SEISMIC SOIL-STRUCTURE INTERACTION ANALYSIS IN TIME-DOMAIN BASED ON COMBINATION COMPUTATION METHOD OF INTERACTION FORCE

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ABSTRACT :
Combining lumped-parameter model and time-domain difference recursive filter model, a time-domain difference approach of interaction force is proposed. The system stability is studied in this paper. Based on general FEM software and the proposed time-domain computation model, history analysis programs with the SSDI effect is developed. After all, numerical example is presented to verify the validity of proposed method. The effect of ground impedance frequency dependency on the dynamic response of soil-structure system is preliminary discussed.

KEYWORDS:
soil-structure dynamic interaction, impedance function, frequency dependent, lumped-parameter model, pseudo-force, time domain, Combination Computation Method

1. INTRODUCTION

To analyze structure dynamic response with consideration to soil–structure dynamic interaction(SSDI) and the non-linear behavior of the structure, it is necessary to calculate the interaction force considering the frequency dependent of ground impedance function. Currently there are two kinds of method dealing with it(Wolf.1985, Du Xiuli et al.2008): one is the direct integrity approach, the other is substructure approach. The former needs high computational cost while the latter ignores the nonlinear effect of dynamic interaction between soil and structure. Due to far less computational cost is needed than the former, the substructure approach is very important in engineering field. Frequency domain substructure method has the merit in dealing with the frequency-dependent of impedance but can not take into account the nonlinearity of structure. Hybrid-frequency-time domain procedure (Darbre.1990, Darbre.1992) and hybrid-time-frequency domain procedure (Bernal et al.1998) need repetitious Fourier Transform and iterative fitting. Key problem in time-domain substructure approach is the calculation of time-domain interaction force.
From the mathematical viewpoint, time-domain foundation resisting force can be expressed by foundation impedance convolution with the frequency-domain function of corresponding input physics quantity (displacement, velocity or acceleration). When the frequency dependency of impedance functions is weak, they can be easily and accurately be transformed to the time domain using the Voigt model (combination of the constant stiffness and the constant damping coefficient). However, due to the effect of inhomogeneity nonlinearity within the soil and the different shape of foundation, usually impedance functions vary quite complicatedly and show strong frequency dependency. This makes the calculation of time-domain foundation resisting force complicated and time-consuming. The available method is simplifying foundation impedance reasonably, and makes it not only can reflect impedance functions frequency dependency but also can be conveniently realized in time-domain.
Currently research on frequency dependent impedance time-domain transform mainly focused on two main subjects: the first is lumped parameter models (mass-spring-dashpot system) approximately fitting to impedance

