Centralized and Distributed Millimeter Wave Massive MIMO based Data Fusion with Perfect and Bayesian Learning (BL)-based Imperfect CSI

Apoorva Chawla, *St. Member, IEEE*, Palla Siva Kumar, *St. Member, IEEE*, Suraj Srivastava, *St. Member, IEEE* and Aditya K. Jagannatham, *Member, IEEE*

I. Data Fusion for a known parameter θ with perfect CSI

This section derives the data fusion techniques for a known parameter scenario, for both the massive array configurations.

A. Decision rule for the C-MIMO Architecture

Employing the two-step architecture, the combined output $\mathbf{y}_{\rm C} \in \mathbb{C}^{M\times 1}$, under both the hypotheses, is distributed as

$$\begin{aligned}\mathcal{H}_0 &: \mathbf{y}_{\rm C} \sim \mathcal{CN}\left(\mathbf{0}, \mathbf{\Sigma}_{\rm C}\right),\\ \mathcal{H}_1 &: \mathbf{y}_{\rm C} \sim \mathcal{CN}\left(M\mathbf{\Psi}\mathbf{F}\mathbf{a}\theta, \mathbf{\Sigma}_{\rm C}\right),\end{aligned} \tag{1}$$

where the covariance matrix $\mathbf{\Sigma}_{\rm C} = \sigma_\eta^2 M^2 \mathbf{\Psi}\mathbf{F}\mathbf{F}^H\mathbf{\Psi}^H + \mathbf{C}_{\tilde{\mathbf{v}}}$ is diagonal with the kth diagonal element $[\mathbf{\Sigma}_{\rm C}]_{k,k} = \sigma_k^2 = \sigma_\eta^2 M^2 \psi_k^2 |f_k|^2 + \sigma_v^2 M \psi_k$.

Adopting the NP criterion [1], which maximizes the probability of detection for a given probability of false alarm, the log likelihood ratio (LLR) test for the binary hypothesis testing problem in (1), can be formulated as

$$\begin{aligned} T_{\rm C,KP}(\mathbf{y}_{\rm C}) &= \ln\left[\frac{p(\mathbf{y}_{\rm C}|\mathcal{H}_1)}{p(\mathbf{y}_{\rm C}|\mathcal{H}_0)}\right] \\ &= \ln\left[\frac{\exp\left(-(\mathbf{y}_{\rm C}-M\mathbf{\Psi}\mathbf{F}\mathbf{a}\theta)^H\mathbf{\Sigma}_{\rm C}^{-1}(\mathbf{y}_{\rm C}-M\mathbf{\Psi}\mathbf{F}\mathbf{a}\theta)\right)}{\exp\left(-\mathbf{y}_{\rm C}^H\mathbf{\Sigma}_{\rm C}^{-1}\mathbf{y}_{\rm C}\right)}\right] \\ &= \Re(\mathbf{y}_{\rm C}^H\mathbf{\Sigma}_{\rm C}^{-1}\mathbf{\Psi}\mathbf{F}\mathbf{a}) \\ &= \sum_{k=1}^{K}\Re\left(\frac{y_{{\rm C},k}^*\psi_k f_k a_k}{\sigma_k^2}\right). \end{aligned}$$

Substituting the expression of σ_k^2 in the above expression, followed by simplification, one obtains

$$T_{\text{C,KP}}(\mathbf{y}_{\text{C}}) = \sum_{k=1}^{K} \Re\left(\frac{y_{\text{C},k}^{*} f_k a_k}{M\psi_k |f_k|^2 \sigma_\eta^2 + \sigma_v^2}\right) \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \tilde{\gamma}, \tag{2}$$

where $\tilde{\gamma}$ is the detection threshold. Notice that, $T_{\text{C,KP}}(\mathbf{y}_{\text{C}})$ is the weighted linear combination of symmetric complex Gaussian random variables $y_{\text{C},k}$. Thus, $T_{\text{C,KP}}(\mathbf{y}_{\text{C}})$ is also symmetric complex Gaussian. The distribution of $T_{\text{C,KP}}(\mathbf{y}_{\text{C}})$, under both the hypotheses, can be expressed as

