Thawing Quintessence with Nearly Flat Potentials

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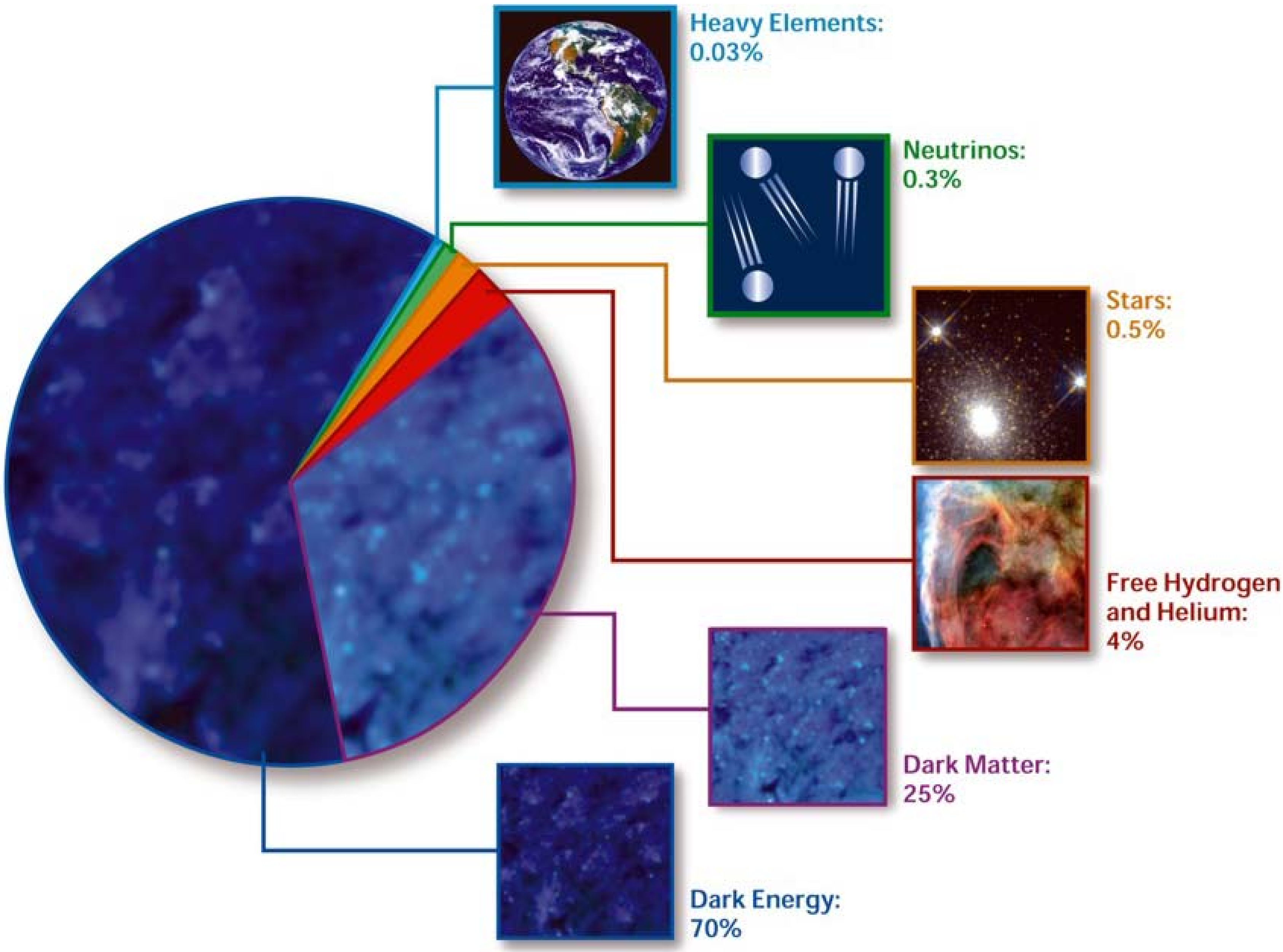
References: **Scherrer and Sen, Phys.Rev.D 77,083515 (2008)**
Scherrer and Sen Phys.Rev.D 78,067303 (2008)
Ali, Sami and Sen Phys.Rev.D 79,123501 (2009)
Sen, Sen and Sami arXiv:0907.2814 (2009).

COMPOSITION OF THE COSMOS



What's the Problem with Cosmological Constant?

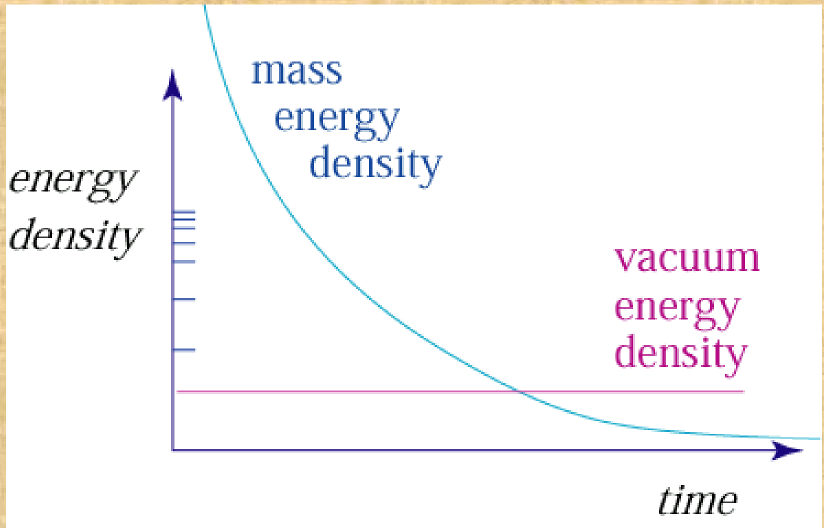
Why not just bring back the cosmological constant (Λ)?
When we calculate how big Λ should be, we don't quite get it right.