functions, and the other is proposing fast integral algorithms based on the properties of Fourier inverse transform. Lysmer and Richart (1969) proposed a lumped parameter model consisting of mass, spring and dashpot, from which time-domain analysis of soil-structure interaction was realized. Further improvement and a large number of results were obtained by scholars at home and abroad. Such as Wolf and Somaini (1986) introduced a double DOF model consisting of 5 lumped parameters: 2 masses, 2 dashpots and one spring for embedded foundation; Barros et al (1990) presented another 5 lumped parameters model include one mass, 2 dashpots and 2 springs; Jean et al (1990) developed a 3 DOF 10 lumped parameters model; Luan Maotian et al (1996) showed a 2 DOF 8 lumped parameters model; Based on wolf's second-order models, second-order lumped parameters models of surface rectangle foundation under three conditions were respectively discussed by Hou Xingmin et al (1999), which separately consist of 6 parameters and 2 internal DOF; Jin Feng et al (1999) transformed impedance to frequency independent equivalent spring-damper-mass model approximatively in interested frequency band by two impedance value at the frequency band extreme points and applied in time-domain analysis of dam-foundation interaction; Similarly Wu WH et al (2002) developed a lumped parameters model to availably substitute infinite soil based on polynomial-fraction; Wolf (1989) showed more complicated models. As a whole, due to clearly physics concepts these lumped parameters models have considerable accuracy at interested frequency value and can deal with the nonlinear effect in structure conveniently. At the same time, the common problems they had were that they can only represent actual impedance at one or two frequencies value at most in a narrow frequency band and have difficulty to fit the actual impedance functions in broad frequency band accurately. In fact, lumped parameters models can only reflect the singular component of foundation impedance which is not square integrable and correspond to simultaneous effect. Nevertheless the regular component can't be reflected, which can correspond to time-delay effect and be square integrable. This makes it difficult to guarantee computation accuracy in abroad applicability. It is also the reason of having inconspicuous effect on increasing the fitting accuracy for impedance functions by introducing more parameters. On the fast integral algorithms based on the properties of Fourier inverse transform, Wolf et al. (1985)proposed a method for performing response analyses in the time domain by using the impulse response obtained from the inverse Fourier transform of the soil impedance. Furthermore, Wolf et al.(198) developed a method for obtaining a recursive representation of the convolution integral from the soil impedance; Nakamura N.(2006) proposed a method to transform the frequency dependent impedance to the impulse response in the time domain. The impulse response of this method is formulated considering the terms concerning both the past displacement and velocity. To increasing the fitting accuracy, Yan Junyi (2003) advanced a formulation to represent the interaction force by acceleration impulse response, moreover decomposed acceleration impulse response into a linear function and the a residual function which attenuates to zero. The time-domain difference recursive model proposed by Safak(2006) used for representing time-domain foundation resisting force can simulate the regular component of foundation impedance and under certain condition is accuracy controllable through adjust the number of filer parameter. But the whole essential of foundation impedance can not be reflected in this model because the corresponding filter function has intrinsically limitation at Nyquist frequency, namely the imaginary part of filter must be zero at Nyquist frequency. To increase the fitting accuracy at low frequency, Gaussian weighting functions were used, but this reduce the fitting range and match the impedance faultily at the higher frequency.

In this paper, a computation method of interaction force is discussed by combining lumped-parameter model and time-domain difference recursive filter model, which can take into account the singular and regular component of foundation impedance completely. Then system stability is studied. Based on general FEM software and the proposed time-domain computation model, history analysis programs with the SSDI effect is developed. Numerical examples results demonstrate that the proposed procedure can perfectly fit the frequency dependent impedance in interested frequency band; furthermore the proposed procedure can expediently be incorporated into general FEM software.

2. COMBINATION COMPUTATION METHOD OF INTERACTION FORCE

In frequency domain, the interactive force-displacement relationship between soil and structure can be described as

$$F(\omega) = S(\omega)u(\omega) \tag{1}$$

$S(\omega)$ is foundation impedance function, $F(\omega)$ and $u(\omega)$ are the interactive force and the displacement of the structure, respectively. Generally foundation impedance function $S(\omega)$ can be decomposed completely into the singular and the regular components, showed as follows

$$S(\omega) = S_s(\omega) + S_r(\omega) \tag{2}$$

Where, $S_s(\omega) = K + i\omega C$ is the singular component which is not square integrable; the regular component $S_r(\omega)$ is square integrable.

Based on linearity of the Fourier transform, the relationship between interaction force and displacement of soil-structure can be expressed as

$$F(t) = Ku(t) + C\dot{u}(t) + \int_0^t S_r(t-\tau)u(\tau)d\tau \tag{3}$$

In which, the first two terms on the right-hand side of previous equation correspond to the simultaneous effect and the third term corresponds to time-delay effect.

$$F_d(t) = \int_0^t S_r(t-\tau)u(\tau)d\tau \tag{4}$$

Where $F_d(t)$ is the interaction force caused by time-delay effect.

