

Indian Institute of Technology Kanpur
Department of Mathematics
Proposal for New Course

Title: Stochastic Analysis and Application to PDEs

Course No.: MTH6XX

Credits: 3-0-0

Semester: Any, **Duration:** Full semester

Department: Mathematics and Statistics

Proposer: Indranil Chowdhury

Prerequisite: Basics of measure theory and probability, MTH424, Instructor consent

Other faculty members interested in teaching the proposed course: Mrinmay Biswas

Total lectures: 39–40, 1 lecture = 50 minutes

Course Objective

This course develops stochastic analysis as a mathematical method for studying local and nonlocal partial differential equations. Probability, martingales, Brownian motion, Itô integration and SDEs provide the foundations for diffusion generators, stopped processes, Dynkin's formula, Kolmogorov equations and Feynman–Kac representations. An introductory treatment of Lévy processes connects jump generators to integro-differential equations and the fractional Laplacian. The emphasis is on precise regularity and integrability hypotheses, selected proofs, and stochastic representations of initial and boundary/exterior value problems.

Syllabus and Lecture Allocation

The allocation assumes **40 teaching sessions of 50 minutes** (33 hours 20 minutes). Counts are indicative; worked examples and consolidation are included within the topic allocations.

Module / Topic	Course Content	Lectures
1. Review of measure theory and probability	σ-algebras, measures and signed measures, Lebesgue integration, convergence theorems and L^1/L^2 estimates. review of Radon–Nikodym and Fubini–Tonelli theorems. - Review of probability spaces, random variables, distributions, expectation, conditional expectation: definition, L^2 projection interpretation, existence, uniqueness and basic properties.	4
2. Processes, martingales and stopping times	- Stochastic processes and finite-dimensional distributions. - Filtrations and adapted processes. Discrete- and continuous-time martingales, examples. - Stopping times and stopped processes, examples.	3
3. Random walks and Brownian motion	- Random-walk motivation; Brownian motion. - Gaussian finite-dimensional distributions and existence via the Lévy construction with selected details. Continuity, scaling and quadratic variation. - Brownian martingales, Markov property and an introduction to the strong Markov property, including Brownian motion in $\mathbb{R}^n$.	6
4. Itô integration and stochastic calculus	- Construction for elementary adapted integrands and extension in L^2; Itô isometry. - Martingale and continuity properties; stopped integrals and the localization needed subsequently. - Itô processes and Itô's formula in one and several dimensions; cross variations and product rule. - Explicit calculations. Itô–Stratonovich comparison may be included briefly if time permits.	8

Syllabus and Lecture Allocation (continued)

Module / Topic	Course Content	Lectures
5. Stochastic differential equations	- Strong and weak solutions: definitions and examples. - Strong existence and pathwise uniqueness under global Lipschitz and linear-growth assumptions; Picard iteration, moment estimates and stability with respect to initial data. - Linear SDEs and the integrating-factor method, including the Ornstein–Uhlenbeck process. - Diffusion solution families and their Markov property.	5
6. Diffusion processes and classical PDE connections	- Transition semigroups, generators and stopped Dynkin’s formula. - Stochastic representations for Dirichlet/Poisson problems, exit probabilities and expected exit times. - Kolmogorov backward equation; weak forward evolution of diffusion laws and the Fokker–Planck density equation under additional hypotheses. - Feynman–Kac with forcing and potential terms. Verification, uniqueness, positivity and comparison for suitable classical solutions; resolvents as time permits.	7-8
7. Lévy processes, generators and nonlocal PDEs	- Poisson and compound Poisson processes: characteristic functions, semigroups and generator derivations. - General Lévy processes, Lévy measures and the Lévy–Khintchine formula; integral generators and small-jump compensation. - Isotropic symmetric stable processes, the fractional Laplacian and fractional heat equations via Fourier transforms and semigroups. - Jump exits and overshoot; exterior-value problems, selected maximum principles and stochastic representations.	6
Total (indicative allocation)		39-40

References

Readings will be drawn from selected chapters and sections of the following references. Øksendal is the main introductory text; the other references support the diffusion–PDE and nonlocal blocks.

1. B. Øksendal. *Stochastic Differential Equations: An Introduction with Applications*. 6th edition. Springer, 2003.
2. R. F. Bass. *Diffusions and Elliptic Operators*. Springer, 1998.
3. D. Applebaum. *Lévy Processes and Stochastic Calculus*. 2nd edition. Cambridge University Press, 2009.
4. R. Cont and P. Tankov, *Financial Modelling with Jump Processes*, Chapman & Hall/CRC, 2004.
5. Krzysztof Bogdan, Tomasz Byczkowski, Tadeusz Kulczycki, Michał Ryznar, Renming Song, and Zoran Vondracek. *Potential Analysis of Stable Processes and its Extensions*. Lecture Notes in Mathematics, Vol. 1980. Springer, 2009.
6. X. Ros-Oton. “Nonlocal elliptic equations in bounded domains: a survey.” *Publicacions Matemàtiques* **60**(1) (2016), 3–26.

Date: 07.10.2026

Signature of the Proposer: _____

Indranil Chowdhury

Signature of DPGC Convener: _____

Course Approved / Not Approved _____

Signature of Chairman SPGC _____