

$$r_1^2[m_1 C^* + m_3 C_p^*]^2 < \frac{1}{3}m_1 m_3 \delta_0 r_0 \qquad \text{(B.3)}$$

$$m_1(\pi_{10}\theta_{10})^2 < \frac{2}{9} m_4 \delta_0 \theta_{10} \quad \text{(B.4)}$$

$$m_4(\theta_1 C_i)^2 < \frac{2}{9} m_1 \delta_0 \theta_{10} \quad \text{(B.5)}$$

$$m_1(\pi_{20}\theta_{20})^2 < \frac{2}{9} m_5 \delta_0 \theta_{20} \quad \text{(B.6)}$$

$$m_5(\theta_2 C_p)^2 < \frac{2}{9} m_1 \delta_0 \theta_{20} \quad \text{(B.7)}$$

$$m_4(\theta_1 C^*)^2 < \frac{2}{3} m_2 \lambda_0 \theta_{10} \quad \text{(B.8)}$$

$$m_5(\theta_2 C^*)^2 < \frac{2}{3} m_3 r_0 \theta_{20} \quad \text{(B.9)}$$

Maximizing LHS and minimizing RHS and choosing $m_1 = m_2 = m_3 = 1$, $\dot{U}$ will be negative definite provided the conditions (16) – (19) are satisfied. Hence E^* is globally stable inside the region of attraction Ω.

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Modeling the Removal of Carbon Dioxide from the Near Earth Atmosphere Using External Species and Greenbelts Plantation Around Sources of Emissions.

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Abstract

The temperature of the earth's environment has increased in recent decades due to the emission of global warming gases such as carbon dioxide (CO_2), etc. in the atmosphere leading to climate change. This may lead to serious consequences in the future such as melting of glaciers, rise in the sea level and even flooding of coastal cities in future. Therefore, some mechanisms have to be found to remove global warming gases from the atmosphere by using methods such as the plantation of leafy greenbelts. In this paper, a nonlinear model is proposed to study the feasibility of removing carbon dioxide from the near earth regional atmosphere by introducing external species such as H_2O, C_aO, etc. which may react with these gases to remove them by gravity. Further, this gas can also be removed by photosynthesis using the plantation of leafy greenbelts in and around its ground sources of emissions. Four dependent variables namely, the concentrations of carbon dioxide and the externally introduced species into the atmosphere, the biomass density of leafy greenbelts in the plantation and the density of resulting particulate matters has been taken to describe the dynamics of carbon dioxide removal. The model is analyzed using the stability theory of differential equations and computer simulation. The model analysis shows that the concentration of carbon dioxide decreases from the near earth atmosphere, if the rate of introduction of external species increases and its concentration decreases further if the rate of absorption of carbon dioxide by greenbelts becomes very large. Computer simulations of the model confirm these s results.

Key words: Global warming, liquid drops, leafy greenbelts, mathematical model, stability

1. Introduction

There has been an increase in the concentration of green house gases in the earth's atmosphere over the past century leading to warming of the earth's environment and causing climate change. There are various consequences of this change such as rising sea levels, decreased snow cover, the spread of vector borne diseases in the northern hemisphere, etc., UNFCCC (1992), Gregory and Oerlemans (1998), Vicki (2001), WHO (2003), Roger (2004), Cao and Caldeira (2011). It is predicted that future climate changes will include further rise in sea levels and an increase in the frequency of extreme weather conditions. The United Nations Framework Convention on Climate Change (1992) calls for the stabilization of the concentrations of green house gases in the atmosphere at a level that is not dangerous to the climate system. The carbon dioxide, CO_2, is the largest contributor to global warming with a share of 60 percent.

In view of these considerations, a nonlinear mathematical model is proposed to study the feasibility of the removal of carbon dioxide by introducing some external species (such as infusing liquid drops or introducing particulate matters) into the atmosphere, which may chemically react with carbon dioxide and removed it by gravity. The following reactions may be noted in this regard, Balen (2005), Nikulshina et al (2007), Wu and Lan (2012),

$$CO_2 + H_2O \rightarrow H_2CO_3$$

$$CO_2 + CaO \rightarrow CaCO_3$$

Carbon dioxide (CO_2) gas can also be reduced from by planting leafy greenbelts across the world, which use this gas in the respiratory process during photosynthesis. A simple chemical equation for photosynthesis in plant leaves which transform carbon dioxide into glucose, Hopkins and Norman (2009),

carbon dioxide + water + light energy = glucose + oxygen

$$6CO_2 + 6H_2O + \text{light energy} \rightarrow C_6H_{12}O_6 + 6O_2$$

In recent years some studies have been conducted for removing air pollutants and particulate matters from the atmosphere by rain but this concept has not been used for decreasing global warming gases from the atmosphere, Ramanathan et al (2001), Lennart (2006), Naresh et al (2007), Shukla et al (2008a, b), Zhang and Wang (2011). In this paper, we apply this concept to model the phenomenon of removing carbon dioxide from the near earth atmosphere by introducing external species and planting green belts around the sources of emission.

2. Mathematical model

Consider a near earth local atmosphere around the source of emission. It is considered that the externally introduced species is dispersed in this regional atmosphere. It is considered further that the leafy trees are planted in the greenbelt around the source of emission.

Let C be the concentration of carbon dioxide gas emitted in the atmosphere at a constant rate Q, C_i is the concentration of externally introduced species discharged with a constant rate q (may be proportional to Q) into the atmosphere, C_p is the concentration of particulate matters formed in the atmosphere due to interaction between carbon dioxide and externally introduced species. Let B be the biomass density of leafy trees in the greenbelt planted around the sources of emission. The natural depletion rate of carbon dioxide as well as those of externally introduced species are assumed to be proportional to their respective concentrations, i.e. $\alpha_0 C$ and $\lambda_0 C_i$ respectively where α_0 and λ_0 are constants. The rate of decrease in concentration of carbon dioxide is assumed to be directly proportional to its concentration and the concentration of externally introduced species (i.e. $\lambda_1 CC_i$) leading to a decrease in the concentration of externally introduced species in the atmosphere (i.e. $\lambda_1 CC_i$). Similarly, the rate of decrease of concentration of carbon dioxide by leafy trees is proportional to its concentration and the biomass density of leafy trees in the greenbelt (i.e. $\lambda_2 BC$). It is further assumed that the formation of particulate matter is proportional to the concentration of carbon dioxide and the number density or concentration of externally introduced species (i.e. θCC_i) and the natural depletion rate of particulate matter is proportional to its concentration (i.e. $\theta_0 C_p$). In the model, r is the intrinsic growth rate of plant species and $\frac{Kr}{r_0}$ is its carrying capacity as compared to the usual logistic model, where K is the measure of plant biomass and r_0 is coefficient related to intra-specific competition $\frac{r_0}{K}$.

With the above considerations, the phenomenon is governed by the following nonlinear ordinary differential equations,

$$\frac{dC}{dt} = Q - \alpha_0 C - \lambda_1 CC_i - \lambda_2 BC \qquad (1)$$

$$\frac{dC_i}{dt} = q - \lambda_0 C_i - \lambda_1 CC_i \qquad (2)$$

$$\frac{dB}{dt} = rB - \frac{r_0 B^2}{K} + \lambda_2 BC \qquad (3)$$

$$\frac{dC_p}{dt} = \theta CC_i - \theta_0 C_p \qquad (4)$$

with $C(0) > 0,\ C_i(0) > 0,\ B(0) > 0, C_p(0) > 0$

To analyze the model (1) – (4), we need to know bounds for the dependent variables. For this, we state the region of attraction Ω in the following lemma (without proof),

2.1. Lemma

The region of attraction for all solutions initiating in the positive octant given by the set $\Omega = \left\{ (C, C_i, B, C_p) : 0 \le C \le \frac{Q}{\alpha_0},\ 0 \le C_i \le \frac{q}{\lambda_0},\ 0 \le B \le B_m,\ 0 \le C_p \le C_m \right\}$,

where $B_m = \frac{K}{r_0}\left(r + \lambda_2 \frac{Q}{\alpha_0} \right)$ and $C_m = \frac{\theta q Q}{\theta_0 \lambda_0 \alpha_0}$

3. Equilibrium analysis

The model (1) – (4) has two nonnegative equilibria, namely $\overline{E}(\overline{C}, \overline{C}_i, 0, \overline{C}_p)$ and $E^*(C^*, C_i^*, B^*, C_p^*)$, These are obtained by solving the algebraic equations:

$$Q - \alpha_0 C - \lambda_1 C C_i - \lambda_2 B C = 0 \tag{5}$$

$$q - \lambda_0 C_i - \lambda_1 C C_i = 0 \tag{6}$$

$$rB - \frac{r_0 B^2}{K} + \lambda_2 B C = 0 \tag{7}$$

$$\theta C C_i - \theta_0 C_p = 0 \tag{8}$$

From (7), we obtain the values of B

$$B = 0 \text{ and } B = \frac{K}{r_0}(r + \lambda_2 C)$$

From these values we get two equilibria corresponding to each B as shown in the following.

3.1. Existence of $\overline{E}(\overline{C}, \overline{C}_i, 0, \overline{C}_p)$

For $\overline{E}(\overline{C}, \overline{C}_i, 0, \overline{C}_p)$, the value of $\overline{C}$ is given by the following algebraic equation

$$\alpha_0 \lambda_1 \overline{C}^2 + (\alpha_0 \lambda_0 + \lambda_1 q - \lambda_1 Q)\overline{C} - Q\lambda_0 = 0 \tag{9}$$

Equation (9) has two roots, one is positive and the other is negative. The positive solution of $\overline{C}$ is

$$\overline{C} = \frac{-(\alpha_0 \lambda_0 + \lambda_1 q - \lambda_1 Q) + \sqrt{(\alpha_0 \lambda_0 + \lambda_1 q - \lambda_1 Q)^2 + 4\alpha_0 Q \lambda_0 \lambda_1}}{2\alpha_0 \lambda_1} \tag{10}$$

Using $\overline{C}$, the other equilibrium values are

$$\overline{C}_i = \frac{q}{(\lambda_0 + \lambda_1 \overline{C})} \qquad (11))$$

$$\overline{C}_p = \frac{\theta \overline{C}\,\overline{C}_i}{\theta_0} \tag{12}$$

It will be shown in section 4 that steady state solution $\overline{E}(\overline{C}, \overline{C}_i, 0, \overline{C}_p)$ is always unstable.

3.2. Existence of $E^*(C^*, C_i^*, B^*, C_p^*)$

For $E^*(C^*, C_i^*, B^*, C_p^*)$, the value of C^* is given by the algebraic equation

$$aC^2 + bC - c = 0 \tag{13}$$

where $a = \alpha_0\lambda_1 + \lambda_2^2 \dfrac{K}{r_0}$, $b = \alpha_0\lambda_0 + \lambda_2 \dfrac{rK}{r_0} + \lambda_1 q - \lambda_1 Q$ and $c = Q\lambda_0$

It is easy to show that the steady-state solution E^* is always within the invariant region. Let

$$G(C) = aC^2 + bC - c \tag{14}$$

Then

$$G(0) = -c = -Q\lambda_0 < 0$$

and

$$G\left(\frac{Q}{\alpha_0}\right) = \frac{Q(KQ\lambda_2^2 + K\alpha_0\lambda_2 r + \alpha_0\lambda_1 q r_0)}{\alpha_0^2 r_0} > 0$$

This shows that the steady state value for C^* is within the invariant region.

We have

$$C_i^* = \frac{q}{(\lambda_0 + \lambda_1 C^*)} < \frac{q}{\lambda_0} \tag{15}$$

Thus the steady-state value for C_i^* is within the invariant region.

We have

$$B^* = \frac{K}{r_0}(r + \lambda_2 C^*) < \frac{K}{r_0}\left(r + \frac{\lambda_2 Q}{\alpha_0}\right) \tag{16}$$

using the fact that the steady-state value for C^* is within the invariant region. Thus the steady-state value for B^* is within the invariant region.

We have

$$C_p^* = \frac{\theta C^* C_i^*}{\theta_0} < \frac{\theta}{\theta_0}\left(\frac{Q}{\alpha_0}\right)\left(\frac{q}{\lambda_0}\right) \tag{17}$$

using the fact that the steady-state value for C^* and C_i^* are both within the invariant region. Thus the steady-state value for C_p^* is within the invariant region.