From above, If we take displacement as input signal and $F_d(t)$ as output signal, and then displacement and $F_d(t)$ make up a linear time-invariant (LTI) system and the regular component $S_r(\omega)$ is the system function. According to digital signal processing knowledge (Oppenheim, 2003), this linear time-invariant (LTI) system is a frequency shaping filter. The frequency shaping filter together with frequency selecting filter are digital filter. Generally digital filter can be discreted as difference equation:

$$y(t) = \sum_{i=0}^{M} b_i x(t - i\Delta t) - \sum_{i=1}^{N} a_i y(t - i\Delta t) \tag{5}$$

Where $x(t)$ and $y(t)$ are the original (i.e. input) and the filtered (i.e. output) signals, respectively; a_i and b_j denote the filter factors, M and N are the filter orders, the parameter t in Eq.(6) is used to denote time, Δt is the sampling interval. On account that the regular part corresponds to time-delay effect and is foreign to the simultaneous effect, let the filter parameter $b_0 = 0$.

By discrete signal Z-transform we can obtain the system function of the filter:

$$H(z) = Y(z)/X(z) = \sum_{i=1}^{M} b_i z^{-i} \Big/ (1 + \sum_{i=1}^{N} a_i z^{-i}) \tag{6}$$

Where: $z = e^{i\omega\Delta t}$.

The system defined by above difference equation is linear time-invariant and causal relation, so the determined output corresponding to the preset input.

From forgoing discussions, impedance functions can be transformed to a spring-dashpot model paralleled by a discrete-time recursive filter. In time-history analysis, the foundation resisting force at the currently step can be computed by the sum of two items which have clearly physics concepts: one is the simultaneous effect showed by spring-dashpot model, and the other is the time-delay effect showed by pseudo-force which is expressed by the terms concerning both the past several displacements and the pseudo-forces. The model can easily realize frequency dependent foundation impedance in time-domain.

After fitting the foundation frequency dependent impedance function $S(\omega)$ in Eq.(2) with $K + i\omega C + H(\omega)$, the impedance function can be completely represented by spring-dashpot model together with the system function of filter in frequency-domain. After obtained the parameters K, C along with filter factor a_j, b_j in $H(\omega) = \dfrac{b_1 z^{-1} + b_2 z^{-2} + \cdots + b_M z^{-M}}{1 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_N z^{-N}}$ (where $z = e^{i\omega\Delta t}$), the foundation resisting force at time t is

$$F(t) = Ku(t) + C\dot{u}(t) + F_d(t) \tag{7}$$

Where the pseudo-force $F_d(t)$ is

$$F_d(t) = -a_1F_d(t-\Delta t) - a_2F_d(t-2\Delta t) - \cdots - a_NF_d(t-N\Delta t) + b_1u(t-\Delta t) + b_2u(t-2\Delta t) + \cdots + b_Mu(t-M\Delta t) \quad (8)$$

Then the determination of recursive parameters K , C , a_j , b_j is the key problem to transform frequency-dependent complex stiffness to time domain. This can be transferred to a error minimization by selecting optimized recursive parameters K, C, a_j, b_j to approximate the impedance function $S(\omega)$.

3. STABILITY CONDITION

The stability condition of LTI system is proverbial the region of converge (ROC) of transfer function $H(z)$ include unit circle $|z|=1$. From the knowledge of digital signal processing (DSP) (Oppenheim, 2003), if all the poles of transfer function $H(z)$ locate in the unit circle, the LTI system stable. Alternatively, if we want to have a stable discrete-time recursive filter (ie. bounded filter output for bounded input), all the roots of the denominator polynomial $1+\sum a_i z^{-i} = 0$ should inside the unit circle in the complex plane. That is

$$|z_k| < 1 \qquad k = 1,2,\cdots,N \quad (9)$$

The roots z_k are known as the poles of the filter, and for stable systems (i.e. no negative damping) they are in complex-conjugate pairs. This condition can be satisfied by introduce a proper penalty factor when calculate the iterative parameters.

