



Indian Institute of Technology, Kanpur

Proposal for a New Course

Course No. : MTH XXXXX

Course title: An Introduction to Geometric Group Theory

Course Objective : Geometric group theory (GGT) is a relatively new and fast emerging area of Mathematics in India. It is devoted to study of finitely generated groups by looking at its action on various geometric objects. GGT got popularized by the seminal work of Mikhael Gromov, in 1987, on Hyperbolic groups. GGT has close interactions with low dimensional topology, hyperbolic geometry, algebraic topology and differential geometry. Some recent developments connect GGT with measure theory and functional analysis. This course is intended to give an exposition of central objects and concepts used in GGT. A student who has basic knowledge of topology, group theory and elementary geometry will be able to grasp the subject.

Prerequisites: Topology (MTH304), Abstract Algebra (MTH 204)

Credits: 3-0-0-0 [9]

Semester: Even Semester

Department/IDP: Mathematics and Statistics

Course Type : Elective Course

Instructor: Abhijit Pal

Course Contents:

Free Groups, Finitely Generated groups, Presentation of a group, Word Metric, Cayley Graphs, Quasi-Isometries, Quasi-isometry invariant, Properly discontinuous and cocompact group action, Svarc-Milnor Lemma, Ends of a Group, Growth of a group, Growth Series. Amalgamated Product of groups, HNN extension, Group acting on a tree, Bass-Serre Tree, FA property.

Hyperbolic Plane, Geodesics & Isometries of Hyperbolic plane, Surface groups, Slim and Thin triangles, Hyperbolic metric spaces, Hyperbolic Groups, Divergence of geodesics, Stability of quasi-geodesics, Algorithmic properties of Hyperbolic Groups : Word Problem, Dehn's Algorithm Conjugacy Problem, Cone Types, Subgroups of Hyperbolic groups, Quasiconvex subgroups, Ping-Pong Lemma. Gromov Boundary of a hyperbolic group.

Tentative Lecture wise Distribution :

Topics	Number of 50 minutes Lectures
Free Groups, Finitely Generated groups, Presentation of a group, Word Metric, Cayley Graphs, Quasi-Isometries, Quasi-isometry invariant	5
Properly discontinuous and cocompact group action, Svarc-Milnor Lemma	2
Ends of a Group, Growth of a group, Growth Series	3
Amalgamated Product of groups, HNN extension, Group acting on a tree, Bass-Serre Tree, FA property	5
Hyperbolic Plane, Geodesics & Isometries of Hyperbolic plane, Surface groups	4
Slim and Thin triangles, Hyperbolic metric spaces, Hyperbolic Groups, Divergence of geodesics, Stability of quasi-geodesics	6
Algorithmic properties of Hyperbolic Groups : Word Problem, Dehn's Algorithm Conjugacy Problem, Cone Types	7
Subgroups of Hyperbolic groups, Quasiconvex subgroups, Ping-Pong Lemma	5
Gromov Boundary of a hyperbolic group	3

Recommended books:

- (1) Metric Spaces of Non-Positive Curvature; Martin R. Bridson, Andre Haefliger; Springer,
- (2) A course on geometric group theory; Brian H. Bowditch, preprint, <http://homepages.warwick.ac.uk/~masgak/papers/bhb-ggtcourse.pdf>
- (3) Topics in Geometric Group Theory; Pierre de la Harpe, Chicago Lectures in Mathematics,
- (4) Fuchsian Groups; Svetlana Katok; The University of Chicago Press,
- (5) Hyperbolic Groups; M. Gromov, Essays in Group Theory Pg. 75- 263.

Students who can enroll: M.Sc & PhD students and a well trained B.S student in Mathematics with required prerequisites can do the course.

Departments to which the proposed course will be of interest: MTH

Dated: 25/09/2024

Proposer: Abhijit Pal

Dated: _____ DPGC Convener: _____

The course is approved / not approved

Chairman, SPGC

Dated: _____