3.3. Variation of C^* with respect to λ_1 and λ_2

From (13), it can easily be shown that $\frac{dC^*}{d\lambda_1} < 0$ and $\frac{dC^*}{d\lambda_2} < 0$. These conditions show that the steady state concentration of carbon dioxide (C^*) decreases with its interaction with the external species with concentration C_i and density B of leafy trees.

Further, from equation (13), we have

$$C^* = \frac{-\left(\alpha_0\lambda_0 + \lambda_2 \frac{rK}{r_0} + \lambda_1 q - \lambda_1 Q\right) + \sqrt{\left(\alpha_0\lambda_0 + \lambda_2 \frac{rK}{r_0} + \lambda_1 q - \lambda_1 Q\right)^2 + 4\left(\alpha_0\lambda_1 + \lambda_2{}^2 \frac{K}{r_0}\right)Q\lambda_0}}{2\left(\alpha_0\lambda_1 + \lambda_2{}^2 \frac{K}{r_0}\right)} \tag{18}$$

This implies that

$$C^* = \frac{-\left(\frac{\alpha_0\lambda_0}{\lambda_1} + \frac{\lambda_2}{\lambda_1}\frac{rK}{r_0} + q - Q\right) + \sqrt{\left(\frac{\alpha_0\lambda_0}{\lambda_1} + \frac{\lambda_2}{\lambda_1}\frac{rK}{r_0} + q - Q\right)^2 + 4\left(\frac{\alpha_0}{\lambda_1} + \frac{\lambda_2{}^2}{\lambda_1{}^2}\frac{K}{r_0}\right)Q\lambda_0}}{2\left(\alpha_0 + \frac{\lambda_2{}^2}{\lambda_1}\frac{K}{r_0}\right)}$$

Thus, we have,

$$\lim_{\lambda_1\to\infty} C^* = 0 \tag{19}$$

Again, from (18), we have

$$C^* = \frac{-\left(\frac{\alpha_0\lambda_0}{\lambda_2} + \frac{rK}{r_0} + \frac{\lambda_1(q-Q)}{\lambda_2}\right) + \sqrt{\left(\frac{\alpha_0\lambda_0}{\lambda_2} + \frac{rK}{r_0} + \frac{\lambda_1(q-Q)}{\lambda_2}\right)^2 + 4\left(\frac{\alpha_0\lambda_1}{\lambda_2{}^2} + \frac{K}{r_0}\right)Q\lambda_0}}{2\lambda_2\left(\frac{\alpha_0\lambda_1}{\lambda_2{}^2} + \frac{K}{r_0}\right)}$$

From this, we get

$$\lim_{\lambda_2\to\infty} C^* = 0 \tag{20}$$

4. Stability Aanalysis

In the following, we state the theorems for local and nonlinear stability of the nonzero equilibrium of model (1) – (4).

Theorem 4.1 The equilibrium $\overline{E}$ is locally unstable.

(See Appendix A for proof).

Theorem 4.2 The equilibrium E^* is locally asymptotically stable without any condition.

(See Appendix B for proof)

Theorem 4.3 The equilibrium E^* is globally asymptotically stable, provided the following condition is satisfied inside the region of attraction Ω,

$$\lambda_1^{\,2}(C^* + C_i^{\,*})^2 < \frac{2}{3}\lambda_0\alpha_0 \qquad (21)$$

(See Appendix C for proof)

These theorems imply that under certain conditions, the system attains equilibrium of carbon dioxide gas. As already noted the magnitude of this steady state decreases as the discharge rate of external species or growth rate of biomass density increases.

5. Numerical simulation

To check the feasibility of our analysis regarding the existence of E^*, we conduct some simulations of model (1) – (4) by choosing the following set of parameters and using Maple 7,
$Q = 1,\ q = 0.5,\ \alpha_0 = 0.6,\ \lambda_0 = 0.3,\ \lambda_1 = 0.1,$

$r = 0.6,\ r_0 = 0.6, K = 20, \lambda_2 = 0.05, \theta = 0.3, \theta_0 = 0.25$

It is found that under the above set of parameters, equilibrium values of components of $E^*(C^*, C_i^{\,*}, B^*, C_p^{\,*})$ as follows,

$C^* = 0.559566, C_i^{\,*} = 1.404665,\ B^* = 20.932611,\ C_p^{\,*} = 0.943204$

The eigen values corresponding to $E^*(C^*, C_i^{\,*}, B^*, C_p^{\,*})$ are given by

$$-0.25,\ -0.3508, -1.7669, -0.6532$$

Since, all four eigenvalues are negative, hence E^* is locally asymptotically stable. It can also be checked that for above set of parameters the global stability condition is satisfied. The nonlinear stability behavior of E^* in $B - C$ plane is shown in the Figure 1. In figures 2-4 , the variations of concentration (C) of carbon dioxide with time $'t'$ are shown for different values of rate of introduction of external species (i.e. at $q = 0.5,\ 2.0,\ 3.5$), the rate of interaction of carbon dioxide with external liquid species (i.e. at $\lambda_1 = 0.01,\ 0.30,\ 0.70$) and the rate of interaction of carbon dioxide with leafy trees in the greenbelt (i.e. at $\lambda_2 = 0.2,\ 0.4,\ 0.8$) for $Q = 1$ respectively. From these figures, it is noted that the equilibrium values of concentration (C) of carbon dioxide decreases as the value of these parameters increase.

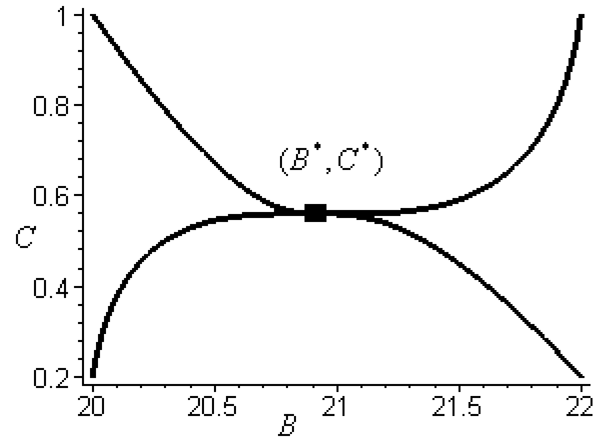

Figure 1. Nonlinear stability in $B-C$ plane

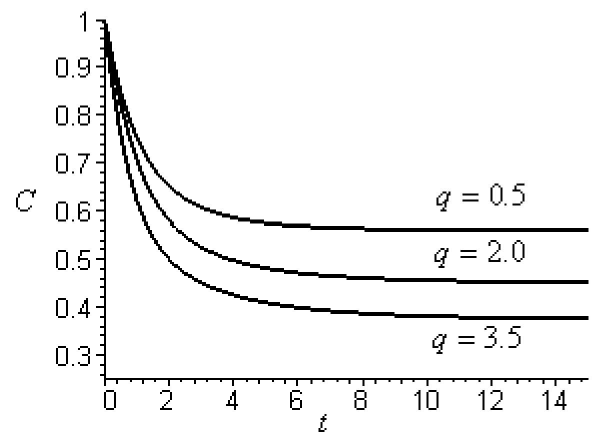

Figure 2. Variation of C with 't' for different values of q

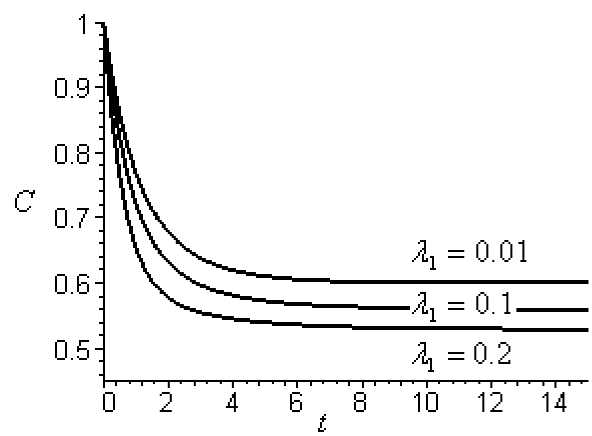

Figure 3. Variation of C with 't' for different values of λ_1

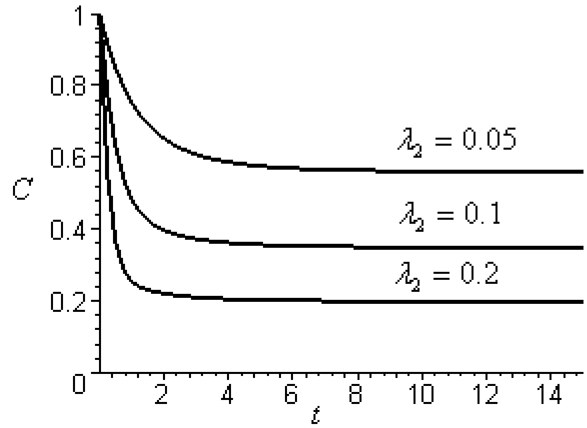

Figure 4. Variation of C with 't' for different values of λ_2

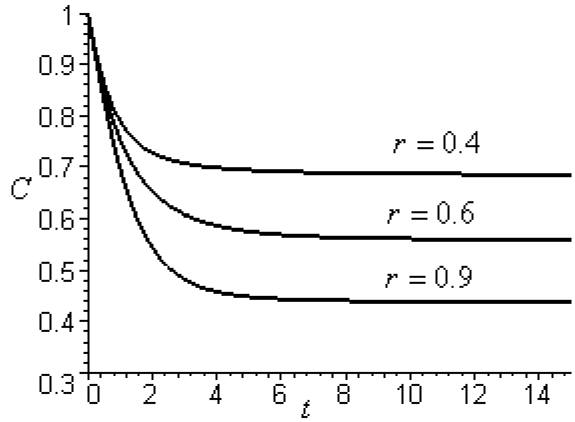

Figure 5. Variation of C with 't' for different values of r

In figure 5, the variation of the concentration (C) of carbon dioxide with time 't' are shown for different values of growth rate of plant biomass (i.e. at $r = 0.5$, 0.6, 0.7) respectively. It is seen that the concentration of carbon dioxide decreases as the growth rate of cumulative biomass density increases.

5. Conclusion

The removal of global warming gas such as carbon dioxide from the atmosphere is a very important problem to be solved by scientists. In this paper a modeling study has been conducted to reduce carbon dioxide from the near earth atmosphere by introducing external species such as liquid drops or appropriate particulate matters which can react with carbon dioxide and removed it by gravity. Carbon dioxide can also be removed by growing leafy trees in the greenbelts surrounding the sources of emission, which use it during photosynthesis. Thus, the phenomenon has been assumed to be governed by four dependent variables, namely, the concentrations of carbon dioxide, the externally introduced species, the cumulative biomass density of trees in the green belts and the concentration of the resulting particulate matters. The rate of introduction of the external species in to the atmosphere has been assumed to be proportional to concentration of carbon dioxide in the regional atmosphere. The cumulative biomass density of plants has been assumed to follow a logistic model with constant intrinsic growth rate and constant carrying capacity. The model analysis has shown that the concentration of carbon dioxide can be decreased considerably and it can almost be eliminated if the rate of introduction of external species as well as cumulative biomass density in the greenbelt is very large.

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Appendix A

Proof of the theorem 4.1 For the steady-state solution $\overline{E}(\overline{C}, \overline{C}_i, 0, \overline{C}_p)$ Jacobian matrix $(\overline{M})$ is

$$\overline{M} = \begin{bmatrix} -Q/\overline{C} & -\lambda_1\overline{C} & -\lambda_2\overline{C} & 0 \\ -\lambda_1\overline{C}_i & -q/\overline{C}_i & 0 & 0 \\ 0 & 0 & r+\lambda_2\overline{C} & 0 \\ \theta\overline{C}_i & \theta\overline{C} & 0 & -\theta_0 \end{bmatrix}$$

The eigenvalues p of $\overline{M}$ are given by

$$(\theta_0 + p)(r + \lambda_2\overline{C} - p)\left[p^2 + p\left(\frac{Q}{\overline{C}} + \frac{q}{\overline{C}_i}\right) + \left(\frac{Q}{\overline{C}}\frac{q}{\overline{C}_i} - \lambda_1^{\,2}\,\overline{C}\,\overline{C}_i)\right)\right] = 0 \qquad \text{(A1)}$$

It is clear that one of the eigenvalues of (A1) is positive. Therefore, equilibrium $\overline{E}$ is unstable.