4. REALIZATION IN FEM

Thus the calculation of foundation resisting force $F(t)$ can be completed in time-domain after determining the parameters. The key of its application in dynamic time-history analysis turns to pseudo-force calculation by the time-delay coefficients and the past displacement and the past pseudo-force. So it is necessary to save former m steps pseudo-force $F(t)$ and former n steps displacement $u(t)$. This makes it essential to modify the general dynamic history programs. Dynamic equilibrium equation in time-domain substructure method accounting for soil-structure dynamic interaction is

$$\begin{bmatrix} M_s & 0 \\ 0 & M_b \end{bmatrix} \begin{Bmatrix} \ddot{u}_s(t) \\ \ddot{u}_b(t) \end{Bmatrix} + \begin{bmatrix} C_{ss} & C_{sb} \\ C_{bs} & \{c_0\} \end{bmatrix} \begin{Bmatrix} \dot{u}_s(t) \\ \dot{u}_b(t) \end{Bmatrix} + \begin{bmatrix} K_{ss} & K_{sb} \\ K_{bs} & \{k_0\} \end{bmatrix} \begin{Bmatrix} u_s(t) \\ u_b(t) \end{Bmatrix} = -\ddot{x}(t) \begin{Bmatrix} M_s \\ M_b \end{Bmatrix} - \begin{Bmatrix} 0 \\ F_d(t) \end{Bmatrix} \quad (10)$$

Subscript s denotes structure and b denotes rigid foundation, $\ddot{x}(t)$ denotes earthquake acceleration.
The parameters in equation (10) can get from the former fitting results: $c_0 = C$, $k_0 = K$,

$$\{F_d(t)\} = -a_1F_d(t-\Delta t) - a_2F_d(t-2\Delta t) - \cdots - a_NF_d(t-N\Delta t) + b_1u(t-\Delta t) + b_2u(t-2\Delta t) + \cdots + b_Mu(t-M\Delta t) \quad (11)$$

Dynamic equilibrium equation (10) can be solved by time-domain finite-difference method or integral method. The key problem in applying is storage of former pseudo-force $F_d(t)$ and displacement $u(t)$. Based on general FEM software ANSYS and the time-domain difference of interaction force, structure history analysis programs with the effect soil-structure dynamic interaction are developed.

5. NUMERICAL EXAMPLES

Calculation module developed in this paper was applied to analyze a simple example with the time-domain analysis approach accounting for soil-structure dynamic interaction. The foundation was considered as a rigid massless disk-shaped foundation. It was also assumed that the foundation is located on the surface of a double layer soil. We presume that the foundation has the welded contact with under layered soil, schematically shown in Fig.1. The superstructure can be simplified to the model shown by Fig.2.

5.1. DESCRIPTION OF MODEL

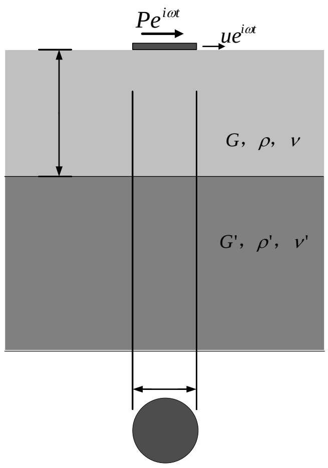


Fig 1.Description of foundation

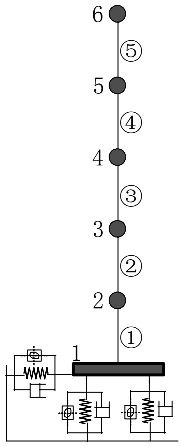


Fig 2. Mechanical model of superstructure

Supposed the groundsill is a soft soil layer on half-space soil. The thickness of the soil covered on the ground is equivalent to the radius of the rigid disk-shaped foundation. The relationship can be expressed as: $H/r_0 = 1$. The material parameters of the soil both top and bottom are: $V/V' = 0.8$, $\rho/\rho' = 0.85$, $\nu = \nu' = 0.25$. The superstructure was made of five lumped masses M and five hollow columns; its height of every storey is L. The interior radius of hollow column is r, the outer radius of the lower three floors is R_1, the upper two floors is R_2. The Young's modulus of superstructure, density are E_p、 ρ_p respectively, damping ratio of materials is 0.05。 The model parameters are shown in Table.1.

