

# Indian Institute of Technology, Kanpur

## Proposal for a New Course

**1. Course No:** MTH4XX/5XX (UG course)

**2. Course Title:** Modular Forms

**3. Per Week Lectures:**  $\_3\_ (L)$ , Tutorial:  $\_0\_ (T)$ , Laboratory:  $\_0\_ (P)$ , Additional Hours [0-2]:  $0\_ (A)$ ,

Credits ( $3*L+2*T+P+A$ ):  $9\_ \_$  Duration of Course: Full Semester

**4. Proposing Department/IDP:** Mathematics and Statistics

Other faculty members interested in teaching the proposed course: Somnath Jha, Saurabh Kumar Singh.

**5. Proposing Instructor:** Sudhanshu Shekhar

### 6. Course Description:

A) Objectives: This course develops the analytic theory of modular forms. Students study the modular group and its subgroups, the fundamental domain, modular forms of integral weight, Eisenstein series, cusp forms, the valence (dimension) formula, Petersson inner product, Hecke operators, and L-functions, including analytic continuation and functional equation.

B) Contents:

S. No	Broad Title	Topics	No. of Lectures
1	The Modular Group	$SL(2, Z)$ ; generators and relations; action on upper half-plane; fundamental domain.	4

2	Congruence Subgroups	$\Gamma(N)$ , $\Gamma_0(N)$ ; cusps; quotient spaces.	3
3	Modular Forms	Definition, transformation law; growth at cusps; $q$ -expansion.	4
4	Eisenstein Series	Definition, convergence; Fourier expansion; classical examples.	4
5	Cusp Forms	Definition, examples, and structure of spaces of modular forms.	4
6	Valence and Dimension Formula	Valence formula; dimension formula for $M_k(\mathrm{SL}(2, \mathbb{Z}))$ .	4
7	Petersson Inner Product	Definition, orthogonality relations; basic properties.	3
8	Hecke Operators	Double cosets; Hecke algebra; eigenforms; multiplicativity of Fourier coefficients.	6
9	L-functions	Dirichlet series attached to eigenforms; analytic continuation; functional equation.	8

C) Pre-requisites: Instructor's consent.

D) Short summary for inclusion in the Courses of Study Booklet

#### Course Description

This course introduces the analytic theory of modular forms. Students study the modular group, the fundamental domain, Eisenstein series, cusp forms, the dimension formula, the Petersson inner product, Hecke operators, and associated L-functions, including their analytic continuation and functional equation.

**7. Recommended books:**

1. F. Diamond and J. Shurman, A First Course in Modular Forms, Springer.
2. T. Miyake, Modular Forms, Springer.
3. S. Lang, Introduction to Modular Forms, Springer.
4. J.S. Milne, Modular Functions and Modular Forms (online notes).

**8. Any other remarks:**

Dated: \_\_\_23/02/26\_\_\_\_\_ Proposer: \_\_Sudhanshu  
Shekhar\_\_\_\_\_

Dated: \_\_\_\_\_ DUGC/DPGC Convener: \_\_\_\_\_

The course is approved / not approved

Chairman, SUGC/SPGC

Dated: \_\_\_\_\_