$$\begin{aligned} \mathcal{H}_0 &: T_{\text{C,KP}}(\mathbf{y}_{\text{C}}) \sim \mathcal{CN}(\mu_{T_{\text{C,KP}}|\mathcal{H}_0}, \sigma^2_{T_{\text{C,KP}}|\mathcal{H}_0}), \\ \mathcal{H}_1 &: T_{\text{C,KP}}(\mathbf{y}_{\text{C}}) \sim \mathcal{CN}(\mu_{T_{\text{C,KP}}|\mathcal{H}_1}, \sigma^2_{T_{\text{C,KP}}|\mathcal{H}_1}), \end{aligned} \tag{3}$$

where $\mu_{T_{\text{C,KP}}|\mathcal{H}_0}, \mu_{T_{\text{C,KP}}|\mathcal{H}_1}$ and $\sigma^2_{T_{\text{C,KP}}|\mathcal{H}_0}, \sigma^2_{T_{\text{C,KP}}|\mathcal{H}_1}$ are the means and variances corresponding to the null and alternate hypotheses, respectively. The mean $\mu_{T_{\text{C,KP}}|\mathcal{H}_0}$, can be found as follows

$$\begin{aligned} \mu_{T_{\text{C,KP}}|\mathcal{H}_0} &= \mathbb{E}\left\{ \sum_{k=1}^{K} \Re\left(\frac{y_{\text{C},k}^{*} f_k a_k}{M\psi_k |f_k|^2 \sigma_\eta^2 + \sigma_v^2}\right) \middle| \mathcal{H}_0 \right\} \\ &= \sum_{k=1}^{K} \Re\left(\frac{(\mathbb{E}\{y_{\text{C},k}^{*}\}|\mathcal{H}_0) f_k a_k}{M\psi_k |f_k|^2 \sigma_\eta^2 + \sigma_v^2}\right) = 0, \end{aligned} \tag{4}$$

where the final expression follows from (1). Similarly, the mean under the alternate hypothesis, $\mu_{T_{\text{C,KP}}|\mathcal{H}_1}$, can be determined as follows

$$\begin{aligned} \mu_{T_{\text{C,KP}}|\mathcal{H}_1} &= \mathbb{E}\left\{ \sum_{k=1}^{K} \Re\left(\frac{y_{\text{C},k}^{*} f_k a_k}{M\psi_k |f_k|^2 \sigma_\eta^2 + \sigma_v^2}\right) \middle| \mathcal{H}_1 \right\} \\ &= \sum_{k=1}^{K} \Re\left(\frac{(\mathbb{E}\{y_{\text{C},k}^{*}\}|\mathcal{H}_1) f_k a_k}{M\psi_k |f_k|^2 \sigma_\eta^2 + \sigma_v^2}\right) \\ &= \sum_{k=1}^{K} \frac{M\psi_k |f_k|^2 |a_k|^2 \theta}{M\psi_k |f_k|^2 \sigma_\eta^2 + \sigma_v^2}. \end{aligned} \tag{5}$$

Now, the variance corresponding to null hypothesis, $\sigma^2_{T_{\text{C,KP}}|\mathcal{H}_0}$, can be obtained as follows

$$\begin{aligned} \sigma^2_{T_{\text{C,KP}}|\mathcal{H}_0} &= \mathbb{E}\left\{T^2_{\text{C,KP}}|\mathcal{H}_0\right\} - \mathbb{E}\left\{T_{\text{C,KP}}|\mathcal{H}_0\right\}^2 \\ &= \mathbb{E}\left\{T^2_{\text{C,KP}}|\mathcal{H}_0\right\} \\ &= \mathbb{E}\left\{ \left[\sum_{k=1}^{K} \left(\frac{\Re(a_k^{*} f_k^{*} y_{\text{C},k})}{M\psi_k |f_k|^2 \sigma_\eta^2 + \sigma_v^2} \right) \right]^2 \middle| \mathcal{H}_0 \right\} \\ &= \sum_{k=1}^{K} \frac{\mathbb{E}\left\{(a_k^{*} f_k^{*} y_{\text{C},k} + a_k f_k y_{\text{C},k}^{*})^2 |\mathcal{H}_0\right\}}{4(M\psi_k |f_k|^2 \sigma_\eta^2 + \sigma_v^2)^2} \end{aligned}$$

$$= \sum_{k=1}^{K} \frac{|a_k|^2 |f_k|^2 \mathbb{E}\{y_{\mathrm{C},k} y_{\mathrm{C},k}^* | \mathcal{H}_0\}}{2(M\psi_k |f_k|^2 \sigma_\eta^2 + \sigma_v^2)^2}$$

$$= \sum_{k=1}^{K} \frac{M\psi_k |f_k|^2 |a_k|^2}{2(M\psi_k |f_k|^2 \sigma_\eta^2 + \sigma_v^2)}. \tag{6}$$