Table 1. Parameters in example

ρ/Kg•m^{-3}	v /m•s^{-1}	r_0 /m	ν	E_p /GPa)	ρ_p /Kg•m^{-3}	r /m	R_1 /m	R_2 /m	m /Kg	L / m
1800	100	2.5	0.25	22.5	2556	0.64	0.96	0.88	48000	5

The foundation dynamic impedance function adopted the results in (Luco, 1974) and nondimensionalized by static stiffness. The rigorous analytical solution of the foundation impedance function can be expressed as:

$$K(\omega) = K_0[K_1(a_0) + iC_1(a_0)] \quad (12)$$

Where dimensionless frequency $a_0 = \omega r_0 / V_s$ and K_0 is static stiffness. Horizontal, vertical and rocking static stiffness are respectively:

$$K_0^H = 8Gr_0/(2-\nu) \quad K_0^V = 4Gr_0/(1-\nu) \quad K_0^R = 8Gr_0^{\ 3}/(3(1-\nu)) \quad (13)$$

Where G is Shear modulus of the soil; r_0 is shear wave velocity of the soil; ν is Poisson's ratio of the soil; r_0 denotes the radius of the circular surface foundation.

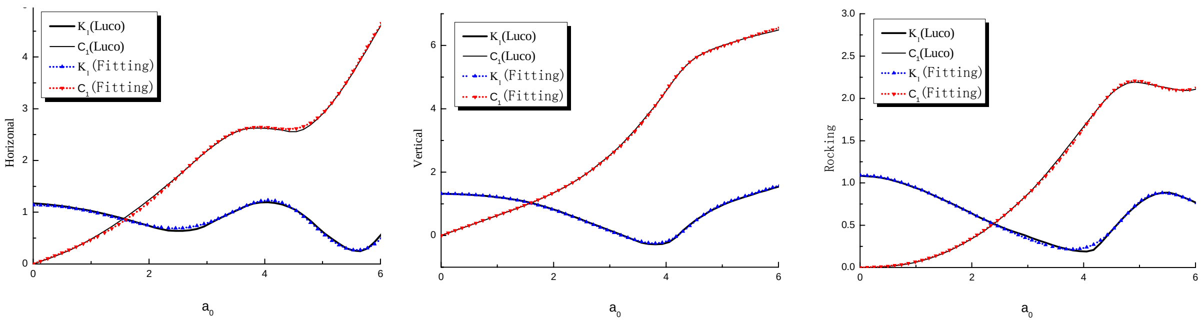
5.2. THE FITTING PARAMETER IN CALCULATION OF TIME-DOMAIN INTERACTION FORCE

Stiffness factors $K_1(a_0)$ and damping factors $C_1(a_0)$ changing with dimensionless frequency a_0 is shown by the solid line in Fig.3. Adopting the computation Matlab program written in (Zhao JF, etc. 2008) and applying the sixth-order filters, we can obtain a series of fitting parameter shown in Table.2. The comparison between impedance function and fitting function in frequency domain is shown in the Fig.3. From Fig.3 we can see sixth-order fitting parameter agrees with the foundation impedance function very well in a wide frequency domain.

Table 2. Sixth-order fitting parameter

Parameter	Horizontal	Vertical	Rocking
K	0.967076108	0.932551948	0.621598354
C	0.016094645	0.024315938	0.009534369
a_1	1.342369456	1.384414061	0.681766503
a_2	0.808201749	1.117937998	0.892367586
a_3	0.375110651	0.641819091	0.168408504
a_4	0.100842318	0.200529189	0.081673619
a_5	-0.007962431	0.024602946	-0.059555536
a_6	-0.007039317	-0.000113300	0.001284421
b_1	-0.109377633	-0.305508737	0.118435652
b_2	0.044598486	0.264487593	0.281896920
b_3	0.037119189	0.869969853	0.480817940
b_4	0.223570463	0.611869927	0.237678400
b_5	0.371025595	0.255624558	0.165381464
b_6	0.006807555	0.000105658	-0.000798394

