Indian Institute of Technology, Kanpur

Proposal for a New Course

Course No.: MTH6XX (PG Level)

Course title: Theory of Hardy Spaces

Pre-requisite(s): Instructor's Consent (knowledge of complex analysis and functional analysis, is desirable)

Credits: L-T-P-A [C] -- 3-0-0-0 [9]

Semester: Even

Department: Mathematics and Statistics

Proposer: Muthukumar P

Other faculty members interested in teaching the proposed course:

Prof. Parasar Mohanty, Prof. Sameer Chavan.

Course Objectives:

The study of Hardy spaces lies at the interface of complex analysis and functional analysis. One of the attractive features of this subject is that the prerequisites are minimal. Hardy spaces have connections to various mathematical fields, including complex analysis, operator theory and harmonic analysis.

This course mainly focuses on the study of Hardy spaces on the open unit disk of the complex plane and its various applications in diverse fields. Theory of Hardy spaces considered to be the starting point in the study of Banach spaces of analytic functions and operators therein. This course will introduce new research direction to our PhD/Master students.

Course Contents:

Topics	No. of Lectures
Hardy Hilbert Space: Review of Analytic Functions – Hardy Space H^2 – Examples – Basic Properties – Growth Estimate – Consequences – Reproducing Kernels – H^2 via Integral Means – Hardy Algebra H^∞.	05
Hardy Spaces H^p: Subharmonic Functions – Monotonic Integral Means – Harmonic Hardy Space h^1 – Boundary Behavior of Poisson-Stieltjes Integrals – Hardy Spaces H^p – Completeness – Geometric Properties.	06

Basic Structure: Nevanlinna Class – Radial Limit – Infinite Products – Blaschke Product – Riesz Factorization Theorem – Mean Convergence to Boundary Values – Inner Function – Outer Function – Singular Inner Function – Canonical Factorization Theorem – Harmonic Majorants.	09
Operators on H^p: Toeplitz Operators – Hankel Operators – Composition Operators – Multiplication Operators – Basic Properties.	06
Applications: Beurling Theorem – Müntz-Szász Theorem – Poisson Integrals and H^1 – H^p Spaces on Unit Circle – Fourier Coefficients – Cauchy-Stieltjes Integrals – F. and M. Riesz Theorem – Characterization of Absolutely Continuous Functions – Rectifiable Jordan Curve – Univalent Functions.	09
Further Reading: Hardy Spaces over Half Plane – Hardy Spaces of General Domains in Complex Plane – Hardy Space on Polydisk and Unit Ball in C^n – Other Analytic/Harmonic Function Spaces – Research Problems.	06
Total Lectures:	41

Text/Reference books:

1. Nikolaï Nikolski, Hardy Spaces, Cambridge University Press, 2019.
2. Ruben A. Martinez-Avendano, Peter Rosenthal, An Introduction to Operators on the Hardy-Hilbert Space, Springer New York, 2010.
3. Peter L. Duren, Theory of H^p Spaces, Academic Press, New York and London, 1987.
4. Ronald G. Douglas, Banach Algebra Techniques in Operator Theory, Springer New York, 2012.
5. Kehe Zhu, Operator Theory in Function Spaces, American Mathematical Society, 2007.
6. Javad Mashreghi (Editor), Lectures on analytic function spaces and their applications, Fields Institute Monographs 39, Springer, 2023.

Dated: 16, May 2025 Proposer: Muthukumar

(Muthukumar P)

Dated: _____ DPGC Convener: _____

Dated: _____ The course is approved / not approved

Chairman, SPGC