Similarly, along similar lines, it can be proved that, $\sigma^2_{T_{\mathrm{C,KP}}|\mathcal{H}_1} = \sigma^2_{T_{\mathrm{C,KP}}|\mathcal{H}_0}$. Taking the above expressions into account, the probabilities of detection (P_D) and false alarm (P_{FA}), can be obtained as

$$P_D = Q\left(\frac{\tilde{\gamma} - \mu_{T_{\mathrm{C,KP}}|\mathcal{H}_1}}{\sigma_{T_{\mathrm{C,KP}}|\mathcal{H}_1}}\right),$$

$$P_{FA} = Q\left(\frac{\tilde{\gamma} - \mu_{T_{\mathrm{C,KP}}|\mathcal{H}_0}}{\sigma_{T_{\mathrm{C,KP}}|\mathcal{H}_0}}\right). \tag{7}$$

B. Decision rule for the D-MIMO Architecture

Similar to the C-MIMO scenario, a two-step architecture is employed at each FC. The combined output at the BPU, under both the hypotheses, follows the distribution

$$\begin{aligned} \mathcal{H}_0 &: \mathbf{y}_{\mathrm{D}} \sim \mathcal{CN}\left(\mathbf{0}, \mathbf{\Sigma}_{\mathrm{D}}\right), \\ \mathcal{H}_1 &: \mathbf{y}_{\mathrm{D}} \sim \mathcal{CN}\left(N_d \mathbf{\Psi}_{\mathrm{D}} \mathbf{F} \mathbf{a} \theta, \mathbf{\Sigma}_{\mathrm{D}}\right), \end{aligned} \tag{8}$$

where $\mathbf{\Sigma}_{\mathrm{D}} = \sigma_\eta^2 N_d^2 \mathbf{\Psi}_{\mathrm{D}} \mathbf{F}\mathbf{F}^H \mathbf{\Psi}_{\mathrm{D}}^H + \mathbf{C}_{\mathbf{v}'} \in \mathbb{C}^{K\times K}$ is a diagonal matrix with the kth diagonal entry $[\mathbf{\Sigma}_{\mathrm{D}}]_{k,k} = \sigma^2_{\mathrm{D},k} = \sigma_\eta^2 N_d^2 \psi_{k,j_k}^2 |f_k|^2 + \sigma_v^2 N_d \psi_{k,j_k}$. The LLR test statistic, $T_{\mathrm{D,KP}}(\mathbf{y}_{\mathrm{D}})$, for the known parameter scenario, can be formulated as

$$\begin{aligned} T_{\mathrm{C,KP}}(\mathbf{y}_{\mathrm{D}}) &= \ln\left[\frac{p(\mathbf{y}_{\mathrm{D}}|\mathcal{H}_1)}{p(\mathbf{y}_{\mathrm{D}}|\mathcal{H}_0)}\right] \\ &= \ln\left[\frac{\exp\left(-(\mathbf{y}_{\mathrm{D}} - N_d \mathbf{\Psi}_{\mathrm{D}} \mathbf{F}\mathbf{a}\theta)^H \mathbf{\Sigma}_{\mathrm{D}}^{-1} (\mathbf{y}_{\mathrm{D}} - N_d \mathbf{\Psi}_{\mathrm{D}} \mathbf{F}\mathbf{a}\theta)\right)}{\exp\left(-\mathbf{y}_{\mathrm{D}}^H \mathbf{\Sigma}_{\mathrm{D}}^{-1} \mathbf{y}_{\mathrm{D}}\right)}\right] \\ &= \Re(\mathbf{y}_{\mathrm{D}}^H \mathbf{\Sigma}_{\mathrm{D}}^{-1} \mathbf{\Psi}_{\mathrm{D}} \mathbf{F}\mathbf{a}) \\ &= \sum_{k=1}^{K} \Re\left(\frac{y_{\mathrm{D},k}^* \psi_{k,j_k} f_k a_k}{\sigma^2_{\mathrm{D},k}}\right). \end{aligned}$$

