

Indian Institute of Technology Kanpur

Proposal for a New Course

1. **Course No:** ME62X
2. **Course Title:** Mechanics of random elastic media
3. **Per Week Lectures:** 3(L), Tutorial: 0(T), Laboratory: 0(P), Additional Hours [0–2]: 0(A), Credits ($3*L+2*T+P+A$): 9, Duration of Course: Semester
4. **Proposing Department/IDP:** ME
Departments/IDPs which may be interested in the proposed course in the future: MSE, CHE, AE, CE, Mathematics, Statistics and Data Science.
5. **Proposing Instructor(s):** Basant Lal Sharma
6. **Course Description:** Elective for Ph.D., Masters, and senior UG students in mechanical engineering, and other students interested in mechanics, applied mathematics, materials science, and related areas.

(A) Objectives:

Real elastic solids are rarely perfectly homogeneous. Their mass density, elastic moduli, microstructure, geometry, defects, and interfaces exhibit spatial variability. When this variability is modeled probabilistically, the equations of elasticity become stochastic, leading to a mechanics of *random elastic media*. Such models arise in composites, porous and heterogeneous materials, polycrystals, metamaterials, granular media, damaged solids, and materials with uncertain microstructure.

- To introduce the role of randomness and uncertainty in mechanical systems and to distinguish deterministic mechanics from stochastic mechanics and random-media models.
- To introduce random mechanical structures, including random walks, graphs, lattices, point processes, random fields, random tensors, random matrices, and random interfaces.
- To connect microscopic structural randomness and material heterogeneity with macroscopic mechanical response through equilibrium, wave propagation, homogenization, instability, fracture, and reliability concepts.
- To introduce computational and data-driven tools for stochastic mechanics, including Monte Carlo simulation, random-field representations, and physics-informed/data-driven modelling.

The proposed course develops mechanics starting from the equations of elasticity and uses random media as the central theme. This connects probability with elastic equilibrium, wave propagation, homogenization, effective constitutive laws, scattering, localization, and computational mechanics.

(B) Course Content: 40 Lectures

S. No.	Broad Title	Topics	No. Lectures*
1	Introduction to Random Elastic Media	Deterministic versus random elastic media; sources of randomness.	3
2	Random Fields and Microstructure	Random variables and spatial random fields; mean, covariance and correlation; stationarity, ergodicity, homogeneity and isotropy; Gaussian fields and correlation lengths; random inclusions, interfaces, porous media and two-phase microstructures.	6
3	Random Elasticity	Random stiffness and compliance tensors; material symmetry and positive definiteness; isotropic and anisotropic random elasticity; elastic boundary-value problems with random coefficients; statistics of displacement, strain and stress; ensemble averages and fluctuations.	6
4	Effective Elasticity and Homogenization	Volume averaging; effective stiffness and compliance; Voigt, Reuss and Hashin–Shtrikman bounds; RVE and scale separation; periodic and stochastic homogenization; effective constitutive equations and size effects.	7
5	Waves in Random Elastic Media	Random mass–spring chains and discrete elastic media; wave propagation, dispersion and scattering; reflection, transmission and attenuation; coherent and incoherent waves; localization; effective wave equations and long-wave limits.	7

S. No.	Broad Title	Topics	No. Lectures*
6	Random Lattices, Defects and Failure	Random lattices and elastic networks; defects, disorder and stress concentrations; fluctuations and finite-size effects; damage, fracture and fatigue; reliability and weakest-link effects.	6
7	Computational Random Elasticity, Advanced Topics and Applications	Monte Carlo simulation; random microstructure generation; random-field discretization; computational homogenization; Random composites, porous and cellular solids, and polycrystals; inverse characterization; Bayesian and data-driven constitutive modeling.	5

*50-minute lecture each week; the above syllabus is delivered as a block of 40 lecture units.

#No separate tutorial hours are proposed. Worked examples, computational demonstrations, and problem-solving activities are integrated into the lectures.

(C) Prerequisites, if any: Instructor consent; familiarity with basic mechanics, multivariable calculus, differential equations, linear algebra, and elementary probability is desirable.

(D) Short summary for inclusion in the Courses of Study Booklet:

This course develops the mechanics of elastic media with random microstructure, geometry, material properties, or loading. It introduces random fields and correlations and studies their effects on elasticity, waves, and effective properties through homogenization. Topics include random lattices, scattering, localization, fluctuations, defects, and fracture, with computational tools.

7. Recommended Textbooks/References

- (a) Torquato, S. (2002). *Random Heterogeneous Materials: Microstructure and Macroscopic Properties*, Springer.
- (b) Beran, M. J. (1965). *Statistical Continuum Theories*, Interscience/Wiley.
- (c) Mura, Toshio. *Micromechanics of defects in solids*. Springer Science & Business Media, 2013.
- (d) Treloar, L. R. G. (1975). *The Physics of Rubber Elasticity*, 3rd Edition, Oxford University Press.
- (e) Selected recent literature on random lattices, wave scattering, stochastic elasticity, Bayesian methods, and data-driven methods will be provided by the instructor. Example: *Effective dynamics in lattices with random mass perturbations*, arXiv:2309.03090.

8. Any other remarks:

The course is interdisciplinary. It is designed to serve as a common platform connecting solid mechanics, materials, lattice dynamics, computational mechanics, and data-driven modelling. The content can be adapted for a more mathematical or more engineering-oriented offering depending on the background of the enrolled students.

Dated: 19th September 2026

Proposer: Basant Lal Sharma

Dated:

DPGC Convener:

The course is approved/not approved.

Chairman, SUGC/SPGC