Appendix B

Proof of the theorem 4.2 To study local stability character of E^* we need the following Jacobian matrix for model system (1) – (4) about E^*,

$$J^* = \begin{bmatrix} -(\alpha_0 + \lambda_1 C_i^* + \lambda_2 B^*) & -\lambda_1 C^* & -\lambda_2 C^* & 0 \\ -\lambda_1 C_i^* & -(\lambda_0 + \lambda_1 C^*) & 0 & 0 \\ \lambda_2 B^* & 0 & -\dfrac{r_0 B^*}{K} & 0 \\ \theta C_i^* & \theta C^* & 0 & -\theta_0 \end{bmatrix}$$

Clearly, one eigenvalue of J^* (i.e. $-\theta_0$) is negative. The other three eigen values of Jacobian matrix J^* corresponding to E^* are given by the following characteristic polynomial,

$$x^3 + a_1 x^2 + a_2 x + a_3 = 0 \qquad \text{(B1)}$$

Where,

$$a_1 = (\alpha_0 + \lambda_1 C_i^* + \lambda_2 B^*) + (\lambda_0 + \lambda_1\, C^*) + \frac{r_0}{K} B^*$$

$$a_2 = (\alpha_0 + \lambda_1 C_i^* + \lambda_2 B^*)\left(\lambda_0 + \frac{r_0}{K} B^*\right) + (\alpha_0 + \lambda_2 B^*)\lambda_1\, C^* + (\lambda_0 + \lambda_1\, C^*)\frac{r_0}{K} B^* + \lambda_2^{\,2} B^* C^*$$

$$a_3 = \{(\alpha_0 + \lambda_1 C_i^* + \lambda_2 B^*)\lambda_0 + (\alpha_0 + \lambda_2 B^*)\lambda_1\, C^*\}\frac{r_0}{K} B^* + (\lambda_0 + \lambda_1\, C^*)\lambda_2^{\,2} B^* C^*$$

It can easily seen that a_1, $a_2 > 0$ and $a_3 > 0$. Further,

$$a_1 a_2 - a_3 = \left(\alpha_0 + \lambda_1 C_i^* + \lambda_2 B^* + \frac{r_0}{K} B^*\right)\left((\lambda_0 + \lambda_1 C^* + \alpha_0 + \lambda_1 C_i^* + \lambda_2 B^*)\frac{r_0}{K} B^* + \lambda_2{}^2 B^* C^*\right)$$

$$+ (\lambda_0 + \lambda_1 C^*)\left((\lambda_0 + \lambda_1 C^* + \alpha_0 + \lambda_1 C_i^* + \lambda_2 B^*)\frac{r_0}{K} B^*\right)$$

Since, $a_1, a_2, a_3 > 0$ and $a_1 a_2 - a_3 > 0$, implying that all the conditions of Routh-Hurwitz criterion are satisfied and therefore the eigen values of the Jacobian matrix corresponding to E^* will either be negative or have negative real part. Thus, the equilibrium E^* is locally asymptotically stable without any condition.

Appendix C

Proof of the theorem 4.3 Consider a positive definite function

$$U = \frac{1}{2} m_1 (C - C^*)^2 + \frac{1}{2} m_2 (C_i - C_i^*)^2 + m_3 \left[B - B^* - B^* \log \frac{B}{B^*} \right] + \frac{1}{2} m_4 (C_p - C_p{}^*)^2 \qquad \text{(C1)}$$

Differentiating with respect to $'t'$ we have

$$\dot{U} = m_1 (C - C^*)\dot{C} + m_2 (C_i - C_i^*)\dot{C}_i + m_3 (B - B^*)\frac{\dot{B}}{B} + m_4 (C_p - C_p{}^*)\dot{C}_p$$

$$= -m_1 (\lambda_1 C_i + \lambda_2 B)(C - C^*)^2 - m_2 \lambda_1 C (C_i - C_i^*)^2$$

$$- m_1 \alpha_0 (C - C^*)^2 - m_2 \lambda_0 (C_i - C_i^*)^2 - m_3 \frac{r_0}{K}(B - B^*)^2 - m_4 \theta_0 (C_p - C_p{}^*)^2$$

$$- [m_1 \lambda_1 C^* + m_2 \lambda_1 C_i^*)](C - C^*)(C_i - C_i^*)$$

$$- (m_1 \lambda_2 C^* - m_3 \lambda_2)(C - C^*)(B - B^*)$$

$$+ (m_4 \theta C_i^*)(C - C^*)(C_p - C_p{}^*)$$

$$+ (m_4 \theta C)(C_i - C_i^*)(C_p - C_p{}^*)$$

$\dot{U}$ will be negative definite, if,

$$[m_1 \lambda_1 C^* + m_2 \lambda_1 C_i^*)]^2 < \frac{2}{3} m_1 m_2 \lambda_0 \alpha_0 \qquad \text{(C2)}$$

$$[m_1\lambda_2 C^* - m_3\lambda_2]^2 < \frac{4}{3} m_1 m_3 \frac{\alpha_0 r_0}{K} \quad \text{(C3)}$$

$$m_4(\theta C_i^*)^2 < \frac{2}{3} m_1 \alpha_0 \theta_0 \quad \text{(C4)}$$

$$m_4(\theta C)^2 < m_2 \lambda_0 \theta_0 \quad \text{(C5)}$$

By choosing $m_1 = m_2 = 1$ and $m_3 = C^*$ the inequality (C3) is satisfied.

To satisfy inequalities (C4) and (C5), we choose m_4 such that,

$$m_4 < \min\left\{ \frac{2}{3} \frac{\alpha_0 \theta_0}{(\theta C_i^*)^2}, \frac{\lambda_0 \theta_0}{\theta^2} \left(\frac{\alpha_0}{Q} \right)^2 \right\}.$$

Now, the inequality (C2) reduces to, $\lambda_1^{\,2}(C^* + C_i^*)^2 < \dfrac{2}{3}\lambda_0 \alpha_0$ (C6)

Thus, $\dot{U}$ is negative definite when the condition (21) is satisfied and the equilibrium E^* is globally asymptotically stable.

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Effects of Emissions of Hot Pollutants from Thermal Power Stations on Suppression of Rainfall: A Non Linear Modeling Study

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Abstract

The decrease in rainfall has been noted in local areas near and around thermal power stations due to discharge of hot gases and particulate matters (including nano carbon particles) in the atmosphere. This phenomenon is studied by proposing a new mathematical model, where interactions of hot gases with cloud droplets and hot particulate matters with raindrops are considered in the atmosphere. Thus, the proposed model consists of five nonlinearly interacting phases i.e. the vapour phase, the phase of cloud droplets, the raindrops phase, the phase of hot gases and the phase of hot particulate matters. The model analysis shows that as the rates of emissions of hot pollutants increase, the number densities of cloud droplets and raindrops decrease leading to decrease in rainfall. The computer simulation of the model confirms the analytical results.

Key words*:* Precipitation, raindrops, water vapour, cloud droplets

1. Introduction

The decrease in rainfall has been found, in local areas near and around thermal power stations, due to discharge of hot gases and particulate matters from chimneys. The cloud droplets and raindrops get vapourized, when they come in contact with hot gases and particulate matters including, nano carbon particles, causing decrease in rainfall. This phenomenon needs to be studied scientifically in a systemic manner by considering the role of vapour clouds, Ashworth (1929,) Rosenfeld (2000, 2007), Ramanathan (2001), Givati and Rosenfeld (2004), Feingold et al (2005), Khain et al (2005), (2008), Ashfaq et al (2009), Tao et al (2012), Diem (2013), Kharol et al (2013). In this regard, Rosenfeld (2000) studied the suppression of rain and snow by urban air pollution. Givati and Rosenfeld (2004) studied the effect of industrial pollution on precipitation and have shown that precipitation decreases due to pollution. Tao et al. (2012) studied the impact of aerosols on convective clouds and precipitation. They found considerable decrease in precipitation.

Some other investigations have also been conducted to study the removal of gases and particulate matters by rain, Flossmann et al (1987, Naresh et al (2007), Shukla et al(2008 a, b), Shukla et al (2014). In this regard, Shukla et al (2008a) proposed a nonlinear mathematical model to study the effect of cloud density on the removal of gaseous pollutants and particulate matters from the atmosphere of a city by rain and found considerable decrease in the concentration of gaseous pollutant and particulate matters. Some other researchers studied the phenomenon related to interactions of aerosols with cloud droplets causing precipitation Pandis et al (1990), Shukla et al (2010). For example, Shukla et al (2010) proposed a mathematical model to produce artificial rain from water vapor. In their analysis, it has been shown that by introducing aerosol particles artificially in the atmosphere, the intensity of rainfall can be increased. Sundar et al (2009) proposed a nonlinear mathematical model to study the interactions of hot gases with cloud droplets and raindrops and shown that the number densities of cloud droplets and raindrops decrease as the concentration of hot gases in the atmosphere increases. But they have not considered the effect of hot particulate matters on the process of precipitation. In this paper, therefore, we have proposed a mathematical model to see the effect of hot gases as well as hot particulate matters on cloud droplets and raindrops respectively leading to decrease in rainfall.

2. Mathematical model

To model the phenomenon, we assume that C_v be the volume density of water vapour, C_d and C_r are the number densities of cloud droplets and raindrops respectively in the atmosphere. C_1 and C_2 are the cumulative concentrations of hot gases and particulate matters discharged from chimneys from industrial sources such as thermal power stations in the atmosphere which interact with cloud droplets and raindrops respectively.

The following assumptions are made in the modeling process,

(i) The water vapour is formed continuously with the constant rate Q.
(ii) The rates of emission of hot gases and particulate matters are Q_1 and Q_2 respectively, which are also constant.
(iii) The rate of formation of cloud droplets is assumed to be in the direct proportion of the volume density of water vapour (i.e. λC_v), λ being a constant coefficient.
(iv) The rate of formation of raindrops is assumed to be in the direct proportion of the number density of cloud droplets (i.e. $r C_d$), r being a constant coefficient.
(v) It is assumed that the hot gases are able to reach as high as the base of cloud droplets making them vapourise during interaction process and leading to decrease in their number density which is considered in the direct proportion of the number density of cloud droplets and the cumulative concentration of hot gases (i.e. $\mu_1\lambda_1 C_1 C_d$) and thus increasing the growth rate of vapours by the same amount.

(vi) It is assumed that particulate matters are not able to reach cloud base and hence interacts only with raindrops and thus decreasing the growth rate of raindrops. This decrease in number density of raindrops is considered to be in the direct proportion of the number density of raindrops and the concentration particulate matters (i.e. $\mu_2\lambda_2 C_2 C_r$) and thus again increasing the growth rate of vapours by the same amount.

(vii) The interaction rate coefficients μ_1 and μ_2 may be function of temperature but for the simplicity of the model system we have taken these as constants.

In view of the above, the equations governing the problem are proposed as follows,

$$\frac{dC_v}{dt} = Q_v - \mu_0 C_v + \mu_1\lambda_1 C_1 C_d + \mu_2\lambda_2 C_2 C_r \tag{1}$$

$$\frac{dC_d}{dt} = \lambda C_v - \lambda_0 C_d - \mu_1\lambda_1 C_1 C_d \tag{2}$$

$$\frac{dC_r}{dt} = rC_d - r_0 C_r - \mu_2\lambda_2 C_2 C_r \tag{3}$$

$$\frac{dC_1}{dt} = Q_1 - \delta_1 C_1 - \lambda_1 C_1 C_d \tag{4}$$

$$\frac{dC_2}{dt} = Q_2 - \delta_2 C_2 - \lambda_2 C_2 C_r \tag{5}$$

with $C_v(0) > 0,\ C_d(0) > 0, C_r(0) > 0, C_1(0) > 0, C_2(0) > 0$

The constants $\mu_0, \lambda_0, r_0, \delta_1, \delta_2$ are the natural depletion rate coefficients of water vapour, cloud droplets, raindrops and hot gases with concentration C_1 and particulate matters with concentration C_2 respectively. All the constants, taken here, are positive. To keep the application of the model in view, it is noted from (1) – (3) that $\mu_0 \geq \lambda$, $\lambda_0 \geq r$.

