

Indian Institute of Technology Kanpur

Ph.D. (Economics) Admission Test – Sample Questions

Candidates with Economics Background

1. Suppose that Apple (US company) sells an iPhone in the US retail market for \$475 where Apple's intermediate production cost is \$385. Of these intermediate costs, \$100 was wholly produced in US and rest was produced by foreign producers. Suppose the production chain is: China sells microchip for \$160 to Japan, Japan adds other parts and sells to Apple for \$285. Using the production (value-added approach), the GDP in US is (a) and in Japan is (b)

2. Suppose the goods market is given by:

$$Y = 0.75 * (Y - \tau Y) + G$$

where $\tau = 0.2$ is the proportional tax rate that the government levies, Y is output and G is government expenditure.

Moreover, government budget deficit (expenditure minus revenue) is defined as:

$$Deficit = G - \tau Y$$

Initially the economy is at steady-state where $Y=100$, $G=40$, and $Deficit=20$. Suppose government increases its expenditures to 60, the new steady-state budget Deficit using the concept of Keynesian multipliers will be

3. The production function of a certain country is given by:

$$Y = K + 2N$$

where Y is output, K is capital and N is labour. The break-even investment function is $(\delta + n)k$ where δ is the depreciation rate of capital, n is the growth rate of labour and $k = \frac{K}{N}$ is capital per worker. Assume that both the depreciation rate and rate of growth of labour is 10% each, and saving rate is 10%. The steady state level of capital per worker will be

4. Let us say we live in a two-commodity world, where the only two goods are x and y . Let us think of a consumer who has a money income M , and faces prices p_x and p_y . Consider the following utility functions and answer the questions given below:

i. $u(x, y) = \min\{x, y, x + y\}$

ii. $u(x, y) = x^2 + y^2$

(ai) The demand function for the utility function $u(x, y) = \min\{x, y, x + y\}$ is given by $x = \dots$ and $y = \dots$

(Write the answer in the grid as $x = \dots$, $y = \dots$; both x and y should be correct to get full credit.)

(aii) The demand function for the utility function $u(x, y) = x^2 + y^2$ is given by $x = \dots$ and $y = \dots$.
(Write the answer in the grid as $x = \dots, y = \dots$; both x and y should be correct to get full credit.)

b. Which of the utility function(s) above violate(s) the convexity assumption? (Consider the weakest version of the convexity assumption in answering this question.)

Mention (i)/(ii)/both/ none to indicate the utility function(s).

5. The long-run cost function of each firm that supplies Q in a perfectly competitive market is $C = q^3 - 4q^2 + 8q$. Assume that the market demand function is $D = 2000 - 100p$.

a. The aggregate quantity in equilibrium is

b. The number of firms are

6. A family of short-run total cost curves is generated by $C = 0.04q^3 - 0.9q^2 + (10 - \ln k)q + 8k^2$, where $k > 1$ denotes firm size and q denotes the level of output. The firm's long-run total cost curve is

(Write down the expression that you obtain. You do not need to simplify it.)

7. The solution for the optimization problem $\max_{x,y} -2(x - 3)^2 - 4(y - 6)^4$ subject to the constraints $x, y \geq 0$ and $5x + 10y \leq 100$ is given by $x = \dots$ and $y = \dots$

(Write the answer in the grid as $x = \dots, y = \dots$; both x and y should be correct to get full credit.)

8. The derivative of $y(x) = 2^x + x^2$ with respect to x is

9. The highest value that the function $f(x) = x^3 - 2x^2 + x$ obtains on the interval $[0, 2]$ is

10. A student appears in the examinations of four subjects Biology, Chemistry, Physics and Mathematics. Suppose that the probabilities of the student clearing examinations in these subjects are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ and $\frac{1}{5}$ respectively. Assuming that the performances of the student in four subjects are independent:

(a) the probability that the student will clear examination(s) of all the subjects is
.....(answer in fraction).

(b) the probability that the student will clear the examination of at least one subject is
.....(answer in fraction).

11. Let X be a random variable with distribution function

$$F_X(x) = \begin{cases} 0, & \text{if } x < 0 \\ \frac{2}{3}, & \text{if } 0 \leq x < 1 \\ \frac{7-6c}{6}, & \text{if } 1 \leq x < 2 \\ \frac{4c^2-9c+6}{4}, & \text{if } 2 \leq x \leq 3 \\ 1, & \text{if } x > 3 \end{cases}$$

where c is a real constant.