On substituting $\sigma^2_{\mathrm{D},k}$ in the above expression, the test can be simplified as

$$T_{\mathrm{D,KP}}(\mathbf{y}_{\mathrm{D}}) = \sum_{k=1}^{K} \Re\left(\frac{y_{\mathrm{D},k}^* f_k a_k}{N_d \psi_{k,j_k} |f_k|^2 \sigma_\eta^2 + \sigma_v^2}\right) \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \check{\gamma}, \tag{9}$$

where $\breve{\gamma}$ is the detection threshold. The test statistic $T_{\text{D,KP}}(\mathbf{y}_{\text{D}})$ follows the complex Gaussian distribution under both the hypotheses, given as

$$\begin{aligned}\mathcal{H}_0 &: T_{\text{D,KP}}(\mathbf{y}_{\text{D}}) \sim \mathcal{CN}(\mu_{T_{\text{D,KP}}|\mathcal{H}_0}, \sigma^2_{T_{\text{D,KP}}|\mathcal{H}_0}),\\ \mathcal{H}_1 &: T_{\text{D,KP}}(\mathbf{y}_{\text{D}}) \sim \mathcal{CN}(\mu_{T_{\text{D,KP}}|\mathcal{H}_1}, \sigma^2_{T_{\text{D,KP}}|\mathcal{H}_1}),\end{aligned} \tag{10}$$

where $\mu_{T_{\text{D,KP}}|\mathcal{H}_0}, \mu_{T_{\text{D,KP}}|\mathcal{H}_1}$ and $\sigma^2_{T_{\text{D,KP}}|\mathcal{H}_0}, \sigma^2_{T_{\text{D,KP}}|\mathcal{H}_1}$ are the means and variances corresponding to the null and alternate hypotheses, respectively. These can be derived along similar lines as that of the C-MIMO architecture and can be expressed as

$$\mu_{T_{\text{D,KP}}|\mathcal{H}_0} = 0, \tag{11}$$

$$\mu_{T_{\text{D,KP}}|\mathcal{H}_1} = \sum_{k=1}^{K} \frac{N_d \psi_{k,j_k} |f_k|^2 |a_k|^2 \theta}{N_d \psi_{k,j_k} |f_k|^2 \sigma_\eta^2 + \sigma_v^2}, \tag{12}$$

$$\sigma^2_{T_{\text{D,KP}}|\mathcal{H}_0} = \sigma^2_{T_{\text{D,KP}}|\mathcal{H}_1} = \sum_{k=1}^{K} \frac{N_d \psi_{k,j_k} |f_k|^2 |a_k|^2}{2(N_d \psi_{k,j_k} |f_k|^2 \sigma_\eta^2 + \sigma_v^2)}. \tag{13}$$

II. Data fusion for a known parameter θ with imperfect CSI

The mmWave massive MIMO channel is first estimated using the SBL-based approach, followed by the determination of the decision rules for distributed detection of the known parameter.

A. Decision rule for the C-MIMO Architecture

The hybrid combined output at the FC follows the distribution

$$\begin{aligned}\mathcal{H}_0 &: \tilde{\mathbf{y}}_{\text{C}} \sim \mathcal{CN}\left(\mathbf{0}, \mathbf{C}_{\tilde{\mathbf{v}}}\right),\\ \mathcal{H}_1 &: \tilde{\mathbf{y}}_{\text{C}} \sim \mathcal{CN}(\breve{\mathbf{G}}\mathbf{F}\mathbf{a}\theta, \mathbf{C}_{\tilde{\boldsymbol{\eta}}}),\end{aligned} \tag{14}$$

where the covariance matrices $\mathbf{C}_{\tilde{\mathbf{v}}}$ and $\mathbf{C}_{\tilde{\boldsymbol{\eta}}}$ are diagonal, with their kth diagonal entries as $[\mathbf{C}_{\tilde{\mathbf{v}}}]_{k,k} = \sigma^2_{\tilde{v},k}$ and $[\mathbf{C}_{\tilde{\boldsymbol{\eta}}}]_{k,k} = \sigma^2_{\tilde{\eta},k}$. Employing the above quantities, the test statistic for the detection of a known parameter, with imperfect CSI, can be expressed as

$$T_{\text{C,KIP}}(\tilde{\mathbf{y}}_{\text{C}}) = \tilde{\mathbf{y}}_{\text{C}}^H (\mathbf{C}_{\tilde{\mathbf{v}}}^{-1} - \mathbf{C}_{\tilde{\boldsymbol{\eta}}}^{-1}) \tilde{\mathbf{y}}_{\text{C}} + 2\Re(\tilde{\mathbf{y}}_{\text{C}}^H \mathbf{C}_{\tilde{\boldsymbol{\eta}}}^{-1} \breve{\mathbf{G}}\mathbf{F}\mathbf{a}\theta). \tag{15}$$