It is, further, noted here that, if the interaction rate coefficients μ_1 and μ_2 are very large, $\frac{dC_d}{dt}$ and $\frac{dC_r}{dt}$ may become negative and no rain formation occurs.

It may be noted here that the model proposed in (1) – (5) is applicable only in local areas near and around chimneys of thermal power stations discharging hot gases and particulate matters.

In the following, we state, without proof, the lemma related to bounds of solutions of the proposed model (1) – (5).

Lemma 2.1 The region of attraction for all solutions initiating in the positive octant given by the set

$$\Omega = \left\{ (C_v^*, C_d^*, C_r^*, C_1^*, C_2^*) : 0 \le C_v + C_d + C_r \le \frac{Q_v}{\delta_m}, 0 \le C_1 < \frac{Q_1}{\delta_1},\ 0 \le C_2 < \frac{Q_2}{\delta_2} \right\}$$

where $\delta_m = \min(\mu_0 - \lambda,\ \lambda_0 - r,\ r_0)$

3. Equilibrium Analysis

The model (1) – (5) has only one nonnegative equilibrium namely $E^*(C_v^*, C_d^*, C_r^*, C_1^*, C_2^*)$, This is obtained by solving the following algebraic equations:

$$Q_v - \mu_0 C_v + \mu_1 \lambda_1 C_1 C_d + \mu_2 \lambda_2 C_2 C_r = 0 \tag{6}$$

$$\lambda C_v - \lambda_0 C_d - \mu_1 \lambda_1 C_1 C_d = 0 \tag{7}$$

$$r C_d - r_0 C_r - \mu_2 \lambda_2 C_2 C_r = 0 \tag{8}$$

$$C_1 = \frac{Q_1}{\delta_1 + \lambda_1 C_d} \tag{9}$$

$$C_2 = \frac{Q_2}{\delta_2 + \lambda_2 C_r} \tag{10}$$

Adding (6) – (8), we get,

$$Q_v - (\mu_0 - \lambda) C_v - (\lambda_0 - r) C_d - r_0 C_r = 0 \tag{11}$$

From (7), we have

$$C_d = \frac{\lambda C_v}{\lambda_0 + \mu_1 \lambda_1 C_1} = \frac{\lambda C_v}{\lambda_0 + \mu_1 \lambda_1 \dfrac{Q_1}{\delta_1 + \lambda_1 C_d}}$$

Or $C_d \left(\lambda_0 + \mu_1 \lambda_1 \dfrac{Q_1}{\delta_1 + \lambda_1 C_d} \right) = \lambda C_v$ (12)

From (8), we have

$$C_r = \frac{r C_d}{r_0 + \mu_2 \lambda_2 C_2} = \frac{r C_d}{r_0 + \mu_2 \lambda_2 \dfrac{Q_2}{\delta_2 + \lambda_2 C_r}}$$

Or $C_r\left(r_0+\mu_2\lambda_2\dfrac{Q_2}{\delta_2+\lambda_2 C_r}\right)=rC_d$ (13)

From (6), $\mu_0 C_v = Q_v + \mu_1\lambda_1 C_1 C_d + \mu_2\lambda_2 C_2 C_r$

From (11), Let

$$F(C_v)=Q_v-(\mu_0-\lambda)C_v-(\lambda_0-r)C_d-r_0 C_r \quad (14a)$$

Or $$F(C_v)<Q_v-\delta_m C_v-\delta_m C_d-\delta_m C_r \quad (14b)$$

Since for $C_v=0$, both C_d and C_r are zero, see equations (12) and (13).

We get from (14a), $F(0)=Q_v>0$ (15)

From (14b), we also have,

$$F\left(\frac{Q_v}{\delta_m}\right)<-\delta_m(C_d+C_r)<0 \quad (16)$$

It is noted from (12) and (13) respectively that

$\dfrac{dC_d}{dC_v}=\dfrac{\lambda}{\left\{\lambda_0+\dfrac{\mu_1\lambda_1\delta_1 Q_1}{(\delta_1+\lambda_1 C_d)^2}\right\}}>0$ and $\dfrac{dC_r}{dC_d}=\dfrac{r}{\left\{r_0+\dfrac{\mu_2\lambda_2\delta_2 Q_2}{(\delta_2+\lambda_2 C_r)^2}\right\}}>0$, giving $\dfrac{dC_r}{dC_v}>0$ Thus,

differentiating (14a), we get

$$F'(C_v)<0 \quad (17)$$

In view of (15) – (17), there exist a unique root (say C_v^*) in $0\le C_v\le\dfrac{Q_v}{\delta_m}$. By knowing the value of C_v^*, we can find the values of C_d^*, C_r^*, C_1^* and C_2^* from (7), (8), (9) and (10) respectively.

Remark: From equations (9) and (10), it can be easily shown that $\dfrac{dC_d}{dC_1}<0$ and $\dfrac{dC_r}{dC_2}<0$. This implies that the densities of cloud droplets and raindrops decrease as the concentrations of hot gases and particulate matters increase respectively.

4. Stability Analysis

In the following, we state the local and nonlinear stability of the equilibrium E^* of the proposed model (1) – (5)

Theorem 4.1 If the following inequalities hold in,

$$(k_1\mu_1\lambda_1 C_1^* + \lambda)^2 < \frac{k_1}{3}\mu_0(\lambda_0 + \mu_1\lambda_1 C_1^*) \tag{18}$$

$$r^2 < \frac{4}{9}(\lambda_0 + \mu_1\lambda_1 C_1^*)(r_0 + \mu_2\lambda_2 C_2^*) \tag{19}$$

$$(\mu_1\lambda_1 C_d^* + \lambda_1 C_1^*)^2 < \frac{2}{3}(\lambda_0 + \mu_1\lambda_1 C_1^*)(\delta_1 + \lambda_1 C_d^*) \tag{20}$$

$$(\mu_2\lambda_2 C_r^* + \lambda_2 C_2^*)^2 < \frac{2}{3}(r_0 + \mu_2\lambda_2 C_2^*)(\delta_2 + \lambda_2 C_r^*) \tag{21}$$

where, $k_1 < \mu_0 \min\left\{\frac{(r_0 + \mu_2\lambda_2 C_2^*)}{3(\mu_2\lambda_2 C_2^*)^2}, \frac{(\delta_1 + \lambda_1 C_d^*)}{2(\mu_1\lambda_1 C_d^*)^2}, \frac{(\delta_2 + \lambda_2 C_r^*)}{2(\mu_2\lambda_2 C_r^*)^2}\right\}$

then equilibrium E^* is locally asymptotically stable (See appendix A for proof).

Theorem 4.2 If the following inequalities hold in Ω,

$$(m_1\mu_1\lambda_1 C_1^* + \lambda)^2 < \frac{m_1}{3}\mu_0\lambda_0 \tag{22}$$

$$r^2 < \frac{4}{9}\lambda_0 r_0 \tag{23}$$

$$(\mu_1\lambda_1 C_d^* + \lambda_1 C_1^*)^2 < \frac{2}{3}\lambda_0\delta_1 \tag{24}$$

$$(\mu_2\lambda_2 C_r^* + \lambda_2 C_2^*)^2 < \frac{2}{3}r_0\delta_2 \tag{25}$$

Where, $m_1 < \mu_0 \min\left\{\frac{r_0}{3(\mu_2\lambda_2 C_2^*)^2}, \frac{\delta_1}{2(\mu_1\lambda_1 C_d^*)^2}, \frac{\delta_2}{2(\mu_2\lambda_2 C_r^*)^2}\right\}$

then equilibrium E^* is globally asymptotically stable in the region Ω (See appendix B for the proof).

Remark

It is noted from (22) – (25), if λ, λ_1, r and λ_2 tend to zero, these stability conditions are satisfied. It is also noted from these conditions that if λ_1 and λ_2, the interaction rate coefficients of hot gases with cloud droplets and hot particulate matters with raindrops respectively, are very large then the possibility of satisfying these conditions decreases. Hence, these parameters have destabilizing effect on the system.

5. Numerical simulation

In this section, we have checked the feasibility of system analysis associated with existence of E^* by simulating the model proposed in (1) – (5). For this, we provide some numerical simulation of model (1 – 5) by choosing the following set of parameters,

$Q = 1,\ \mu_0 = 0.8,\ \mu_1 = 0.2,\ \mu_2 = 0.1,\ \lambda_1 = 0.01, \lambda_2 = 0.01, \lambda = 0.06, \lambda_0 = 0.05$

$r = 0.02, r_0 = 0.019,\ Q_1 = 1, Q_2 = 1.2, \delta_1 = 0.2, \delta_2 = 0.5$

It is found that under the above set of parameters, equilibrium values of $E^*(C_v^*, C_d^*, C_r^*, C_1^*, C_2^*)$ are given by

$C_v^* = 1.268571, C_d^* = 1.281431,\ C_r^* = 1.200757,\ C_1^* = 4.698932,\ C_2^* = 2.343715$

The eigenvalues corresponding to $E^*(C_v^*, C_d^*, C_r^*, C_1^*, C_2^*)$ are given by

$$-0.800772,\ -0.213511, -0.058023, -0.021190, -0.512065$$

Since all the eigen values are negative. Hence E^* is locally asymptotically stable.

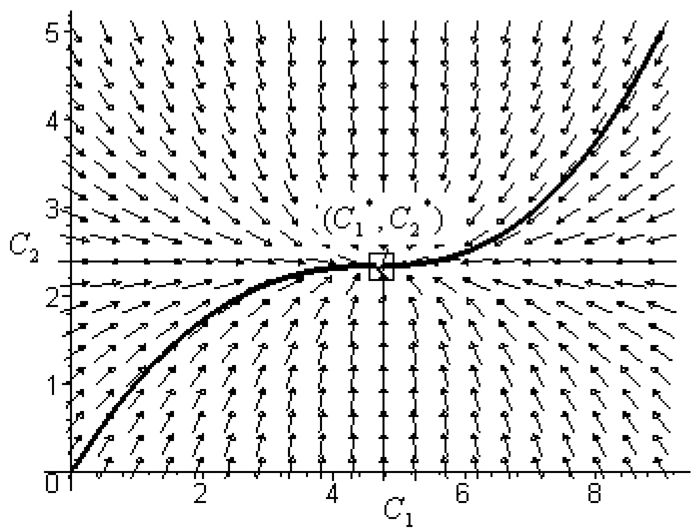

Figure 1. Nonlinear stability in $C_1 - C_2$ plane

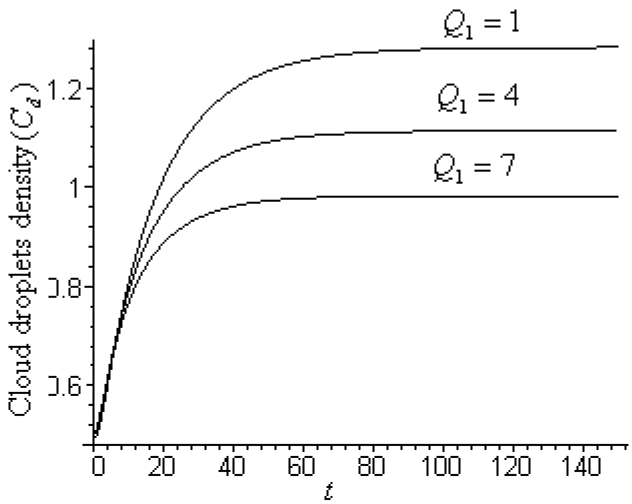

Figure 2. Variation of C_d with 't' for different values of Q_1

The nonlinear stability behavior of E^* in $C_1 - C_2$ plane is shown in the figure 1.It has also been noted that local and nonlinear stability conditions are satisfied by these set of parameters. In figure 2, the variation of number density of cloud droplets (C_d) with time 't' is shown for different values of Q_1. From this figure, it is seen that the density of cloud droplets (C_d) decreases as the rate of introduction of hot gases into the atmosphere increases. This implies that hot gases interact with cloud droplets and vapourized them increasing the growth rate of volume density of vapours. Further, due to decrease in density of cloud droplets, density of raindrops may also be decreased as the growth of raindrops formation is assumed in the direct proportion of density of cloud droplets (i.e. $r\,C_d$). In figure 3, the variation of density raindrops (C_r) with time 't' is shown for different values of Q_2. From this figure, it is noted that the density of raindrops (C_r) decreases as the rate of introduction of hot particulate matters into the atmosphere increases. In figures 4 and 5, the variation of density of cloud droplets (C_d) and vapour density (C_v) with time 't' is shown for different values of μ_1. From these figures, it is noted that the density of cloud droplets (C_d) decreases while the vapour density increases as the depletion rate coefficient of cloud droplets (μ_1) increases. Figures 6 and 7 shows the variation of density raindrops (C_r) and vapour density (C_v) with time 't' is shown for different values of μ_2. From these figures, we note that the density of raindrops (C_r) decreases while the vapour density increases as the depletion rate coefficient (μ_2) increases.

