Indian Institute of Technology Kanpur
Department of Mathematics

Proposal for New Course

Title:	Regularity Theory for Elliptic PDEs
Course No.:	MTH7XX
Credits:	3-0-0-0(9)
Semester:	Any
Duration:	Full semester
Department:	Mathematics
Proposer:	Vedansh Arya
Prerequisites:	Desirable: Contents of MTH301, MTH404, MTH424, and instructor consent
Other faculty members interested in teaching the proposed course:	Kaushik Bal, Indranil Chowdhury
Total lectures:	40, 1 lecture = 50 minutes

Course Objective

This is an advanced course on the regularity theory of elliptic partial differential equations. The course develops the principal analytical techniques used to obtain qualitative and quantitative regularity estimates for weak and viscosity solutions.

The course begins with a review of Sobolev spaces, weak derivatives, weak solutions, and fundamental inequalities, followed by the study of H^2-regularity for elliptic equations. The classical Schauder theory is then introduced, providing regularity estimates in Hölder spaces.

We next develop the De Giorgi–Nash–Moser theory for uniformly elliptic equations in divergence form. The course emphasizes Caccioppoli inequalities, De Giorgi and Moser iteration, measure-to-point estimates, oscillation decay, Hölder continuity, and Harnack inequalities. The connection with Hilbert's 19th problem is also discussed.

The final part of the course is devoted to the Krylov–Safonov theory for uniformly elliptic in non-divergence form. Topics include viscosity solutions, the Aleksandrov–Bakelman–Pucci estimate, the Krylov–Safonov measure estimate, Hölder regularity, and Harnack inequalities. If time permits, we will also discuss analogous results for parabolic PDEs.

Syllabus and Lecture Allocation

The allocation assumes 40 teaching sessions of 50 minutes each (33 hours 20 minutes). Counts are indicative; worked examples, proofs, and consolidation are included within the topic allocations.

Module / Topic	Course Content	Lectures
1. Review of Sobolev spaces and weak solutions	• Weak derivatives and Sobolev spaces $W^{k,p}$, H^k. • Approximation by smooth functions and density results. • Poincaré and Sobolev inequalities. • Morrey's inequality and embeddings. • Weak formulations of elliptic equations.	4-5
2. H^2-regularity for elliptic equations	• Difference quotient method. • H^2-estimates for Poisson's equation.	4
3. Schauder theory for elliptic equations	• Hölder spaces and basic properties. • Schauder estimates for variable-coefficient elliptic equations.	6
4. De Giorgi–Nash–Moser theory: elliptic equations	• Connection to Hilbert's 19th problem. • Caccioppoli inequality and local boundedness. • De Giorgi iteration. • Measure-to-point estimates and oscillation decay. • Hölder continuity of weak solutions. • Moser iteration and local boundedness. • Harnack inequality for uniformly elliptic equations in divergence form.	9
6. Krylov–Safonov theory	• Uniformly elliptic operators in non-divergence form. • Viscosity solutions. • Maximum principles and the Aleksandrov–Bakelman–Pucci estimate. • Krylov–Safonov measure estimate. • Weak Harnack inequality. • Oscillation decay and Hölder continuity. • Harnack inequality for non-divergence form elliptic equations.	12
5. Regularity theory for parabolic equations	• Parabolic Sobolev spaces and weak solutions. • Parabolic cylinders and natural scaling. • Parabolic Sobolev inequalities. • De Giorgi iteration for parabolic equations. • Hölder continuity and Harnack inequalities for parabolic equations.	4-5

Total: 40 lectures

References

1. L. A. Caffarelli and X. Cabré, *Fully Nonlinear Elliptic Equations*, American Mathematical Society, 1995.

2. L. C. Evans, *Partial Differential Equations*, 2nd edition, American Mathematical Society, 2010.

3. Xavier Fernández-Real and Xavier Ros-Oton, *Regularity Theory for Elliptic PDE*, DOI:10.4171/ZLAM/28, 2022.

4. D. Gilbarg and N. S. Trudinger, *Elliptic Partial Differential Equations of Second Order*, Springer.

5. C. Imbert and L. Silvestre, *An Introduction to Fully Nonlinear Parabolic Equations*, in *An Introduction to the Kähler–Ricci Flow*, Lecture Notes in Mathematics, Vol. 2086, Springer, 2013, pp. 7–88.

6. N. V. Krylov, *Lectures on Elliptic and Parabolic Equations in Hölder Spaces*, American Mathematical Society.

Date: November 2, 2026 Signature of the Proposer: Vedansh Arya

Signature of DPGC Convener: _____

Course Approved / Not Approved

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Signature of Chairman SPGC