* Nondimensionalized by foundation static stiffness

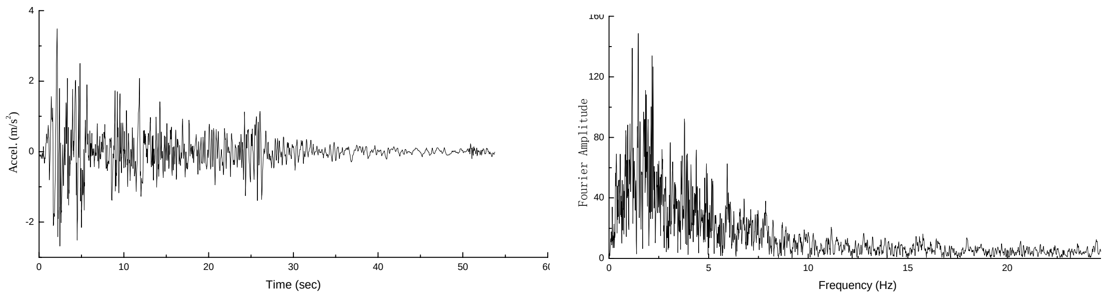


(a) Horizontal (b) Vertical (c) Rocking

Fig 3. Dimensionless foundation impedance function and fitting function

5.3. DYNAMIC RESPONSE OF THE STRUCTURE

Input the 1940 El Centro Site 3-dimensional earthquake wave as the ground acceleration. Two horizontal seismic acceleration peaks were made to 0.3569g and 0.2142g respectively. And vertical seismic acceleration peak as 0.2468g. The two horizontal dominant frequencies were 1.46Hz and 2.14Hz. The vertical dominant frequency was 8.44Hz and the lasting time as 53.78s. Owing to space constraints of this paper, time-domain analysis of seismic acceleration only in the x direction and its Fourier spectrum was introduced. Both of them are shown in Fig.4.



(a) (b)

Fig 4. Ground acceleration and associated Fourier spectra of x direction

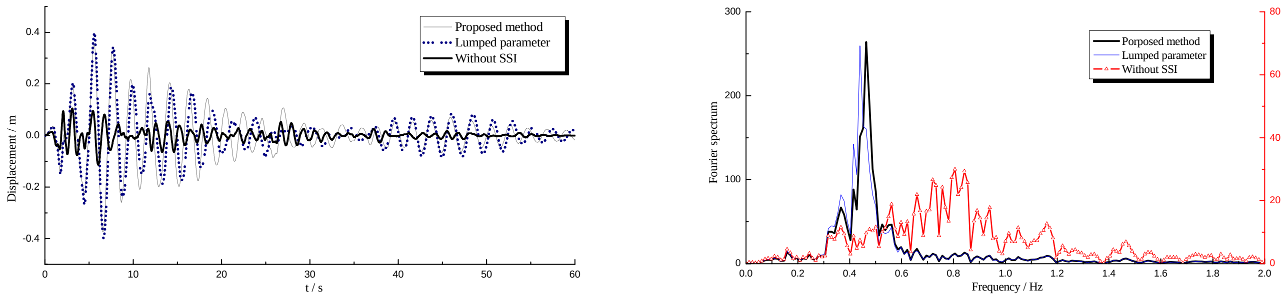
Calculate the dynamic response of every mass respectively under 3-dimensional seismic load and compare them to results of lumped parameter model. Stiffness and damping of lumped parameter model in low frequency domain is shown in Table.3.