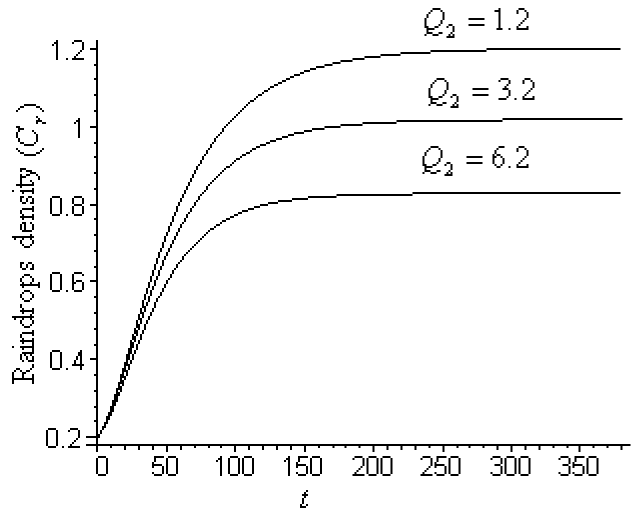

Figure 3. Variation of C_r with 't' for different values of Q_2

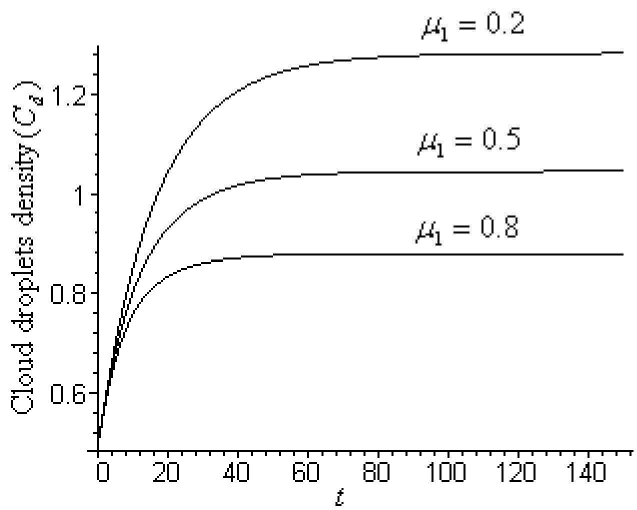

Figure 4. Variation of C_d with 't' for different values of μ_1

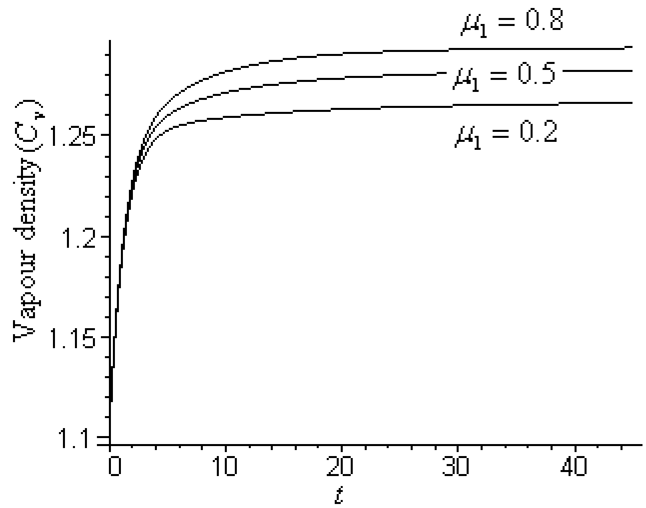

Figure 5. Variation of C_v with 't' for different values of μ_1

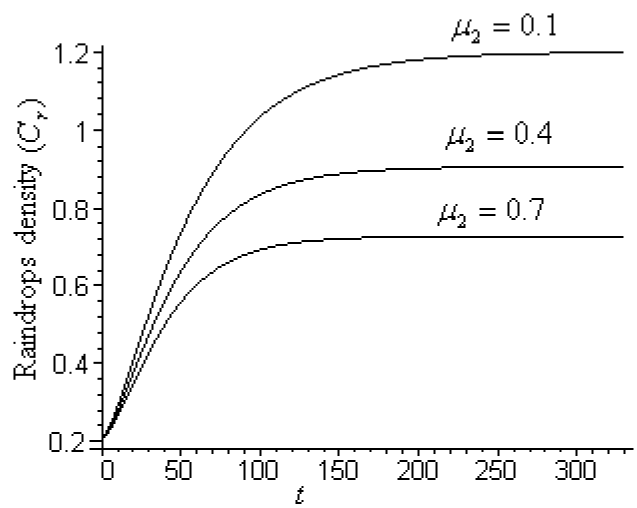

Figure 6. Variation of C_r with 't' for different values of μ_2

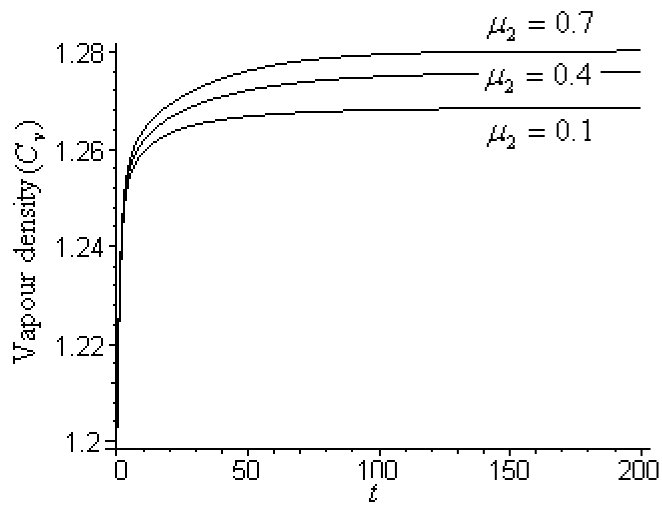

Figure 7. Variation of C_v with 't' for different values of μ_2

6. Conclusions

In this paper, a nonlinear mathematical model is proposed and analyzed to study the effects of emission of hot pollutants discharged from chimneys of thermal power stations in the form of hot gases and particulate matters on rainfall in the local areas near and around these power stations. The problem is assumed to be governed by five nonlinearly dependent variables namely; the densities of vapours, cloud droplets and raindrops, the concentrations of hot gases and particulate matters including nano carbon particles. The proposed model is analyzed using stability theory and numerical simulations. It has been shown, analytically and numerically, that the densities of cloud droplets and raindrops decrease as the rates of emissions of hot gases and particulate matters into the atmosphere increase. Further, as the interaction rate coefficient of hot gases and particulate matters with cloud droplets and raindrops increase respectively the corresponding densities of cloud droplets and raindrops decrease leading to decrease in rainfall in local areas near and around thermal power stations. The numerical simulations of the model confirm the analytical results.

Appendix A

Proof of the theorem 4.1

Consider a positive definite function

$$V = \frac{1}{2}[k_1 C_{v1}{}^2 + k_2 C_{d1}{}^2 + k_3 C_{r1}{}^2 + k_4 C_{11}{}^2 + k_5 C_{21}{}^2] \qquad \text{(A1)}$$

Where,

$C_v = C_v{}^* + C_{v1}, C_d = C_d{}^* + C_{d1}, C_r = C_r{}^* + C_{r1}, C_1 = C_1{}^* + C_{11}, C_2 = C_2{}^* + C_{21}$

Differentiating with respect to $'t'$ we get

$$\dot{V} = k_1 C_{v1}\dot{C}_{v1} + k_2 C_{d1}\dot{C}_{d1} + k_3 C_{r1}\dot{C}_{r1} + k_4 C_{11}\dot{C}_{11} + k_5 C_{21}\dot{C}_{21}]$$

$$\begin{aligned}\dot{V} = &-k_1\mu_1 C_{v1}{}^2 - (\lambda_0 + \mu_1\lambda_1 C_1{}^*)k_2 C_{d1}{}^2 - (r_0 + \mu_2\lambda_2 C_2{}^*)k_3 C_{r1}{}^2 \\ &- (\delta_1 + \lambda_1 C_d{}^*)k_4 C_{11}{}^2 - (\delta_2 + \lambda_2 C_r{}^*)k_5 C_{21}{}^2 \\ &+ (k_1\mu_1\lambda_1 C_1{}^* + \lambda k_2)C_{v1}C_{d1} - (k_1\mu_2\lambda_2 C_2{}^*)\,C_{v1}C_{r1} \\ &+ (k_1\mu_1\lambda_1 C_d{}^*)C_{v1}C_{11} + (k_1\mu_2\lambda_2 C_r{}^*)\,C_{v1}C_{21} \\ &+ k_3 r\,C_{d1}C_{r1} - (k_2\mu_1\lambda_1 C_d{}^* + k_4\lambda_1 C_1{}^*)C_{d1}C_{11} + (k_3\mu_2\lambda_2 C_r{}^* + k_5\lambda_2 C_2{}^*)C_{r1}C_{21}\end{aligned}$$

$\dot{V}$ will be negative definite, if,

$$(k_1\mu_1\lambda_1 C_1{}^* + \lambda k_2)^2 < \frac{k_1 k_3}{3}\mu_0(\lambda_0 + \mu_1\lambda_1 C_1{}^*) \qquad \text{(A2)}$$

$$k_1(\mu_2\lambda_2 C_2{}^*)^2 < \frac{k_3}{3}\mu_0(r_0 + \mu_2\lambda_2 C_2{}^*) \qquad \text{(A3)}$$

$$k_1(\mu_1\lambda_1 C_d{}^*)^2 < \frac{k_4}{2}\mu_0(\delta_1 + \lambda_1 C_d{}^*) \qquad \text{(A4)}$$

$$k_1(\mu_2\lambda_2 C_r{}^*)^2 < \frac{k_5}{2}\mu_0(\delta_2 + \lambda_2 C_r{}^*) \qquad \text{(A5)}$$

$$k_3 r^2 < \frac{4k_2}{9}(\lambda_0 + \mu_1\lambda_1 C_1{}^*)(r_0 + \mu_2\lambda_2 C_2{}^*) \qquad \text{(A6)}$$

$$(k_2\mu_1\lambda_1 C_d^* + k_4\lambda_1 C_1^*)^2 < \frac{2}{3}k_2k_4(\lambda_0 + \mu_1\lambda_1 C_1^*)(\delta_1 + \lambda_1 C_d^*) \qquad \text{(A7)}$$

$$(k_3\mu_2\lambda_2 C_r^* + k_5\lambda_2 C_2^*)^2 < \frac{2}{3}k_3k_5(r_0 + \mu_2\lambda_2 C_2^*)(\delta_2 + \lambda_2 C_r^*) \qquad \text{(A8)}$$

Now, choosing, $k_1 < \mu_0 \min\left\{\frac{(r_0 + \mu_2\lambda_2 C_2^*)}{3(\mu_2\lambda_2 C_2^*)^2}, \frac{(\delta_1 + \lambda_1 C_d^*)}{2(\mu_1\lambda_1 C_d^*)^2}, \frac{(\delta_2 + \lambda_2 C_r^*)}{2(\mu_2\lambda_2 C_r^*)^2}\right\}$, $k_2 = k_3 = k_4 = 1$ and $k_5 = 1$, $\dot{V}$ will negative definite provided the conditions (18) – (21) are satisfied and hence E^* is locally asymptotically stable.

