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Special Issue

On

Technology Diffusion

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SECTION I

Original Research Papers

REMARKS FROM THE EDITOR-IN-CHIEF

In the first few volumes of this journal, the aim is to publish papers in mathematical sciences from authors of IIT Kanpur Research group in Mathematical Modeling and the Members of the Indian Academy of Mathematical Modeling and Simulation having Head Quarters at IIT Kanpur.

Researchers from elsewhere will be invited later to publish their research work in this journal. Self submission is not encouraged. But letters to the Editors are invited from researchers on ideas relevant to Nature and Society for publication in the journal.

In the present special volume is devoted to Technology Diffusion. It contains four original research papers, where the effects of demography, social factors, switching from primary to a new secondary technology and media have been proposed by using Nonlinear mathematical models. As in previous volumes, the Stability theory of differential equations has been used to analyse these models.

These papers will provide young researchers a good idea as how to conduct research in technology diffusion.

All the Best.

Prof J.B Shukla

REMARKS FROM THE GUEST EDITOR

This special volume is an exposition in Technology Diffusion, an area that is particularly important in Social Sciences, in general and economics and managerial sciences, in particular. It consists of four research papers, namely, Modeling and Analysis of the Effects of Demographic Factors on Diffusion of Innovation, Modeling and Analysis of the Effects of Demographic and Social Factors on Diffusion of Innovation, Modeling And Analysis Of The Effects Of Demography And Switching On Technology Diffusion and Modeling the Effect of Media on Diffusion of Innovation in a Population.

In the first paper, a model for technology diffusion process under the effects of demographic factors such as immigration, migration and the intrinsic growth of population has been considered. The second paper proposes a model by considering changes in the total population and the cumulative densities of social influence factors. A model that takes into account demographic factors such as immigration, emigration and the intrinsic growth of population has been proposed as the the third paper. Finally, a model that takes into account the effect of media on technology diffusion in a population is given in the fourth paper.

I feel that this volume would be very useful to Modelers in Social Sciences.

I have immensely enjoyed editing this special issue.

Prof. Ajai Shukla

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Modeling and Analysis of the Effects of Demographic Factors on Diffusion of Innovation

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Abstract

In this paper, a new non-linear mathematical model is proposed and analyzed to study the effects of demographic factors such as immigration, emigration and intrinsic growth of population on innovation diffusion. The model is governed by two variables, namely, number of non adopters and adopters.The model is analyzed by stability theory of differential equations.The model analysis shows that the number of adopters increases as the density of population increases by immigration and its intrinsic growth rate but it decreases due to emigration.

Keywords: Diffusion of innovation, Demographic Factors, Mathematical Model, Stability Analysis

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1. Introduction

For many decades a simple Bass model has been in use to predict the diffusion of innovation in a population by emphasizing the rate of communication between adopters and non adopters as an internal process, see Bass (1969), Sharif(1976), Wang(2003),Wang (2006)and Yu (2007).The effect of advertisement on innovation diffusion has also been modeled by Bass(1969 as an external process. In the Bass model, it has been assumed that the number of potential adopters (i.e. the total population) remains constant in the diffusion process but this does not match with real market scenario, where, the changes in the demography of the population may be caused by immigration, emigration as well as due to its intrinsic growth (i.e. due to birth and death processes in a population), Mahajan et al (1978), Sharif(1981).

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In this paper, therefore, we propose a new model to study the effects of demographic factors on diffusion of innovation. The model is analyzed by stability theory of differential equations and computer simulation , Wang and Yu (2006, 2007).

2. Mathematical Model

Let N_0 be the density of non-adopters population, N_a be the density of adopters population of new technology and M be the total population. Let the constant p be the cumulative coefficient of external influences and q be the cumulative coefficient of internal influences, A be the immigration rate, r_0 be the intrinsic growth rate of population, K be the carrying capacity of the population, e be the emigration rate of the population from the market scenario, ν be the rate of adopters going back to non-adopters class. Let M_0 be the density of non-immigrating population from the market and γ be be the coefficient representing emigration. Let α be the rate by which adopters discontinue to use the technology and would neither go back to non-adopters' class nor use any other technology.