Determining a closed-form expression for the test $T_{\text{C,KIP}}(\tilde{\mathbf{y}}_{\text{C}})$ is significantly challenging. Furthermore, determining the distribution of the test thus obtained is mathematically intractable. Thus, in the interest of practical implementation, it is essential to determine simplistic detectors that have a low complexity. The energy detector (ED), which readily meets these criteria, is ideally suited in such systems. Hence, this is employed for distributed sensing in a mmWave massive MIMO WSN with imperfect CSI at the FC. The corresponding test statistic for the centralized antenna topology is given as

$$T_{\text{C,KIP}}(\tilde{\mathbf{y}}_{\text{C}}) = \tilde{\mathbf{y}}_{\text{C}}^{H}\tilde{\mathbf{y}}_{\text{C}} = \sum_{k=1}^{K} |\tilde{y}_{\text{C},k}|^2 \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \gamma'', \tag{16}$$

where γ'' is the detection threshold. The test statistic $T_{\text{C,KIP}}(\tilde{\mathbf{y}}_{\text{C}})$ under both the hypotheses can be expressed as

$$\begin{aligned}\mathcal{H}_0 : T_{\text{C,KIP}}(\tilde{\mathbf{y}}_{\text{C}}) &= \sum_{k=1}^{K} \frac{\sigma^2_{\tilde{v},k}}{2} \frac{|\tilde{y}_{\text{C},k}|^2}{\sigma^2_{\tilde{v},k}/2} = \sum_{k=1}^{K} \frac{\sigma^2_{\tilde{v},k}}{2} \chi_2^2, \\ \mathcal{H}_1 : T_{\text{C,KIP}}(\tilde{\mathbf{y}}_{\text{C}}) &= \sum_{k=1}^{K} \frac{\sigma^2_{\tilde{\eta},k}}{2} \frac{|\tilde{y}_{\text{C},k}|^2}{\sigma^2_{\tilde{\eta},k}/2} = \sum_{k=1}^{K} \frac{\sigma^2_{\tilde{\eta},k}}{2} \chi_2^2(\lambda_{k,1}),\end{aligned} \tag{17}$$

where χ_2^2 and $\chi_2^2(\lambda_{k,1})$ denote independent central and non-central chi-squared random variables with non-centrality parameter $\lambda_{k,1} = \frac{2\left||\hat{h}_{i_k,k}|^2 f_k a_k \theta\right|^2}{\sigma^2_{\tilde{\eta},k}}$, respectively, with two degrees of freedom. For both the hypotheses, the test statistic in (17) can be approximated as the non-central chi-squared random variable

$$\begin{aligned}\mathcal{H}_0 : T_{\text{C,KIP}}(\tilde{\mathbf{y}}_{\text{C}}) &\approx \chi^2_{l_{\text{C,KF}}}(\lambda_{\text{C,KF}}), \\ \mathcal{H}_1 : T_{\text{C,KIP}}(\tilde{\mathbf{y}}_{\text{C}}) &\approx \chi^2_{l_{\text{C,KD}}}(\lambda_{\text{C,KD}}),\end{aligned} \tag{18}$$

where the quantities $l_{\text{C,KF}}$, $l_{\text{C,KD}}$ and $\lambda_{\text{C,KF}}$, $\lambda_{\text{C,KD}}$ are the degrees of freedom and the non-centrality parameters for the null and alternate hypotheses respectively. These can be obtained from the first four cumulants of $T_{\text{C,KIP}}(\tilde{\mathbf{y}}_{\text{C}})$ [2]. For the test statistic in (18), the expressions of P_D and P_{FA} can be obtained as

$$\begin{aligned}P_D &\approx \Pr(\chi^2_{l_{\text{C,KD}}}(\lambda_{\text{C,KD}}) > \gamma'') = Q_{\chi^2_{l_{\text{C,KD}}}(\lambda_{\text{C,KD}})}(\gamma''), \\ P_{FA} &\approx \Pr(\chi^2_{l_{\text{C,KF}}}(\lambda_{\text{C,KF}}) > \gamma'') = Q_{\chi^2_{l_{\text{C,KF}}}(\lambda_{\text{C,KF}})}(\gamma'').\end{aligned} \tag{19}$$