$$\frac{\rho_\Lambda(obs)}{\rho_\Lambda(th)} = 10^{-120}$$ → ***Fine Tuning Problem !!***

- ***Why now?***

  $\rho \propto R^{-3}$

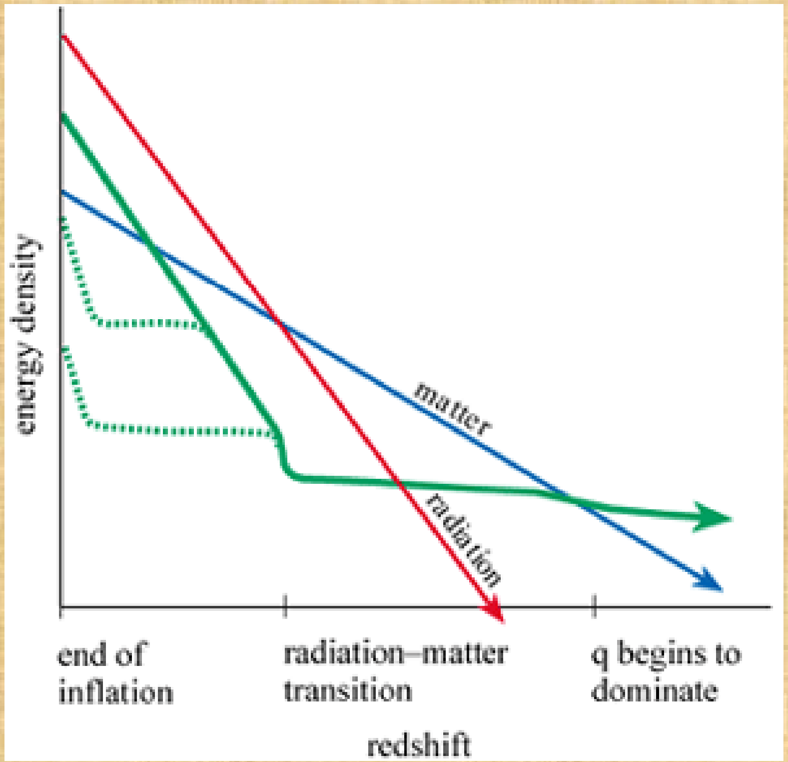
  Vacuum Energy: $\rho \propto constant$



Cosmic Coincidence Problem !!

Quintessence

- Quintessence is another theory for dark energy that involves a dynamic, time-evolving and spatially independent form of energy.
- It makes slightly different predictions for the acceleration



Quintessence

Scalar field ϕ with Lagrangian $\mathcal{L}_\phi = (1/2)(\partial_\mu \phi)^2 - V(\phi)$

Energy density $\rho_\phi = (1/2)\, \phi'^{\,2} + V(\phi)$

Pressure $p_\phi = (1/2)\, \phi'^{\,2} - V(\phi)$

Einstein gravity says gravitating mass ρ+3p, so acceleration if equation of state ratio $w = p/\rho < -1/3$