In view of the above, the model is proposed as follows,

$$\begin{aligned} \frac{dN_0}{dt} &= A + r_0(M - \frac{M^2}{K}) - \gamma(M - M_0) - pN_0 - \frac{qN_a}{M}N_0 + \nu N_a - eN_0 \\ \frac{dN_a}{dt} &= pN_0 + \frac{qN_a}{M}N_0 - eN_a - \nu N_a - \alpha N_a \end{aligned} \tag{2.1}$$

By adding the two equations described in model (2.1), the rate of change in total population ($N_0 + N_a = M$) is given by the following equation,

$$\frac{dM}{dt} = A + r_0(M - \frac{M^2}{K}) - \gamma(M - M_0) - eM - \alpha N_a \tag{2.2}$$

Thus, the equations governing the problem are given by,

$$\begin{aligned} \frac{dN_a}{dt} &= p(M - N_a) + \frac{qN_a}{M}(M - N_a) - (e + \alpha + \nu)N_a \\ \frac{dM}{dt} &= A_1 + rM - r_0\frac{M^2}{K} - \alpha N_a \end{aligned} \tag{2.3}$$

where

$$\begin{aligned} A_1 &= A + \gamma M_0 \\ r &= r_0 - \gamma - e > 0 \end{aligned}$$

Here, we consider two separate cases for analyzing the above mentioned mathematical model (2.3):

(i) Case:1
when $r_0 \neq 0$

(ii) Case:2
when $r_0 = 0$.

3. Case: 1 ($r_0 \neq 0$)

In this case, we note here that actual solution of equation governing M with the same initial condition is bounded by the upper solution (M_u) and the lower solution (M_l) of the following inequalities:

$$A_1 + (r-\alpha)M_l - \frac{r_0 {M_l}^2}{K} = \frac{dM_l}{dt} < \frac{dM}{dt} < A_1 + rM_u - \frac{r_0 {M_u}^2}{K} = \frac{dM_u}{dt} \quad (3.1)$$

On solving (3.1) and as taking $t \to \infty$, we get:

$$\frac{K}{2r_0}[r-\alpha+\sqrt{(r-\alpha)^2+\frac{4r_0A_1}{K}}] = M_l < M < M_u = \frac{K}{2r_0}[r+\sqrt{r^2+\frac{4r_0A_1}{K}}] \quad (3.2)$$

The upper bound of N_a is given as follows:

$$0 \leq N_a \leq M_u \quad (3.3)$$

3.1. Equilibrium Analysis

The model (2.3) has only one non-negative equilibrium ${E_1}^*(N_a^*, M^*)$. This is determined from the following equations obtained by equating the right hand sides of the equations of the model (2.3) to zero,

$$p + \bar{q}\frac{N_a}{M} - q(\frac{N_a}{M})^2 = 0 \quad (3.4)$$

$$g(M) = \frac{N_a}{M} = \frac{1}{\alpha}(\frac{A_1}{M} + r - r_0\frac{M}{K}) > 0 \quad (3.5)$$

where

$$\bar{q} = q - (p + \alpha + e + \nu) \quad (3.6)$$

We note from equation (3.5) that $g(M_u) = 0$ and $g(M_l) = 1$.

From (3.4) and (3.5) , we get:

$$F(M) = p + \frac{\bar{q}}{\alpha}(\frac{A_1}{M} + r - r_0\frac{M}{K}) - \frac{q}{\alpha^2}(\frac{A_1}{M} + r - r_0\frac{M}{K})^2 = 0 \tag{3.7}$$

From equation (3.7), we note the following,
(i) when $M = M_u$

$$F(M_u) = p > 0$$

(ii) when $M = M_l$

$$F(M_l) = -(\alpha + e + \nu) < 0$$

(iii) On differentiating (3.7), we get,

$$F'(M) = -\frac{\bar{q}}{\alpha}(\frac{A_1}{M^2} + \frac{r_0}{K}) + \frac{2q}{\alpha^2}(\frac{A_1}{M} + r - r_0\frac{M}{K})(\frac{A_1}{M^2} + \frac{r_0}{K}) \tag{3.8}$$

Multiplying (3.5) both sides by $(\frac{A_1}{M} + r - r_0\frac{M}{K})$ and using (3.4) again, we have

$$(\frac{A_1}{M} + r - r_0\frac{M}{K})F'(M) = (\frac{A_1}{M^2} + \frac{r_0}{K})[p + \frac{q_2}{\alpha^2}(\frac{A_1}{M} + r - r_0\frac{M}{K})^2]$$

Therefore, $F'(M) > 0$. Hence, $F(M) = 0$ has one and only one positive root say M^* in $M_l < M < M_u$. Therefore, ${E_1}^*(N_a^*, M^*)$ exists and is unique.