Table 3. Stiffness and damping of lumped parameter model in low frequency domain

	Horizonal	Vertical	Rock
Stiffness	1.0617866594	1.2344202364	0.9805762434
Damping	0.0137403991	0.0158374795	0.0026653232

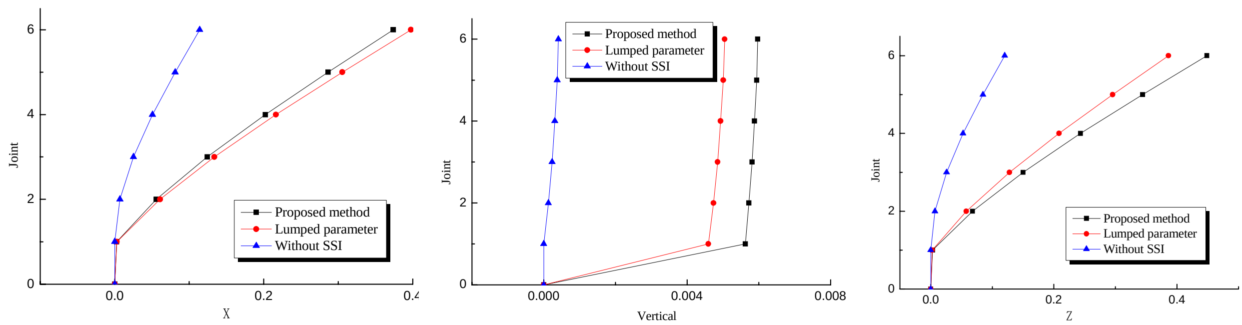
* Nondimensionalized by foundation static stiffness

Relative displacement and associated Fourier spectra of structure top in x direction are shown in Fig.5; Fig.6 is the peak of relative displacement of the mass under 3-dimensions earthquake. From Fig.5 and Fig.6， we know that the dynamic interaction between foundation and structure has a important impact on the dynamic response of the structure. Generally it will increase the flexibility of the structure and magnify the relative displacement of the structure. At the same time it will reduce the natural frequency of the structure. Because both the dominant frequency and the natural frequency of the structure are small, the method applied in this paper has little difference with the spring-dashpot lumped-parameter fitting model in the result. It only in the vertical direction has a certain difference. So frequency's relativity of the impedance function can be ignored and equivalent to spring-dashpot model in low frequency-domain. The relative displacement peak of x direction calculated with the method in this paper is smaller than the result of the spring-dashpot lumped-parameter model, while the relative displacement peak of y and z direction are bigger than the result of the spring-dashpot lumped-parameter model. All of them are connected with the fact that the spring-dashpot lumped-parameter model can't simulate the dynamic foundation impedance function in the whole frequency-domain, the method applied in this paper is not impacted by seismic and structure's features in the analysis of soil-structure dynamic interaction, so it has a higher precision fitting in the whole frequency-domain.



(a) Displacement history (b)Fourier spectrum

Fig 5. Relative displacement and Fourier spectra of structure of x direction



(a) X (b) Vertical (b) Z

Fig.6 Peak of relative displacement under 3-dimensions input

From the numerical result: 1) The calculation method of time-domain interaction force adopted in this paper can account for the frequency dependency of impedance function. 2) Both the nonlinear of the structure and the frequency dependency of impedance can be toke into account. This provides an effective solution method for accurate analysis of the structural dynamic response include SSI. 3) For the low frequency-domain, due to impedance function can be simulated with spring-dashpot model, the approach applied in this paper has less difference with the result of lumped-parameter model, this indicate that lumped-parameter model and the combination calculation method fit well in the low frequency domain.

6. CONCLUSION

This paper realize the calculation of time-domain foundation resisting force accounting for frequency's relativity of the impedance function by introducing virtual force in the finite element software. Based on general FEM software and the time-domain difference of ground resisting force, structure history analysis programs with the effect soil-structure dynamic interaction is developed. Response analysis of multi-storey structure on disk-shaped foundation in the 3-dimensions earthquake input. Numerical example is presented to verify the validity of proposed method. The effect of foundation impedance frequency dependency on the dynamic response of soil-structure system is preliminary discussed.

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