$$w = (K-V) / (K+V)$$

Potential energy dominates (slow roll): $V >> K \Rightarrow w = -1$

Kinetic energy dominates (fast roll): $K >> V \Rightarrow w = +1$

$$\rho(a) \sim e^{3\int d\ln a\,[1+w(a)]} \sim a^{-3(1+w)}$$

Dynamics important! Value and running: *w*, *w′*, Ω_ϕ

Dynamics of Quintessence

Equation of motion of scalar field

$$\ddot{\phi} + 3H\dot{\phi} = -dV(\phi)/d\phi$$

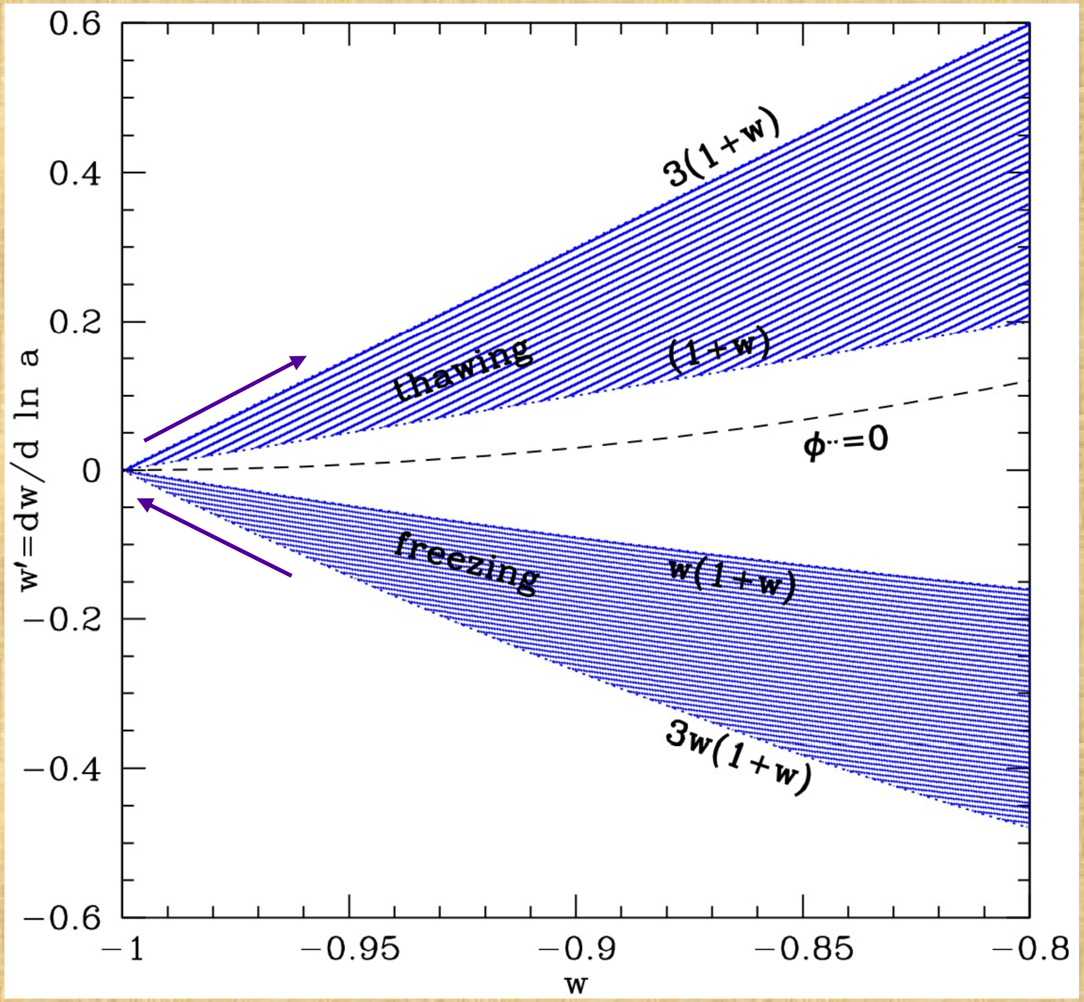
- **driven by steepness of potential**
- **slowed by Hubble friction**

Broad categorization -- which term dominates:

- **field rolls but decelerates as dominates energy**
- **field starts frozen by Hubble drag and then rolls**

Freezers vs. Thawers

Limits of Quintessence



Distinct, narrow regions of w-w′

Caldwell & Linder 2005
PRL; astro-ph/0505494

Thawing Model with Nearly Flat Potential

Let us assume the scalar field with initial
Value ϕ_0 in a nearly flat potential. Specifically we
Assume that the field satisfies the slow-roll
Condition at $\phi = \phi_0$:

$$\left(\frac{1}{V}\frac{dV}{d\phi}\right)^2 \ll 1 \qquad \frac{1}{V}\frac{d^2V}{d\phi^2} \ll 1,$$

One can define variables:

$prime$ → d/dlog(a) $\quad x = \phi'/\sqrt{6} \quad y = \sqrt{V(\phi)/3H^2} \quad \lambda = -\frac{1}{V}\frac{dV}{d\phi}$

With this one can now write:

$$\Omega_\phi = \frac{\rho_\phi}{\rho_c} = x^2 + y^2 \qquad \gamma_\phi \equiv 1 + w_\phi = \frac{2x^2}{x^2+y^2}$$

We can also define: $\Gamma \equiv V\frac{d^2V}{d\phi^2}/\left(\frac{dV}{d\phi}\right)^2$

Scherrer and Sen PRD 2008

Thawing Model with Nearly Flat Potential

One can now construct an autonomous system:

$$\gamma' = -3\gamma(2-\gamma) + \lambda(2-\gamma)\sqrt{3\gamma\Omega_\phi} \qquad \Omega_\phi' = 3(1-\gamma)\Omega_\phi(1-\Omega_\phi)$$

$$\lambda' = -\sqrt{3}\lambda^2(\Gamma - 1)\sqrt{\gamma\Omega_\phi}$$

Assuming

$$\Omega_\phi' \neq 0 \qquad \frac{d\gamma}{d\Omega_\phi} = \frac{\gamma'}{\Omega_\phi'} = \frac{-3\gamma(2-\gamma)+\lambda(2-\gamma)\sqrt{3\gamma\Omega_\phi}}{3(1-\gamma)\Omega_\phi(1-\Omega_\phi)}$$

$$\gamma << 1 \qquad \lambda = \lambda_0 = -(1/V)(dV/d\phi)\Big|_{\phi=\phi_0}$$

$$\frac{d\gamma}{d\Omega_\phi} = -\frac{2\gamma}{\Omega_\phi(1-\Omega_\phi)} + \frac{2}{3}\lambda_0\frac{\sqrt{3\gamma}}{(1-\Omega_\phi)\sqrt{\Omega_\phi}}$$