Differentiating (3.4) with respect to A_1 and using this equation again, we have

$$\frac{1}{N_a}(q{N_a}^2 + pM^2)\frac{dN_a}{dA_1} = (2pM + \bar{q}N_a)\frac{dM}{dA_1} \tag{3.9}$$

This shows that when M increases with A_1, N_a also increases.By differentiating (3.5) and eliminating $\frac{dM}{dA_1}$ using (3.6) we get $\frac{dN_a}{dA_1} > 0$. Similarly, we can show that $\frac{dN_a}{dr_0} > 0, \frac{dN_a}{d\alpha} < 0$ and $\frac{dN_a}{de} < 0$. These imply that, N_a increases as A and r_0 increase and as e and α decrease.

3.2. Stability Analysis

The local stability result of the equilibrium ${E_1}^*$ is stated in the following theorem.

***Theorem* 3.1.** The equilibrium ${E_1}^*({N_a}^*, M^*)$ is locally stable provided the following condition is satisfied.

$$M^* > \frac{rK}{2r_0} \tag{3.10}$$

This theorem can be proved by finding the Jacobian matrix corresponding to this equilibrium.

The proof of the above theorem is given in the **Appendix-A**.

For proof of global stability of $E_1{}^*(N_a{}^*, M^*)$ we need the following lemma which we stated without proof.

***Lemma* 3.1.** The region of attraction for all solutions initiating in the positive octant is given by the set:

$$\Omega_{s1} = \{(N_a{}^*, M^*) : 0 \leq N_a < M_u, and\ max[M_l, \frac{rK}{2r_0}] = M_{lm} \leq M < M_u\} \tag{3.11}$$

***Theorem* 3.2.** *The equilibrium $E_1{}^*(N_a^*, M^*)$ is nonlinearly stable in Ω_{s1} provided the following condition is satisfied:*

$$\frac{4qpr_0}{M^*\alpha N_a{}^*K}(M_{lm} + M^* - \frac{rK}{2r_0}) - (\frac{qM_u}{M_{lm}M^*})^2 > 0 \tag{3.12}$$

The proof of this theorem is given in the **Appendix-B**.

Remarks:

The above discussions imply that under the above conditions, the system variables (N_a, M) would attain equilibrium values and the number of adopters increases as immigration rate and intrinsic growth rate of total population increase but it decreases due to emigration.

4. Case: 2 ($r_0 = 0$)

The model in this case for , $r_0 = 0$ is written as follows, from (2.3)

$$\begin{aligned} \frac{dN_a}{dt} &= p(M - N_a) + q\frac{N_a}{M}(M - N_a) - e_1 N_a \\ \frac{dM}{dt} &= A_1 - (e + \gamma)M - \alpha N_a \end{aligned} \tag{4.1}$$

where $N_a(0) \geq 0$ and $M(0) > 0$ and

$$\begin{aligned} A_1 &= A + \gamma M_0 \\ e_1 &= e + \nu + \alpha \end{aligned} \tag{4.2}$$

In the following we analyzed the model (4.1):

First we note that the solutions of the model (4.1) is bounded, as described in the following lemma which is stated without proof.

***Lemma* 4.1.** The region of attraction for all solutions initiating in the positive octant is given by Ω, Freedman and So(1985), Shukla et al (2012).

$$\Omega = \{(N_a,\ M):\ 0 \leq N_a < M_{max},\ M_{min} < M < M_{max}\} \tag{4.3}$$

where

$$M_{min} = \frac{A_1}{e+\gamma+\alpha};\ M_{max} = \frac{A_1}{e+\gamma} \tag{4.4}$$

4.1. ***Equilibrium Analysis***

The model (4.1) has only one non-negative equilibrium $E_2{}^*(N_a{}^*, M^*)$.