B. Decision rule for the D-MIMO Architecture

The distribution of the hybrid combined output at BPU follows the distribution

$$\begin{aligned}\mathcal{H}_0 &: \tilde{\mathbf{y}}_{\text{D}} \sim \mathcal{CN}\left(\mathbf{0}, \mathbf{C}_{\breve{\mathbf{v}}}\right), \\ \mathcal{H}_1 &: \tilde{\mathbf{y}}_{\text{D}} \sim \mathcal{CN}(\breve{\mathbf{G}}_{\text{D}}\mathbf{F}\mathbf{a}\theta, \mathbf{C}_{\breve{\boldsymbol{\eta}}}),\end{aligned} \tag{20}$$

where the covariance matrices $\mathbf{C}_{\check{\mathbf{v}}}$ and $\mathbf{C}_{\check{\boldsymbol{\eta}}}$ are diagonal, with kth diagonal entries as $[\mathbf{C}_{\check{\mathbf{v}}}]_{k,k} = \sigma^2_{\check{v},k}$ and $[\mathbf{C}_{\check{\boldsymbol{\eta}}}]_{k,k} = \sigma^2_{\check{\eta},k}$. The ED for the data fusion of a known parameter, with imperfect CSI and the D-MIMO configuration can be expressed as

$$T_{\text{D,KIP}}(\tilde{\mathbf{y}}_{\text{D}}) = \tilde{\mathbf{y}}_{\text{D}}^H \tilde{\mathbf{y}}_{\text{D}} = \sum_{k=1}^{K} |\tilde{y}_{\text{D},k}|^2 \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \gamma''. \tag{21}$$

The test statistic under both the hypotheses can be simplified as

$$\begin{aligned}\mathcal{H}_0 : T_{\text{D,KIP}}(\tilde{\mathbf{y}}_{\text{D}}) &= \sum_{k=1}^{K} \frac{\sigma^2_{\check{v},k}}{2} \frac{|\tilde{y}_{\text{D},k}|^2}{\sigma^2_{\check{v},k}/2} = \sum_{k=1}^{K} \frac{\sigma^2_{\check{v},k}}{2} \chi^2_2, \\ \mathcal{H}_1 : T_{\text{D,KIP}}(\tilde{\mathbf{y}}_{\text{D}}) &= \sum_{k=1}^{K} \frac{\sigma^2_{\check{\eta},k}}{2} \frac{|\tilde{y}_{\text{D},k}|^2}{\sigma^2_{\check{\eta},k}/2} = \sum_{k=1}^{K} \frac{\sigma^2_{\check{\eta},k}}{2} \chi^2_2(\lambda_{k,1}),\end{aligned} \tag{22}$$

where χ^2_2 and $\chi^2_2(\lambda_{k,1})$ denote independent central and non-central chi-squared random variables with non-centrality parameter $\lambda_{k,1} = \frac{2\left||\hat{h}_{i_k,k,j_k}|^2 f_k a_k \theta\right|^2}{\sigma^2_{\check{\eta}}}$, respectively, with two degrees of freedom. The test statistic can be well-approximated as non-central chi-squared random variable with $l_{\text{D,KF}}$ and $l_{\text{D,KD}}$ degrees of freedom, and the non-centrality parameters $\lambda_{\text{D,KF}}$ and $\lambda_{\text{D,KD}}$, under the null and alternate hypotheses, respectively. The quantities can be computed using the first four cumulants of the test statistic as shown in [2]. The P_D and P_{FA} expressions for the test statistic in (22) can be derived as

$$\begin{aligned}P_D &\approx \Pr(\chi^2_{l_{\text{D,KD}}}(\lambda_{\text{D,KD}}) > \gamma'') = Q_{\chi^2_{l_{\text{D,KD}}}(\lambda_{\text{D,KD}})}(\gamma''), \\ P_{FA} &\approx \Pr(\chi^2_{l_{\text{D,KF}}}(\lambda_{\text{D,KF}}) > \gamma'') = Q_{\chi^2_{l_{\text{D,KF}}}(\lambda_{\text{D,KF}})}(\gamma'').\end{aligned} \tag{23}$$

References

[1] S. M. Kay, *Fundamentals of statistical signal processing, volume 2: Detection theory*. New Jersey: Prentice-Hall Inc, 1993.

[2] H. Liu, Y. Tang, and H. H. Zhang, "A new chi-square approximation to the distribution of non-negative definite quadratic forms in non-central normal variables," *Computational Statistics and Data Analysis*, vol. 53, no. 4, pp. 853–856, 2009.