One can solve this with boundary condition

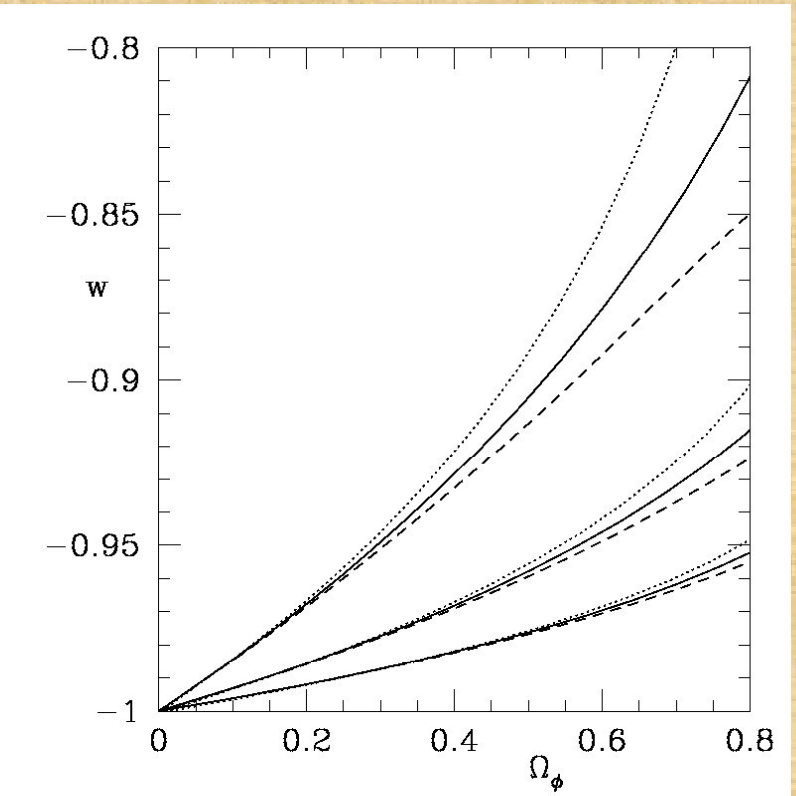
$$\gamma = 0, \quad \Omega_\phi = 0$$

Thawing Model with Nearly Flat Potential

The solution: $\gamma = 1 + w_\phi = \frac{\lambda_0^2}{3}\left[\frac{1}{\sqrt{\Omega_\phi}} - \frac{1}{2}\left(\frac{1}{\Omega_\phi} - 1\right)\ln\left(\frac{1+\sqrt{\Omega_\phi}}{1-\sqrt{\Omega_\phi}}\right)\right]^2$

$V(\phi) = \phi^2, \phi^{-2}$

for dotted and dashed lines. Solid Line is for analytical Approximation. Top to bottom $\lambda_i = 1, 2/3, 1/3$



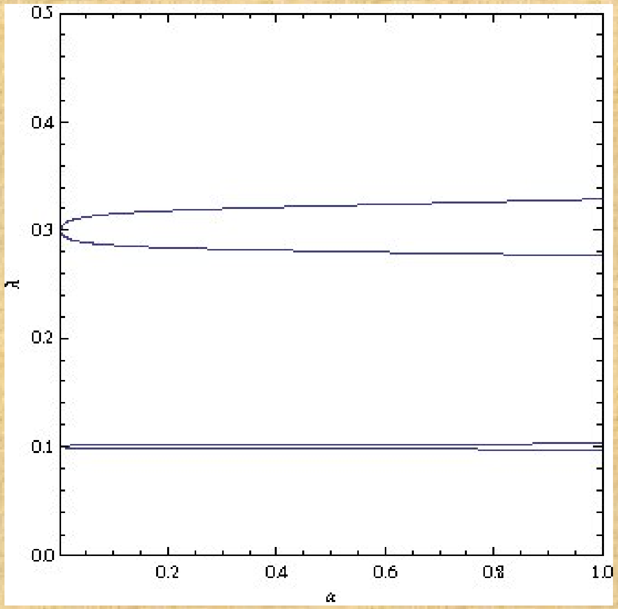
Thawing Model with Nearly Flat Potential

$1+w \sim O(\lambda_0^2)$ The first Slow-Roll condition ensures that $1+w \ll 1$.

One can write

$$\frac{\lambda'}{\lambda} = \frac{1}{V}\frac{d^2V}{d\phi^2} - \left(\frac{1}{V}\frac{dV}{d\phi}\right)^2 \ll 1$$