The existence of $E_2{}^*(N_a{}^*, M^*)$ is determined by equating the right hand sides of equations (4.1) to zero.

$$p(1-\frac{N_a}{M}) + q\frac{N_a}{M}(1-\frac{N_a}{M}) - e_1\frac{N_a}{M} = 0 \tag{4.5}$$

$$A_1 - (e+\gamma)M - \alpha N_a = 0 \tag{4.6}$$

From equation (4.6), we can write:

$$g(M) = \frac{N_a}{M} = \frac{1}{\alpha}(\frac{A_1}{M} - (e+\gamma)) > 0 \tag{4.7}$$

We note from equation (4.7) that :

$$\begin{aligned} g(M_{max}) &= 0, \\ g(M_{min}) &= 1, \end{aligned} \tag{4.8}$$

By differentiating $g(M)$ w.r.t. M, we get

$$g'(M) = -\frac{1}{\alpha}\frac{A_1}{M^2} < 0 \tag{4.9}$$

Now using (4.5) and (4.7) the following function is defined:

$$F(M) = p(1-g(M)) + qg(M)(1-g(M)) - e_1 g(M) = 0 \tag{4.10}$$

From equation (4.10), we note the following,:-

1. when $M = M_{min}$

$$F(M_{min}) = -(\alpha + e + \nu) < 0 \tag{4.11}$$

2. when $M = M_{max}$

$$F(M_{max}) = p > 0 \tag{4.12}$$

3. $F'(M) > 0$

$$F'(M) = -pg'(M) + qg'(M)(1 - 2g(M)) - e_1 g'(M) > 0 \tag{4.13}$$

By using (4.9) in (4.13), we note that $F'(M) > 0$ if

$$g(M) \geq \frac{1}{2}. \tag{4.14}$$

Hence, $F(M) = 0$ has one and only one positive root M^* in $M_{min} < M \leq M_i$.where M_i can be found by solving the equation (4.7) for M after substituting $g(M) = \frac{1}{2}$ as follows,:

$$\frac{A_1}{(e+\gamma) + \frac{\alpha}{2}} = M_i \tag{4.15}$$

Differentiating (3.1) with respect to A_1 and using this equation again, we have

$$\frac{1}{N_a}(q{N_a}^2 + pM^2)\frac{dN_a}{dA_1} = (2pM + \bar{q}N_a)\frac{dM}{dA_1} \tag{4.16}$$

This shows that when M increases with A_1, N_a also increases. Again, differentiating (3.2) and eliminating $\frac{dM}{dA_1}$ using (3.5) we get $\frac{dN_a}{dA_1} > 0$. Similarly, we can show that $\frac{dN_a}{dr_0} > 0, \frac{dN_a}{d\alpha} < 0$ and $\frac{dN_a}{de} < 0$. These imply that, N_a increases as A and r_0 increase and as e and α decrease.

4.2. Stability Analysis

The local stability result of the equilibrium ${E_2}^*({N_a}^*, M^*)$ is stated in the following theorem.

***Theorem* 4.1.** The equilibrium ${E_2}^*({N_a}^*, M^*)$ is locally stable without any condition.

The proof of the above theorem is given in the **Appendix-A**.

For defining the global stability result of the equilibrium ${E_2}^*({N_a}^*, M^*)$ we need following lemma which is stated below without proof:

***Lemma* 4.1.** The region of attraction for all solutions initiating in the positive octant is given by the the following set, a subset of —*Omega*

$$\Omega_{s2} = \{(N_a, M) : 0 \leq N_a \leq M_i < M_{max},\ M_{min} < M \leq M_i < M_{max}\} \qquad (4.17)$$

***Theorem* 4.2.** *The equilibrium* ${E_2}^*({N_a}^*, M^*)$ *is nonlinearly stable in* Ω_{s2}*, provided the following conditions are satisfied:*

$$\frac{4p(e+\gamma)}{\alpha {N_a}^*} - \frac{q}{M^*}(\frac{M_{max}}{M_{min}})^2 > 0 \qquad (4.18)$$

The proof of this theorem is given in the **Appendix-B**.