The value of constant c is (answer in fraction).

12. From a sample of 10 observations, the following results were obtained:

$$\begin{aligned} \sum Y_i &= 1,110 & \sum X_i &= 1,700 & \sum X_i Y_i &= 205,500 \\ \sum X_i^2 &= 322,000 & \sum Y_i^2 &= 132,100 \end{aligned}$$

with coefficient of correlation $r = 0.9758$. But on rechecking these calculations it was found that two pairs of observations were recorded:

Instead of

Y	X
90	120
140	220

Y	X
80	110
150	210

The correct r should be(up to two decimal places).

Candidates with Non-Economics Background

1. Consider two random variables X and Y having a uniform joint probability on pairs of nonnegative integers that satisfy the condition $0 \leq X + Y \leq 2$. Compute the correlation coefficient $\rho(X, Y)$.
2. The completion of a construction job may be delayed because of a strike. It is known that there will be a strike with probability p , the construction job will be completed on time with probability p_1 if there is no strike, and the construction job will be completed on time with probability $p_2 < p_1$ if there is a strike. Given that the construction job was completed on time, what is the probability that there was a strike? Write your answer in terms of p , p_1 , and p_2 .

3. Compute the number of conditions to be satisfied for k events to be independent.

(Hint: Two events A and B are independent if $P(A)P(B) = P(A \cap B)$. So only one condition needs to be satisfied. Three events A , B , and C are independent if (i) $P(A)P(B) = P(A \cap B)$, (ii) $P(B)P(C) = P(B \cap C)$, (iii) $P(C)P(A) = P(C \cap A)$, and (iv) $P(A)P(B)P(C) = P(A \cap B \cap C)$. So four conditions need to be satisfied. You are expected to compute the expression for the number of conditions for a general value k .)

4. Consider a random variable X having a uniform density function in the interval $[\alpha, \beta]$. Let the mean of X , $\mathbb{E}[X] = \mu$. Compute

(a) $\mathbb{E}[(X - \mu)^3]$.

(b) $\mathbb{E}[(X - \mu)^5]$.

5. Consider the matrix $A = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$. It is known that $|A| = 0.5$. Compute

(a) the determinant of $2A^2$.

(b) the determinant of $\begin{pmatrix} a+g & b+h & c+i \\ 3d+a & 3e+b & 3f+c \\ 2g+5d & 2h+5e & 2i+5f \end{pmatrix}$.

6. Evaluate the following integrals:

(a) $\int \frac{1}{x \log(x)} dx$

(b) $\int \frac{1}{x \log^2(x)} dx$

(c) $\int x \cdot 2^x dx$

(d) $\int_0^1 x(1-x)^n dx$

(e) $\int_0^{\infty} xe^{-x^2} dx$

7. Compute the maximum of $x_1^3 + x_2^3$ if (x_1, x_2) belongs to the set $\{x_1 + x_2 = 1\}$. What is the minimum?
8. Consider the curve $xy = 1$. Compute the slope of the curve at the point $(x_0, \frac{1}{x_0})$.
9. A U.S. Pharmaceuticals firm sells its patent-protected drug Levemir in the U.S. market. The domestic demand function is given by $Q_{US} = 120 - 2P_{US}$, where price is measured in U.S. dollars and quantity is measured in vials. The firm's marginal cost is $MC = \$10$. Find out firms' optimal price and number of vials sold in the markets.
10. Define the following terms: (i) GDP, (ii) inflation, (iii) money supply, (iv) price index, (v) VAT.
11. A Survey indicated that chocolate ice cream is India's favorite ice-cream flavor. For each of the following, use diagrams to indicate the possible effects on demand and/or supply and equilibrium price and quantity of chocolate ice cream.
 - A severe drought in Gujarat causes dairy farmers to reduce the number of milk-producing cattle in their herds by a third. These dairy farmers supply cream that is used to manufacture chocolate ice cream.
 - A news report by the Indian Medical Association reveals that chocolate does, in fact, have significant health benefits.
 - The discovery of cheaper synthetic vanilla flavoring lowers the price of vanilla ice cream.
 - New technology for mixing and freezing ice cream lowers manufacturers' costs of producing chocolate ice cream.