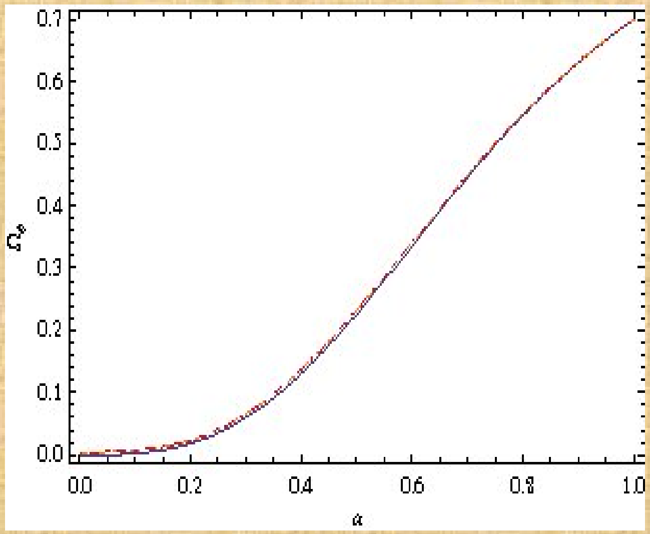
using two slow-roll conditions



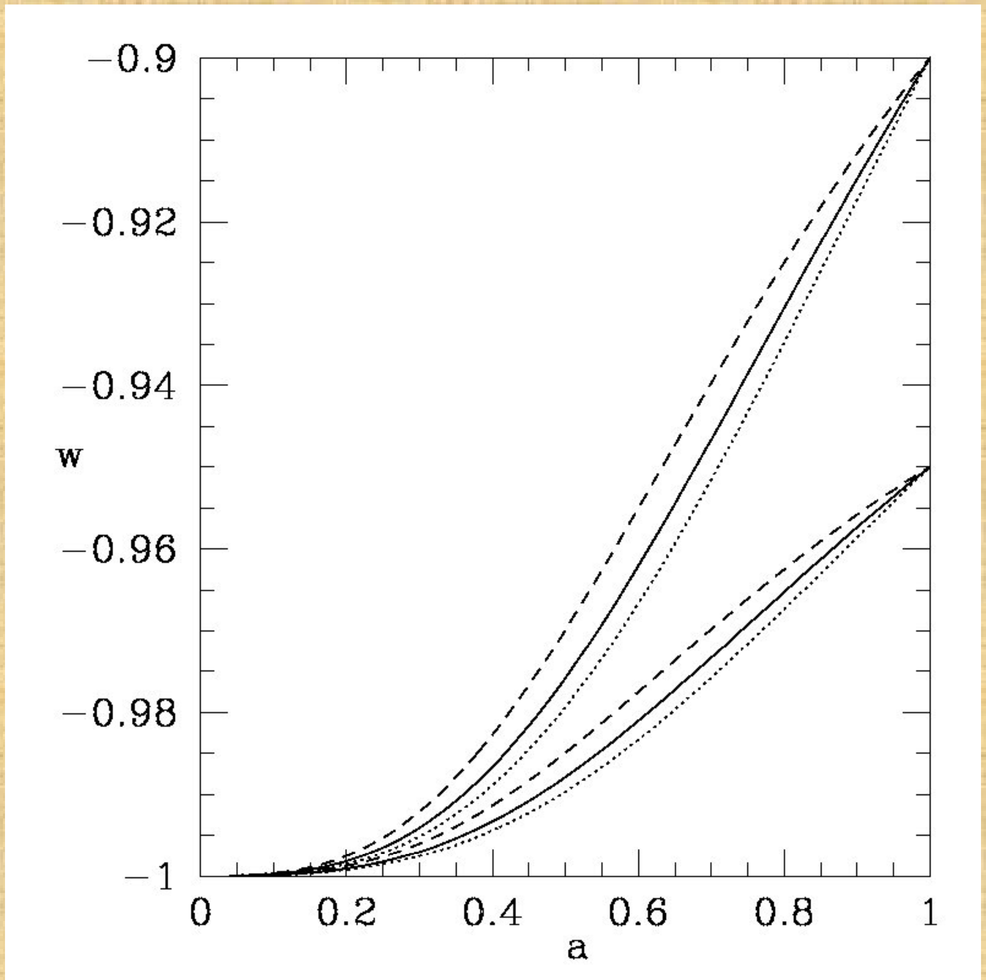
Thawing Model with Nearly Flat Potential

One also should have $\Omega_\phi(a)$. Assuming $\gamma \ll 1$

$$\Omega_\phi = \left[1 + \left(\Omega_0^{-1} - 1\right)(a/a_0)^{-3}\right]^{-1}$$



Thawing Model with Nearly Flat Potential



Thawing Model with Tachyon Field

Action is given by: $S = -\int V(\phi)\sqrt{1-\partial^\mu\phi\partial_\mu\phi}\sqrt{-g}\; d^4x$

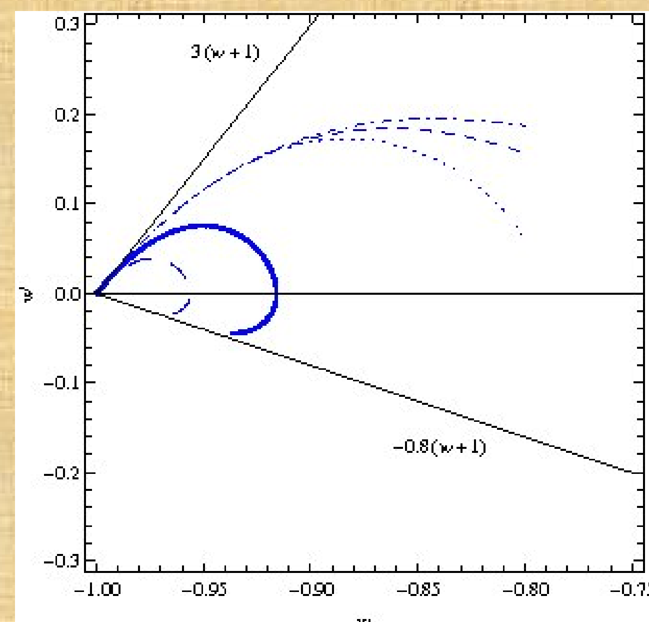
DBI Form

Energy Density $\rho_\phi = \frac{V(\phi}{\sqrt{(1-\dot{\phi}^2)}}$ Pressure $p_\phi = -V(\phi)\sqrt{1-\dot{\phi}^2}$

Equation of state $w = \frac{p_\phi}{\rho_\phi} = -(1-\dot{\phi}^2)$

Equation of motion for φ(t) :

$$\ddot{\phi} + 3H\dot{\phi}(1-\dot{\phi}^2) + \frac{V'}{V}(1-\dot{\phi}^2) = 0$$



Thawing Model with Tachyon Field

Define: $x = H\phi', \ y = \frac{\sqrt{V}}{\sqrt{3}H}, \ \lambda = -\frac{V_\phi}{V^{3/2}}, \ \Gamma = V\frac{V_{\phi\phi}}{V_\phi^2}$

$$\gamma = (1 + \omega_\phi) = x^2$$

$$\Omega_\phi = \frac{y^2}{\sqrt{1-x^2}}$$

Autonomous System :

$$\gamma' = -6\gamma(1-\gamma) + 2\sqrt{3\gamma\Omega_\phi}\lambda(1-\gamma)^{5/4}$$

$$\Omega_\phi' = 3\Omega_\phi(1-\gamma)(1-\Omega_\phi)$$

$$\lambda' = -\sqrt{3}\lambda^2(\Gamma - 3/2)\sqrt{\gamma\Omega_\phi}(1-\gamma)^{1/4}$$