5. Numerical Simulation

To check the feasibility of our analysis regarding the existence of ${E_1}^*$ and ${E_2}^*$, we conduct numerical simulation of model (2.3) and (4.1) by choosing the following values of parameters:-

$p = 0.01; q = 0.3; e = 0.05; \nu = 0.01; \alpha = 0.002; A = 100; r_0 = .03; K = 100000; \gamma = 0.0002; m_0 = 1000;$

It is found that for the above set of parameters, the conditions for the existence of ${E_1}^*({N_a}^*, M^*)$ and ${E_2}^*({N_a}^*, M^*)$ are satisfied and the equilibrium values of the dependent variables come out to be $({E_1}^*(N_a^* = 66563, M^* = 83168))$ and $({E_2}^*({N_a}^* = 1936, M^* = 1553))$.It is pointed out here that for above set of parameters, the conditions for linear and nonlinear stability are also satisfied.

To see the effects of various parameters on the dependent variables, we have further solved the equations of model (2.3) and model (4.1)and plotted these in Fig-[B.1]-Fig-[B.5].

From Figure B.1 and Figure B.2 it is clear that as the rate of immigration increases, the number of adopters population increases in both the cases.

From Figure B.3 and Figure B.4 it is evident that as discontinuance rate increases, the number of adopters population decreases in both the cases.

From Figure B.5 we can easily see that the numbers of adopters' and total population increase as the intrinsic growth rate of total population increases.

6. Conclusions

It has been known for many decades that the diffusion of innovation is governed by Bass model which takes into account the effect of external and internal influences on the diffusion process of a new technology by assuming that the total population is a constant. However, in the real life situations, the total population changes because of immigration,emigration as well as due to birth and death processes..Thus, the model for diffusion process must consider various demographic factors just mentioned. In this paper, therefore, we have considered the effects of demographic factors such as immigration,emigration and intrinsic growth of the population in the modeling process.The proposed non linear model has been analyzed by stability theory of differential equations. The following results have been obtained:

(i) The number of adopters increases as immigration of population increases.

(ii) The number of adopters increases as the intrinsic growth rate of population increases.

(iii) The number of adopters decrease as the coefficient of emigration of adopters increases.

It may be pointed out that our study does not include the variations of external and internal influences. This we leave for future research.

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Appendix A.

Proof of Theorem 3.1. Here we consider the stability of the equilibrium ${E_1}^*({N_a}^*, M^*)$.The characteristic equation for the Jacobian matrix:

$$J = \begin{bmatrix} -p + q - \frac{2q{N_a}^*}{M^*} - (e + \alpha + \nu) - \lambda & p + \frac{q{N_a*}^2}{{M*}^2} \\ -\alpha & -(\frac{2r_0M^*}{K} - r) - \lambda \end{bmatrix}$$

On further manipulations by using equations of model(3.4), we get

$$J = \begin{bmatrix} -((\frac{pM^*}{{N_a}^*}) + \frac{q{N_a}^*}{M^*}) - \lambda & p + \frac{q{N_a*}^2}{{M*}^2} \\ -\alpha & -(\frac{2r_0M^*}{K} - r) - \lambda \end{bmatrix}$$

where λ is given by:

$$\lambda^2 + a_1\lambda + a_2 = 0 \tag{A.1}$$

Where

$$a_1 = ((\frac{pM^*}{{N_a}^*}) + \frac{q{N_a}^*}{M^*}) + (\frac{2r_0M^*}{K} - r)$$
$$a_2 = ((\frac{pM^*}{{N_a}^*}) + \frac{q{N_a}^*}{M^*}) + (\frac{2r_0M^*}{K} - r) + \alpha(p + \frac{q{N_a*}^2}{{M*}^2})$$

Both of the eigenvalues from the equation (A.1) will be negative or will have negative real parts and the equilibrium $E^*({N_a}^*, M^*)$ will be locally asymptotically stable if the following conditions are satisfied,

$$M^* > \frac{rK}{2r_0} \tag{A.2}$$

□

Proof of Theorem 4.1. Here we consider the stability of the equilibrium ${E_2}^*({N_a}^*, M^*)$.The characteristic equation for the Jacobian matrix:

$$J = \begin{bmatrix} -p + q - \frac{2q{N_a}^*}{M^*} - (e + \alpha + \nu) - \lambda & p + \frac{q{N_a*}^2}{{M*}^2} \\ -\alpha & -(e + \gamma) - \lambda \end{bmatrix}$$