Appendix B

Proof of the theorem 4.2

Consider a positive definite function

$$U = \frac{m_1}{2}(C_v - C_v^*)^2 + \frac{m_2}{2}(C_d - C_d^*)^2 + \frac{m_3}{2}(C_r - C_r^*)^2 + \frac{m_4}{2}(C_1 - C_1^*)^2 + \frac{m_5}{2}(C_2 - C_2^*)^2 \quad \text{(B1)}$$

Differentiating with respect to $'t'$ we get

$$U = m_1(C_v - C_v^*)\dot{C}_v + m_2(C_d - C_d^*)\dot{C}_d + m_3(C_r - C_r^*)\dot{C}_r$$
$$+ m_4(C_1 - C_1^*)\dot{C}_1 + m_5(C_2 - C_2^*)\dot{C}_2$$

$$\dot{U} = -m_1\mu_0(C_v - C_v^*)^2 - m_2\mu_1\lambda_1 C_1(C_d - C_d^*)^2 - m_2\lambda_0(C_d - C_d^*)^2$$
$$- m_3\mu_2\lambda_2 C_2(C_r - C_r^*)^2 - m_3 r_0(C_r - C_r^*)^2$$
$$- m_4\lambda_1 C_d(C_1 - C_1^*)^2 - m_5\lambda_2 C_r(C_2 - C_2^*)^2$$
$$- m_4\delta_1(C_1 - C_1^*)^2 - m_5\delta_2(C_2 - C_2^*)^2$$
$$+ (m_1\mu_1\lambda_1 C_1^* + m_2\lambda)(C_v - C_v^*)(C_d - C_d^*) + (m_1\mu_2\lambda_2 C_2^*)(C_v - C_v^*)(C_r - C_r^*)$$
$$+ (m_1\mu_1\lambda_1 C_d)(C_v - C_v^*)(C_1 - C_1^*) + (m_1\mu_2\lambda_2 C_r)(C_v - C_v^*)(C_2 - C_2^*)$$
$$+ m_3 r(C_d - C_d^*)(C_r - C_r^*) - (m_2\mu_1\lambda_1 C_d^* + m_4\lambda_1 C_1^*)(C_d - C_d^*)(C_1 - C_1^*)$$
$$- (m_3\mu_2\lambda_2 C_r^* + m_5\lambda_2 C_2^*)(C_r - C_r^*)(C_2 - C_2^*)$$

$\dot{U}$ will be negative definite, if,

$$(m_1\mu_1\lambda_1C_1^* + \lambda m_2)^2 < \frac{m_1m_3}{3}\mu_0\lambda_0 \tag{A2}$$

$$m_1(\mu_2\lambda_2C_2^*)^2 < \frac{m_3}{3}\mu_0r_0 \tag{A3}$$

$$m_1(\mu_1\lambda_1C_d)^2 < \frac{m_4}{2}\mu_0\delta_1 \tag{A4}$$

$$m_1(\mu_2\lambda_2C_r)^2 < \frac{m_5}{2}\mu_0\delta_2 \tag{A5}$$

$$m_3r^2 < \frac{4m_2}{9}\lambda_0r_0 \tag{A6}$$

$$(m_2\mu_1\lambda_1C_d^* + m_4\lambda_1C_1^*)^2 < \frac{2}{3}m_2m_4\lambda_0\delta_1 \tag{A7}$$

$$(m_3\mu_2\lambda_2C_r^* + m_5\lambda_2C_2^*)^2 < \frac{2}{3}m_3m_5r_0\delta_2 \tag{A8}$$

Now, choosing, $m_1 < \mu_0 \min\left\{\frac{r_0}{3(\mu_2\lambda_2C_2^*)^2}, \frac{\delta_1}{2(\mu_1\lambda_1)^2}\left(\frac{\delta_m}{Q_v}\right)^2, \frac{\delta_2}{2(\mu_2\lambda_2)^2}\left(\frac{\delta_m}{Q_v}\right)^2\right\}$, $m_2 = m_3 = 1$, $m_4 = 1$ and $m_5 = 1$, $\dot{U}$ will negative definite provided the conditions (22) – (25) are satisfied and hence E^* is globally asymptotically stable within the region of attraction Ω.

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Modeling the removal of cement dust particles from a chimney by water sprays and greenbelts plantation

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Abstract

In this paper, a nonlinear mathematical model for removal of dust particles emitted from a cement factory in the near Earth atmosphere is proposed and analyzed. The removal process is conducted by using water sprays above and around the factory. The effect of dust particles on biomass in greenbelts plantation around the factory is also modeled and analyzed. To model the phenomenon, three nonlinearly interacting dependent variables, namely, the concentration of dust particles, the density of plant biomass and the number density of droplets in water sprays, are considered. The model analysis, using stability theory of differential equations, shows that the equilibrium concentration of dust particles in the atmosphere decreases as the rates of removal parameters involving water sprays increase. This concentration, in the absence of plantation, is much higher than its magnitude when the greenbelts plantation is present around the factory. It is also found that the biomass density decreases, as the concentration of dust particles increases, causing closure of leaf stomata.

Keywords: Mathematical model; plant biomass density; water sprays; dust pollution; stability

1. Introduction

Several types of dust particles emitted from cement and other industrial sources in the atmosphere cause various undesirable consequences to human beings and their surroundings including greenbelts plantations. Cement factories alone contribute significantly to the menace of dust pollution by increasing the concentration of dust particles in their regional atmosphere. Therefore, it is necessary to find a mechanism to reduce the concentration of dust particles, particularly from the near earth atmosphere. The composition of cement dust particles consists of calcium oxide (CaO) as the main component whereas K_2O, Na_2O, KCl, MgO and silica (SiO_2) are found in varying amounts. These dust particles with more than 24% calcium oxide has been found more deleterious to plants as it covers the leaf area reducing the amount of light required for photosynthesis and blocking the openings of stomata (Kulshreshth et al. (2009), Gheorghe and Ion (2011)). The dust particles, deposited on the leaves of plants, not only reduce the amount of light required for photosynthesis but also increase the leaf temperature affecting net photosynthesis (Mebrahtu et al. (1991)). This adverse effect mainly depends upon the load, time or duration of effect, chemical and physical characteristic of the components of the dust particles, tolerance level and response of the plants in the greenbelts plantation.

Some investigations have been conducted to study the effect of dust particles emitted from cement industries on plant biomass (Thompson et al. (1984), Anda (1986), Shukla et al. (1990), Hindy et al. (1990), Mutlu et al. (2013). In this regard, Thompson et al. (1984) studied the effect of dust on photosynthesis on roadside plants and found considerable decrease in photosynthesis. Anda (1986) found deterioration in average yield of plants by 16.2% during an observation of three years period. Shukla et al. (1990) studied experimentally the effect of cement dust on the growth and yield of *Brassica campestris* L. In their analysis they have found consistent reduction in growth, photosynthesis, yield and oil content of plants under consideration. Mutlu et al. (2013) studied the effect of cement dust pollution consisting of some crucial elements (such as P, S, K, C_a, F_e and Cl) in wild plants using wavelength-dispersive spectrometer X-ray fluorescence technique. They have found high concentrations of P, S, K and Cl in plants growing within the range in 0-100 m around the cement factory as compared to same plants at 2000 m far from the factory but C_a and F_e contents were found low in plants ranging 0-100 m from the factory. In their study, they have also shown that dust particles have detrimental effect on plant growth.

Thus, mitigation of dust particles in the near Earth atmosphere is crucial due to its adverse effect on plant species. Some observations have also been made to study the removal of air pollution by leafy trees (Beckett et al. (2000 a, b), Nowak et al. (2006, 2013), Tiwary et al. (2009), Tallis et al. (2011), Buffoni and Silli (2013), Kumar et al. (2013)). In this aspect, Kumar et al. (2013) conducted an experiment to study the control of environmental pollution by plant species by calculating APTI (air pollution tolerant index) of some selected plants. They have shown that plant species play an important role to remove the particulate matters from the environment and help to make the cities and towns cleaner. Nowak et al. 2013 have studied the effect of tree species on particulate matters in ten cities of U. S. showing an improvement in air quality. In their analysis, they demonstrated that average air quality improvement ranged between 0.05 % in San Francisco and 0.24% in Atlanta.

Particulate matters (dust particles) can also be reduced by introducing water sprays into the environment near and around the source of emission. Some experimental observations have been made to study the removal of particulate air pollutants from the human environment by water sprays (Tsai et al. (2003), Salaver et al. (2007), Chen et al. (2013), Ran et al. (2014). For example, Huang (2005) developed a theoretical model to investigate absorption of sulfur dioxide by fine water spray and found considerable decrease in sulfur dioxide concentration. Yoo et al. (2014) investigated washout effect of rain on surface air pollutants (CO, NO_2, SO_2 etc.) and revealed that particulate matter is scavenged more efficiently by rainfall. Chen et al. 2013 have conducted a theoretical study for the absorption process of air pollutants in sprays under droplet-droplet interaction. Ran et al. (2014) presented an experimental study using ultrasonic field to increase the scavenging of particles by water sprays. In their studies, they have shown that the use of water sprays in presence of ultrasonic field increase the particle scavenging.

The removal of gaseous pollutants and particulate matters from the atmosphere by rainfall has also been studied (Arora et al. (1991), Ravindra et al. (2003), Naresh et al. (2007), Shukla et al. (2008 a, b, 2012, 2013), Elperin et al. (2011), Sundar et al. (2013)). For example, Ravindra et al. (2003) observed the variation in spatial pattern of air pollutants (SO_2, NO_2, O_3) before and during initial rain of monsoon at Shahdara National Ambient Air Quality Monitoring (NAAQM) station in Delhi, India in 1999. In their observations they concluded that air pollutants concentration is reduced after initial and subsequent rain of the monsoon. Shukla et al. (2008b) presented a mathematical model to study the effect of cloud density on removal of gaseous pollutants and particulate matters from the atmosphere by rain.

Since cement dust pollution has a significant effect on the plant growth, vegetation and human surroundings, its removal from the environment is necessary. As mentioned above it is noted that the greenbelts plantation captures dust particles by depositing them on plant canopies, which ultimately hinders the plant growth by affecting the photosynthesis process. To the best of our knowledge no study has been done to remove cement dust pollution by water sprays, in presence of greenbelts plantation, using a mathematical model. Therefore, in this paper an attempt is made to model the removal of dust particles in the atmosphere around the source of emissions by water sprays in presence of greenbelts plantation.

2. A Mathematical Model

In this section, we propose a mathematical model to study the removal of dust particles emitted from a factory by water sprays and their effect on plant biomass in a greenbelt. To model the phenomenon, let C be the cumulative environmental concentration of various types of dust particles emitted from the cement factory, B be the density of plant biomass in the greenbelts plantation around the factory and W be the number density of water droplets discharged from water sprays.

The following assumptions are made in the modeling process,

(viii) The rate of emission of cement dust particles into the atmosphere is constant (say Q).

(ix) To reduce the concentration of dust particles, water sprays are used such that W is proportional to the concentration of dust particles in the region (i.e. λC).

(x) The depletion of the concentration of dust particles due to water sprays is in direct proportion to the cumulative environmental concentration of dust particles as well as the number density of water droplets (i.e. $\delta_2 WC$), δ_2 being the removal rate coefficient of dust particles.

(xi) The plant biomass in the greenbelts plantation acts as a dust scavenger reducing the concentration of dust particles. Thus, the depletion of dust particles due to plant biomass is taken to be in direct proportion to the environmental concentration of dust particles as well as density of plant biomass (i.e. $\delta_1 BC$), δ_1 being the removal rate coefficient of dust particles.

(xii) The rate of depletion of plant biomass is in direct proportion to the cumulative environmental concentration of dust particles and the density of plant biomass (i.e. $r_1 BC$), r_1 being the depletion rate coefficient of plant biomass. Further $r_2 B^2 C$ denotes the depletion of carrying capacity of plant biomass.

Keeping the above assumptions in view, the dynamics of the system is proposed to be governed by the following system of nonlinear differential equations:

$$\frac{dC}{dt} = Q - \delta_0 C - \delta_1 BC - \delta_2 WC, \tag{1}$$

$$\frac{dB}{dt} = rB\left(1 - \frac{B}{K}\right) - r_1 BC - r_2 B^2 C \tag{2}$$

$$\frac{dW}{dt} = \lambda C - \lambda_0 W - \lambda_1 WC \tag{3}$$

$$C(0) \geq 0, B(0) > 0, W(0) \geq 0$$

In (1) the constant δ_0 is the natural depletion rate coefficient of dust pollutants, r and K are the intrinsic growth rate and carrying capacity of plant biomass in the greenbelts plantation, respectively. The constant λ is the rate of introduction of water droplets in the atmosphere, λ_0 being the natural depletion rate coefficient of water droplets falling on the ground without removing dust particles. The constant λ_1 is the removal rate coefficient of water droplets due to interaction with dust particles. All the constants taken in the model are assumed to be positive.