Thawing Model with Tachyon Field

Compare with standard scalar field:

$$\frac{d\gamma}{d\Omega_\phi} = \frac{\gamma'}{\Omega'_\phi} = \frac{-3\gamma(2-\gamma)+\lambda(2-\gamma)\sqrt{3\gamma\Omega_\phi}}{3(1-\gamma)\Omega_\phi(1-\Omega_\phi)}$$

$$\frac{d\gamma}{d\Omega_\phi} = \frac{\gamma'}{\Omega'_\phi} = \frac{-6\gamma(1-\gamma)+2\lambda(1-\gamma)^{5/4}\sqrt{3\gamma\Omega_\phi}}{3(1-\gamma)\Omega_\phi(1-\Omega_\phi)}$$

Standard scalar field **Tachyon type field**

Assumptions:

$\gamma << 1$ $\lambda = \lambda_0 = -(1/V)(dV/d\phi)^{3/2}\Big|_{\phi=\phi_0}$

$$\frac{d\gamma}{d\Omega_\phi} = -\frac{2\gamma}{\Omega_\phi(1-\Omega_\phi)} + \frac{2}{3}\lambda_0\frac{\sqrt{3\gamma}}{(1-\Omega_\phi)\sqrt{\Omega_\phi}}$$

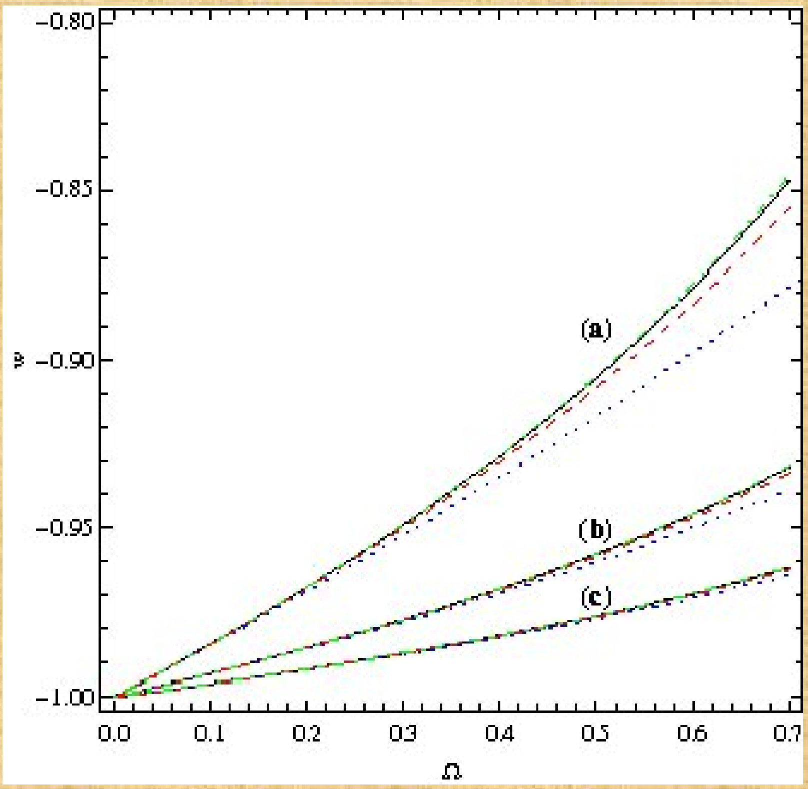
Identical to Standard scalar Field case

Thawing Model with Tachyon Field

$a)\ \lambda = 1$

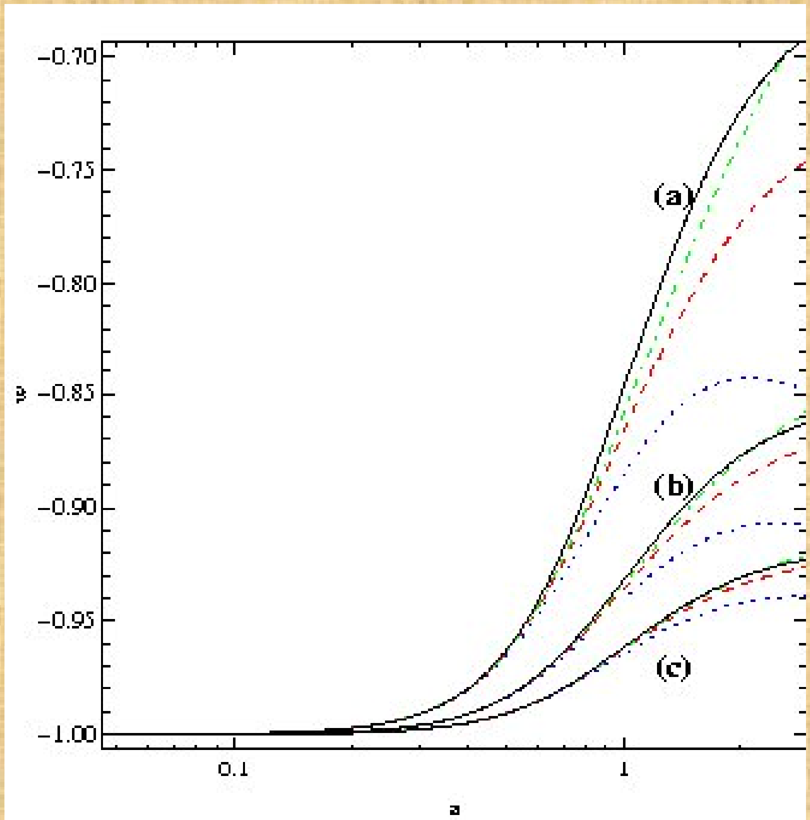
$b)\ \lambda = 2/3$

$c)\ \lambda = 1/2$

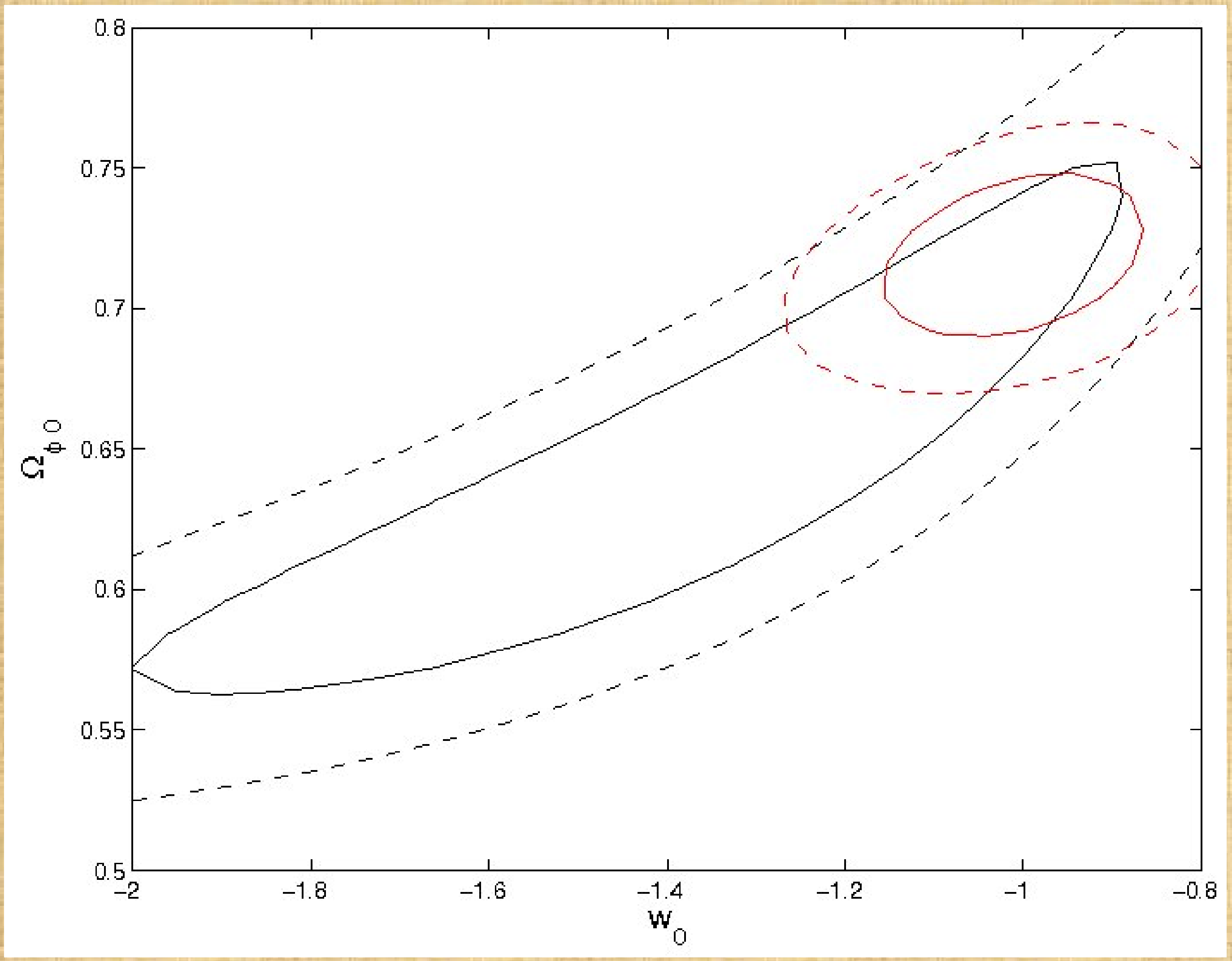


Solid is for approximate result
dot-dashed, dashed, dotted for
$V(\phi) = \phi^{-3}, \phi^{-2}, \phi^{-1}$

Thawing Model with Tachyon Field

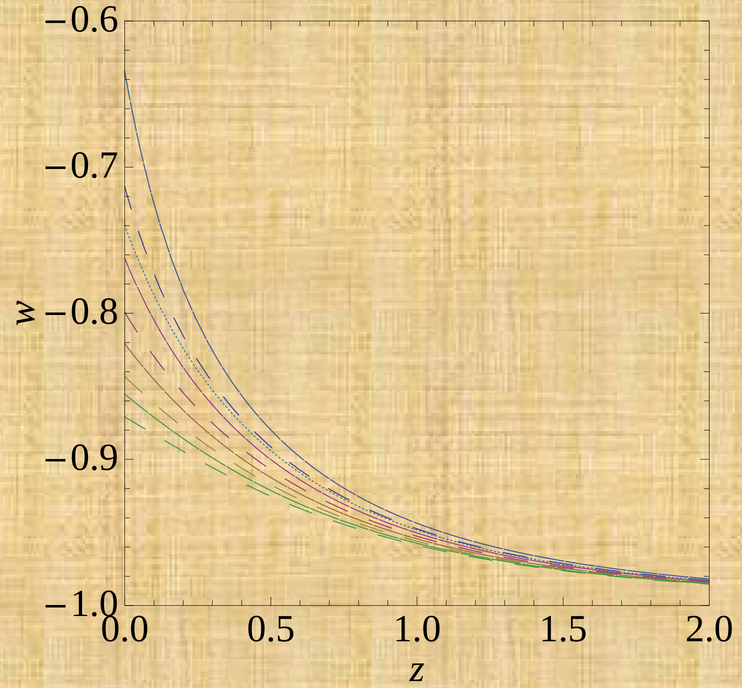


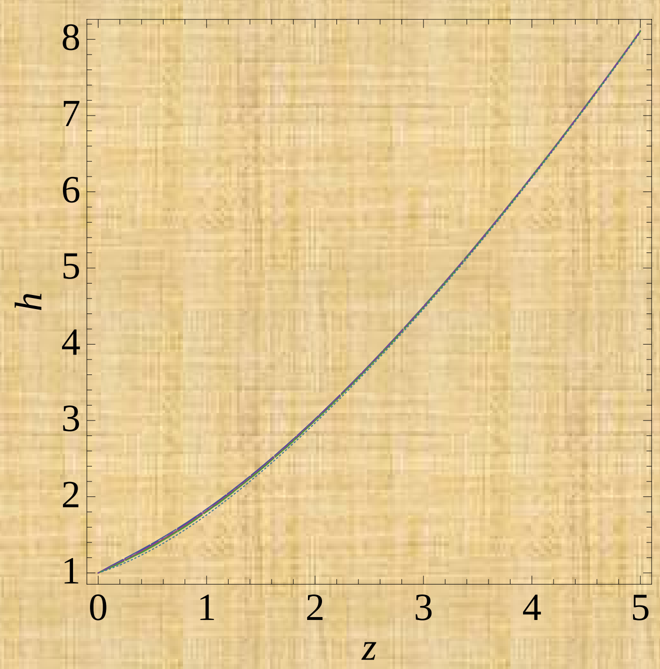
Observational Results



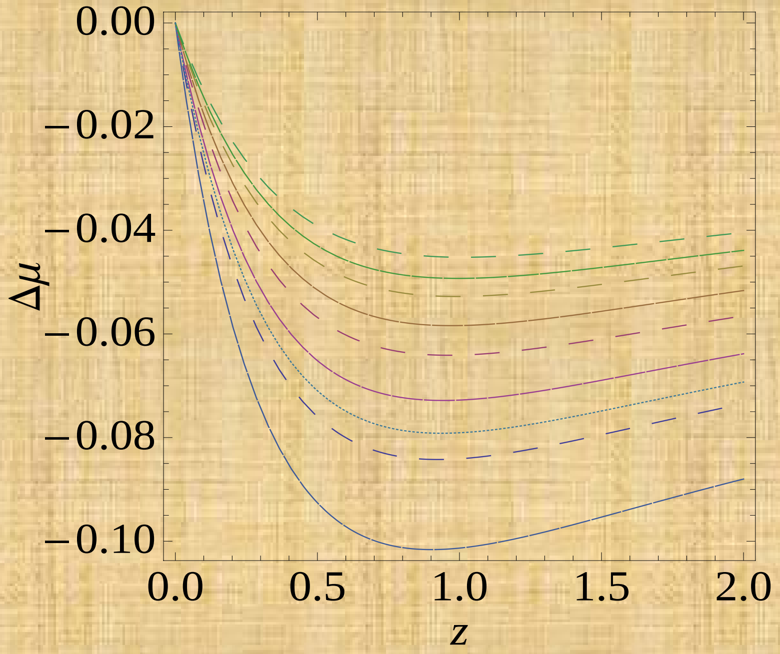
Thawing Model Without Slow-Roll

- Let us assume that $\lambda_i \sim 1$ initially so that the slow-roll condition is not satisfied.





Thawing Models Without Slow-Roll



Error Bars in Constitution Data set for distance modulus range from 0.074 to 1.1 approximately.

Thawing Model Without Slow-Roll

Model	$V(\phi)$	Δ (diff with LCDM)
Scalar field	Φ	-0.023
Scalar field	Φ^2	-0.016
Scalar field	$Exp(\Phi)$	-0.013
Scalar field	Φ^{-2}	-0.0112
Scalar field	PNGB	-0.02156
Tachyon	Φ	-0.033
Tachyon	Φ^2	-0.0195
Tachyon	$Exp(\Phi)$	-0.015
Tachyon	Φ^{-2}	-0.0123

- **For BAO Measurements: the Observational data:**

$$D_V(z=0.35)/D_V(z=0.20) = 1.812 \pm 0.060$$

Conclusions

- **The evidence for a late-time accelerating universe continues to mount as the number of experiments and data grows**.
- **Current observations suggest that either dark energy is exactly C.C and if not very, close to it.**
- **We showed that for generic thawing models with standard canonical type scalar fields or with fields with DBI type kinetic term, under slow-roll conditions, the evolution is unique.**
- **We found this unique e.o.s behaviour, although with current observations one can not distinguish it with C.C.**
- **We also showed that even if do not assume the slow-roll condition thawing models both with quintessence and k-essence type field can not be distinguished with C.C with current errors bars of observational data.**

Thank You