It is remarked here that if the removal rate coefficients of ground level dust pollution (i.e. δ_1 and δ_2) are very high then $\frac{dC}{dt}$ may become negative and entire dust particles can be removed from the near earth atmosphere. Further, if the depletion rate coefficient of plant biomass (r_1, r_2)due to dust particles is very high the plant biomass density tends to zero as $\frac{dB}{dt}$ may become negative.

To analyze the model system (1) – (3), we need the bounds of dependent variables. For this, we state the region of attraction in the following lemma without proof, Freedman and So, (1985).

Lemma 1. The set

$$\Omega = \left\{ (C,B,W) \in R_+^3 : C_0 < C \le \frac{Q}{\delta_0},\ 0 < B \le K,\ 0 \le W \le \frac{\lambda Q}{\lambda_0 \delta_0} \right\} \tag{4}$$

is the region of attraction for all solutions of the model system (1) – (3) initiating in the interior of positive octant.

Remarks.

1. From (1), $Q > \delta_1 BC$. This gives $\delta_0 > \delta_1 K$. (5-i)
2. From equation (2), we note that, for $r > r_1 C$. Thus, we get $r > \dfrac{r_1 Q}{\delta_0}$. (5-ii)
3. From equation (2), $r > r_2 BC$. This gives $r > r_2 CK = \dfrac{r_2 QK}{\delta_0}$. (5-iii)
4. From equation (3), $\lambda - \lambda_1 W \le 0$ then $\dfrac{dW}{dt} < 0$. Thus, we have $\lambda > \lambda_1 W = \dfrac{\lambda_1 \lambda Q}{\lambda_0 \delta_0}$ (5-iv)

Now we analyze the model (1) – (3) under the above conditions.

3. Equilibrium analysis

The model system (1) – (3) has only two non-negative equilibria namely,

(i) $\widetilde{E}(\widetilde{C}, 0, \widetilde{W})$

(ii) $E^*(C^*, B^*, W^*)$

We show the existence of these two equilibria as follows.

(i) Existence of $\widetilde{E}(\widetilde{C}, 0, \widetilde{W})$:

Here $\tilde{C}$ and $\tilde{W}$ are the positive solutions of the following algebraic equations:

$$Q - \delta_0 C - \delta_2 WC = 0,$$

$$W = \frac{\lambda C}{(\lambda_0 + \lambda_1 C)}.$$

Solving these we have

$$C = \frac{(Q\delta_1 - \lambda_0 \delta_0) + \sqrt{(Q\delta_1 - \lambda_0\delta_0)^2 + 4(\lambda_1 \delta_0 + \lambda \delta_2) Q \lambda_0}}{2(\lambda_1 \delta_0 + \lambda \delta_2)} = \widetilde{C} > 0 \text{ (say)}$$

Using the value of $\tilde{C}$, we can find $\tilde{W}$.

(ii) Existence of $E^*(C^*, B^*, W^*)$:

In order to show the existence of the interior equilibrium E^*, we note that C^*, B^*, W^* are the positive solutions of the following algebraic equations:

$$Q - \delta_0 C - \delta_1 BC - \delta_2 WC = 0 \tag{6}$$

$$B = \frac{r - r_1 C}{\frac{r}{K} + r_2 C} = f(C) > 0 \text{ provided } r > \frac{r_1 Q}{\delta_0} \tag{7}$$

$$W = \frac{\lambda C}{(\lambda_0 + \lambda_1 C)} = g(C) > 0 \tag{8}$$

We note here that $f'(C) < 0$ and $g'(C) > 0$

In view of (7) and (8), equation (6) can be written as

$$F(C) = Q - \delta_0 C - \delta_1 C f(C) - \delta_2 C g(C) = 0 \tag{9}$$

From (9), we have

$$F(0) = Q > 0$$

$$F\left(\frac{Q}{\delta_0}\right) = -\left(\frac{Q}{\delta_0}\right)\left\{\delta_1 f\left(\frac{Q}{\delta_0}\right) + \delta_2 g\left(\frac{Q}{\delta_0}\right)\right\} < 0 \tag{10}$$

This implies that $F(C) = 0$ has at least one root (say C^*) in $0 < C \le \frac{Q}{\delta_0}$ provided, $f\left(\frac{Q}{\delta_0}\right)$ is positive. This gives $r > \frac{r_1 Q}{\delta_0}$ which is satisfied under (5-ii)

To prove the uniqueness of this root, we have to show that $F'(C) < 0$. From equation (9), we have

$$F'(C) = -\delta_0 - \delta_1[Cf'(C) + f(C)] - \delta_2[Cg'(C) + g(C)] \tag{11}$$

which is negative if

$$-\delta_1[K + Cf'(C) + f(C)] - \delta_2[Cg'(C) + g(C)] < 0 \tag{12}$$

From (11) we note that coefficient of $(-\delta_2)$ is positive. The coefficient of $(-\delta_1)$ is also always positive since $K+[Cf'(C)+f(C)]$ is positive under conditions (5).

Thus, E^* uniquely exists under the condition (11).

4. Stability analysis

By computing the Jacobian matrix, it can easily be checked that the equilibrium $\widetilde{E}(\widetilde{C},0,\widetilde{W})$ is unstable.

In the following, we state the local stability results of the interior equilibrium $E^*(C^*,B^*,W^*)$ using Lyapunov's second method, LaSalle and Lefschetz, 1961.

Theorem 4.1 The interior equilibrium $E^*(C^*,B^*,W^*)$, if it exists, is locally asymptotically stable without any condition

Proof:

To establish the local stability behaviour of $E^*(C^*,B^*,W^*)$, we consider the following positive definite function

$$U=\frac{1}{2}k_1C_1^{\,2}+\frac{1}{2}k_2B_1^{\,2}+\frac{1}{2}k_3W_1^{\,2} \tag{13}$$

where C_1,B_1,W_1 are small perturbations about $E^*(C^*,B^*,W^*)$ and $k_i\ (i=1,2,3)$ are positive constants to be chosen appropriately.

Differentiating U with respect to $'t'$ along the model system (1) – (3), we get

$$\frac{dU}{dt}=k_1C_1\frac{dC_1}{dt}+k_2B_1\frac{dB_1}{dt}+k_3W_1\frac{dW_1}{dt} \tag{14}$$

Now substituting the values of $\frac{dC_1}{dt}$, $\frac{dB_1}{dt}$ and $\frac{dW_1}{dt}$ from the model system (1) – (3) after linearization about $E^*(C^*,B^*,W^*)$, we get

$$\frac{dU}{dt}=-k_1(\delta_0+\delta_1B^*+\delta_2W^*)C_1^{\,2}-k_2\left(\frac{r}{K}+r_2C^*\right)B^*B_1^{\,2}-k_3(\lambda_0+\lambda_1C^*)W_1^{\,2}$$

$$-k_1\delta_1C^*C_1B_1-k_2(r_1+r_2B^*)B^*C_1B_1+(-k_1\delta_2C^*+k_3(\lambda-\lambda_1W^*)C_1W_1$$

Now choosing $k_1=1$, $k_2=\frac{k_1\delta_1C^*}{(r_1+r_2B^*)B^*}$ and $k_3=\frac{\delta_2C^*}{(\lambda-\lambda_1W^*)}>0$ under 5(iv), then $\frac{dU}{dt}$ is negative definite provided $\delta_1(r_1 + r_2 B^*)C^* < (\delta_0 + \delta_1 B^* + \delta_2 W^*)\left(\frac{r}{K} + r_2 C^*\right)$ which is satisfied under (5).

In the following, we state the nonlinear stability results of the interior equilibrium $E^*(C^*, B^*, W^*)$ in Ω by using Lyapunov's second method, LaSalle and Lefschetz, 1961.

Theorem 4.2 Let the following inequality holds in Ω

$$\delta_1(r_1 + r_2 B^*)C^* < \delta_0 \frac{r}{K} \tag{15}$$

Then the interior equilibrium $E^*(C^*, B^*, W^*)$, if exists, see condition (11), is nonlinearly asymptotically stable.

Proof:

To establish the nonlinear stability behaviour of $E^*(C^*, B^*, W^*)$, we consider the following positive definite function

$$V = \frac{1}{2} m_1 (C - C^*)^2 + m_2 \left(B - B^* - B^* \log \frac{B}{B^*} \right) + \frac{1}{2} m_3 (W - W^*)^2 \tag{16}$$

where m_i $(i = 1, 2, 3)$ are positive constants to be chosen appropriately.

Differentiating V with respect to $'t'$ along the model system (1) – (3), we get

$$\frac{dV}{dt} = m_1 (C - C^*) \frac{dC}{dt} + m_2 (B - B^*) \frac{1}{B} \frac{dB}{dt} + m_3 (W - W^*) \frac{dW}{dt}$$

Now substituting the values of $\frac{dC}{dt}$, $\frac{dB}{dt}$ and $\frac{dW}{dt}$ from the model system (1) – (3) about $E^*(C^*, B^*, W^*)$, we get

$$\frac{dV}{dt} = -m_1(\delta_1 B + \delta_2 W)(C - C^*)^2 - -m_2 r_2 C (B - B^*)^2 - m_1 \delta_0 (C - C^*)^2 - m_2 \frac{r}{K}(B - B^*)^2$$

$$- m_3(\lambda_0 + \lambda_1 C^*)(W - W^*)^2 - m_2(r_1 + r_2 B^*)(C - C^*)(B - B^*)$$

$$- m_1 \delta_1 C^* (C - C^*)(B - B^*) + \{-m_1 \delta_2 C^* + m_3(\lambda - \lambda_1 W^*)\}(C - C^*)(W - W^*)$$

Now choosing $m_1 = 1$, $m_2 = \frac{m_1 \delta_1 C^*}{(r_1 + r_2 B^*)}$ and $m_3 = \frac{\delta_2 C^*}{(\lambda - \lambda_1 W^*)}$, $\frac{dV}{dt}$ will be negative definite inside the region of attraction Ω, provided condition (15) is satisfied and hence the theorem.

This theorem implies that trajectories with every initial start within the region of attraction Ω approach to equilibrium state $E^*(C^*, B^*, W^*)$.

Further, the above two theorems also imply that under a suitable conditions the system would remain in equilibrium within the region of attraction.

Remark. It is noted if $\delta_1 = 0$ then the stability condition (15) is satisfied. This implies that δ_1 has destabilizing effect on the model system.

5. Numerical simulation and discussion

In this section, using MAPLE 7, some numerical simulations have been performed to study the feasibility of the model system (1) – (3) by choosing the following set of parameter values.

$$Q = 2,\ \delta_0 = 0.1,\ \delta_1 = 0.00004,\ \delta_2 = 0.00004, K = 1000, r = 1, r_1 = 0.00002$$

$$r_1 = 0.000002,\ \lambda = 0.1,\ \lambda_0 = 0.002, \lambda_1 = 0.02$$

The equilibrium values of $E^*(C^*, B^*, W^*)$ corresponding to above data are found as,

$$C^* = 14.3813,\ B^* = 971.7618,\ W^* = 4.9654$$

The eigen values of Jacobian matrix corresponding to equilibrium $E^*(C^*, B^*, W^*)$ for the model system (1) – (3) are $-0.1377, -1.0009$ and -0.2896. Thus, all eigenvalues of the Jacobian matrix are negative. Hence the interior equilibrium $E^*(C^*, B^*, W^*)$ is locally asymptotically stable without any condition. It may be noted here that nonlinear stability condition (15) in Theorem 2 is also satisfied for the above set of parameter values. The nonlinear stability behaviour of $E^*(C^*, B^*, W^*)$ in $W - C$ plane has been shown in figure 1 with different initial starts. From this figure, it is apparent that all the trajectories initiating inside the region of attraction Ω approaches towards the equilibrium value (W^*, C^*).

The variations of concentration of dust particles C with time 't' for different values of δ_1 (i.e. at $\delta_1 = 0.00004, 0.00008, 0.00012$) and δ_2 (i.e. at $\delta_2 = 0.00004, 0.00040, 0.00080$) are shown in figures 2 and 4, respectively. From these figures, it is clear that as the rates of interactions of dust particles with plant species as well as with water droplets increase, the equilibrium concentration of the dust particles into the atmosphere decreases. Further, due to reduction in concentration of dust particles, biomass damage decreases and hence its density increases (figures 3 and 5 respectively). The variations of concentration of dust particles C and density of plant species B

with time $'t'$ for different values of r_1 (i.e. at $r_1 = 0.00002, 0.00020, 0.00040$) are shown in figures 6 and 7, respectively. From these figures it is observed that as the depletion rate coefficient of plant species (due to dust particles) increases, the equilibrium concentration of dust particles increases while the density of plant species decreases.

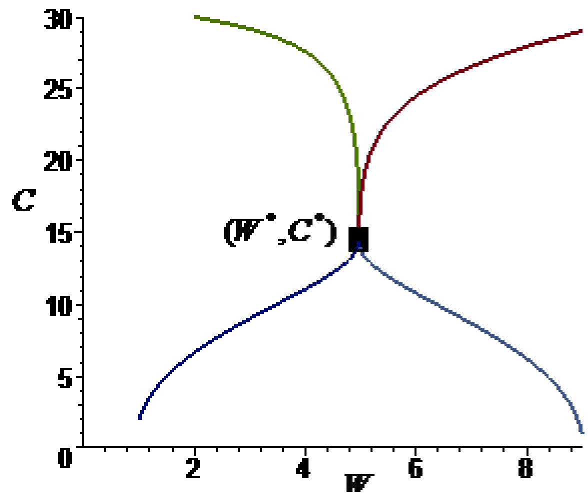

Figure 1: Nonlinear stability in $C-W$ plane

The variation of concentration of dust particles with time $'t'$ for different values of λ (i.e. at $\lambda = 0.1, \lambda = 0.3, \lambda = 0.5$) is shown in figure 8. It shows that as the rate of introduction of water droplets increases the equilibrium concentration of dust particles decreases.

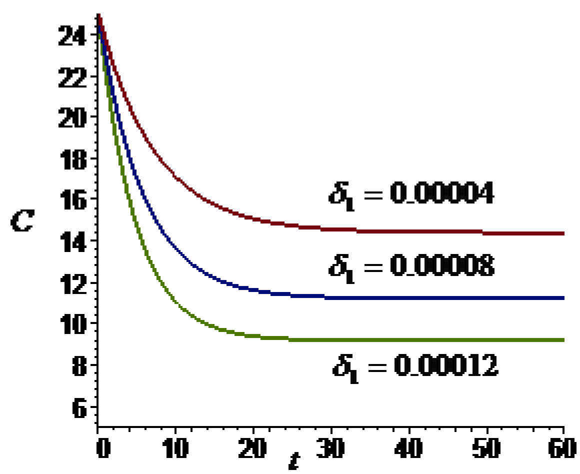

Figure 2: Variation of concentration C with time $'t'$ for different values of δ_1

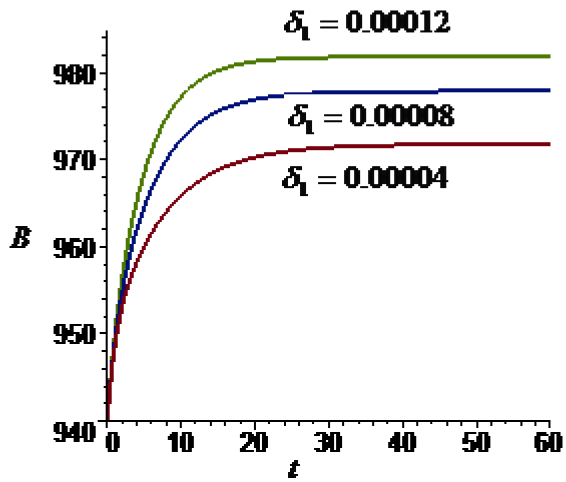

Figure 3: Variation of concentration B with time 't' for different values of δ_1

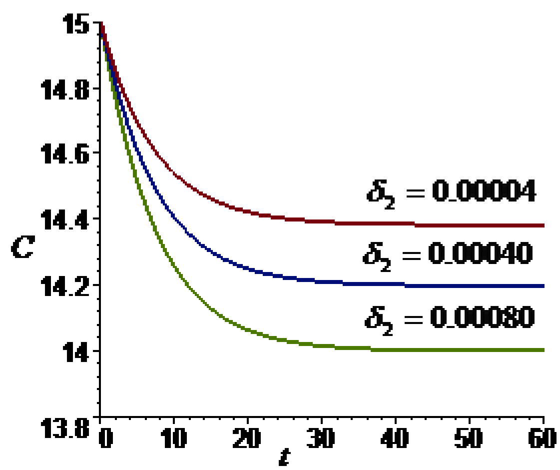

Figure 4: Variation of concentration C with time 't' for different values of δ_2

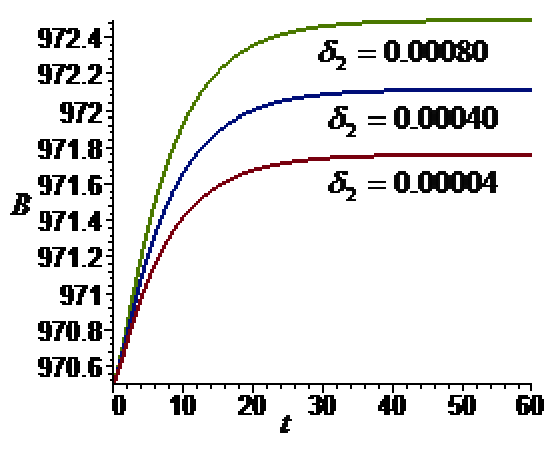

Figure 5: Variation of concentration B with time $'t'$ for different values of δ_2

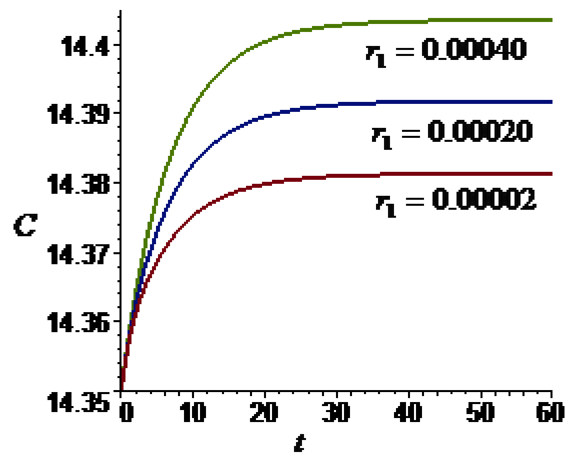

Figure 6: Variation of concentration C with time $'t'$ for different values of r_1

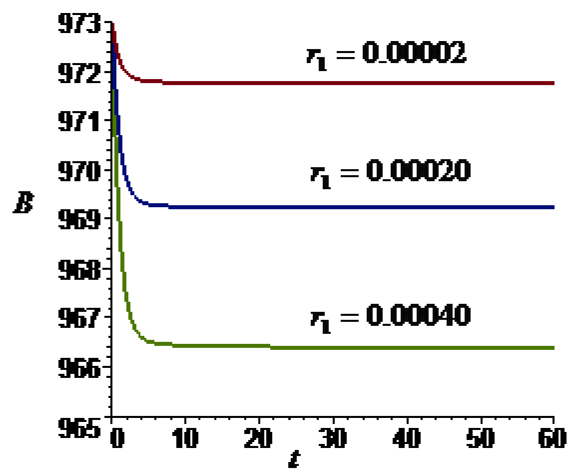

Figure 7: Variation of biomass density B with time $'t'$ for different values of r_1.

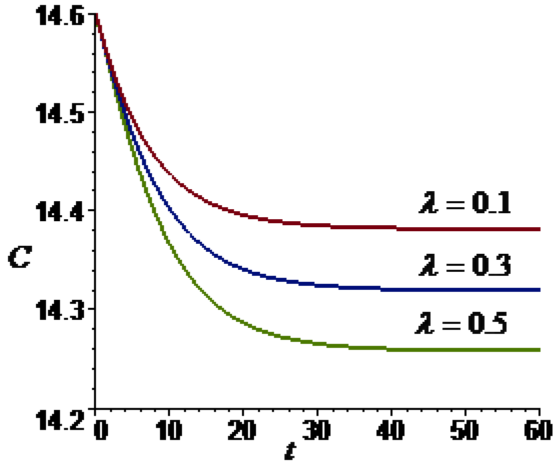

Figure 8: Variation of concentration C with time 't' for different values of λ

7. Conclusions

In this paper, a nonlinear model for the removal of dust particles, emitted from a cement factory, by water sprays has been proposed by considering three interacting variables namely, the concentration of dust particles, the density of plant biomass in the greenbelts plantation and the number density of droplets in water sprays. This study suggests that the water spray and greenbelt plantation play significant role in the removal of dust particles as they get removed by the process of impaction when they interact with water spray. It has been shown that dust particles near in the near earth atmosphere is further reduced by greenbelts plantation. Further, the concentration of dust particles decreases as the number density of water droplets increases. The simulation of the model confirms the analytical results.

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SECTION 2

Editor's Choice

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Robotic Tree As An Air Purifier And For Reducing Global Warming

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It is well known that a green belt plantations in and around forest is very useful for the conservation of forest resources as well as for biodiversity. One of the main functions of a forest is to serve as a sink for CO_2 and thus help in reducing air pollution and global warming. We may note here that green belts plantation requires several years to become functional and therefore an alternative powerful mechanism is required for conservation of forest resources as well for the reduction of the concentration of CO_2 in the atmosphere and thereby reducing air pollution and global warming.

It is proposed here that robotic tree plantations using sensors to monitoring and reducing concentration of CO_2 and othergases as well as particulate matters from the atmosphere.

A robotic tree has to the following two parts:

1. Various sensors for to monitoring concentrations of CO_2 and other gases in the atmosphere as well as particulate matters in the atmosphere. These sensors have to be associated with usual devices measuring these concentration.
2. Robotic tree having configuration like the usual tree with trunks and branches.

A robotic tree is designed to have its trunk fabricated from a large diameter pipe while its branches can be made of small diameter pipes with holes using strong plastic or metallic material. It should be flexible enough to be turned around in any direction. The branches are to be used for the following purposes:

1. For attaching sensors to monitor various air pollutants including CO_2.
2. For attaching the robotic arms, which may be used for the following aspects:
    a. For spraying water, which may contain appropriate chemicals / calcium oxide by a mechanism attached to the branches in the of the robotic tree, to capture particulate matters and various gases.

    b. For hanging cylindrical porous pots containing CaO(calcium oxide powder) to absorb CO_2 and for hanging buckets containing relevant liquid or a mixture of liquid and solids to capture gaseous pollutants and particulate matter from the environment.

These robotic trees can be used for reducing the concentration of CO_2 and other gases and particulate matter in the following cases:

(i) For reducing pollution including CO_2 discharge form vehicular traffic in the road atmosphere. These robotic trees can be planted in a line on the dividers between two roads . These trees should be planted in such a manner so that water can be pumped to these trees and it can be sprayed in all direction in atmosphere

(ii) For reducing pollution emitted from coal based thermal power stations. These robotic trees can be planted around the chimneys to capture air pollutants.

(iii) For reducing pollution including CO_2 The robotic tree belts can be planted in and around deforested zones. These trees will be helpful in conserving forest resources and preserve biodiversity by reducing global warming(1,2,3).

(iv) These robotic trees can also be planted at verandas and roof tops of the houses situated at the highways and expressways and heavy traffic roads

To determine the number of trees and required in a belt , a mathematical model is to be derived relating the concentration of CO_2in the atmosphere and various parameters in the tree plantations. (3) Then the optimal control theory well known in operations research and calculus of variations have to applied in such a way that the concentration of CO_2 and other pollutants are minimized in the atmosphere.

A Modeling Study

The above is being studied by considering the nonlinear interactions of the following variables:

1. The concentration of CO_2
2. The cumulative concentrations of particulate matter
3. The concentrations liquid droplets to capture of particulate matter
4. The concentrations of calcium oxide in a solute from to neutralize CO_2 .

A mathematical model has been proposed and analysed to study the above mentioned problem.

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