On further manipulations by using equations of model(4.1):

$$J = \begin{bmatrix} -((\frac{pM^*}{N_a{}^*}) + \frac{qN_a{}^*}{M^*}) - \lambda & p + \frac{qN_a*^2}{M*^2} \\ -\alpha & -(e+\gamma) - \lambda \end{bmatrix}$$

is given by:

$$\lambda^2 + a_1\lambda + a_2 = 0 \tag{A.3}$$

Where

$$a_1 = ((\frac{pM^*}{N_a{}^*}) + \frac{qN_a{}^*}{M^*}) + (e+\gamma)$$
$$a_2 = ((\frac{pM^*}{N_a{}^*}) + \frac{qN_a{}^*}{M^*}) + (e+\gamma) + \alpha(p + \frac{qN_a*^2}{M*^2})$$

Both of the eigenvalues coming out from the equation (A.3) will be negative or will have negative real parts. and the equilibrium $E_2{}^*(N_a{}^*, M^*)$ will be locally asymptotically stable without any condition. □

Appendix B.

Proof of Theorem 3.2. To prove this theorem, we consider the following positive definite function about $E_1{}^*$,

$$V(N_a, M) = (N_a - N_a{}^* - N_a{}^* ln\frac{N_a}{N_a{}^*}) + \frac{c_1}{2}(M - M^*)^2 \tag{B.1}$$

where c_1 is a positive constants, to be chosen appropriately.

Differentiating above equation with respect to t along the solutions of system (2.3) and after a simple algebraic manipulation we get,

$$\begin{aligned} \frac{\text{dV}}{\text{dt}} &= -(N_a - N_a{}^*)^2(\frac{pM}{N_a N_a{}^*} + \frac{q}{M^*}) \\ &- c_1\frac{r_0}{K}(M + M^* - \frac{rK}{r_0})(M - M^*)^2 \\ &+ [(\frac{p}{N_a{}^*} + \frac{qN_a}{MM^*} - c_1\alpha](M - M^*)(N_a - N_a{}^*) \end{aligned} \tag{B.2}$$

Choosing

$$c_1 = \frac{p}{\alpha {N_a}^*}$$

we have,

$$\begin{aligned}\frac{\mathrm{dV}}{\mathrm{dt}} &= -(\frac{pM}{N_a {N_a}^*} + \frac{q}{M^*})(N_a - {N_a}^*)^2 \\ &- \frac{p}{\alpha {N_a}^*}\frac{r_0}{K}(M + M^* - \frac{rK}{r_0})(M - M^*)^2 \\ &+ [\frac{qN_a}{MM^*}](M - M^*)(N_a - {N_a}^*)\end{aligned} \tag{B.3}$$

Now $\frac{dV}{dt}$ will be negative definite inside the subregion of attraction provided the following inequalities are satisfied:-

$$\frac{4qpr_0}{M^* \alpha {N_a}^* K}(M + M^* - \frac{rK}{2r_0}) > (\frac{qN_a}{MM^*})^2 \tag{B.4}$$

Thus final condition for global stability for the equilibrium ${E_1}^*({N_a}^*, M^*)$ will be:

$$\frac{4qpr_0}{M^* \alpha {N_a}^* K}(M_{lm} + M^* - \frac{rK}{2r_0}) > (\frac{M_u}{M_{lm}})^2$$

□

Proof of Theorem 4.2.2. To prove this theorem, we consider the following positive definite function about ${E_2}^*$,

$$V(N_a, M) = (N_a - {N_a}^* - {N_a}^* ln\frac{N_a}{{N_a}^*}) + \frac{c_2}{2}(M - M^*)^2 \tag{B.5}$$

where c_2 is a positive constants, to be chosen appropriately.

Differentiating above equation with respect to t along the solutions of system (4.1) and after a simple algebraic manipulation we get,

$$\begin{aligned}\frac{\mathrm{dV}}{\mathrm{dt}} &= -(N_a - {N_a}^*)^2(\frac{pM}{N_a {N_a}^*} + \frac{q}{M^*}) \\ &- c_2(e + \gamma)(M - M^*)^2 \\ &+ [(\frac{p}{{N_a}^*} + \frac{qN_a}{MM^*} - c_2\alpha](M - M^*)(N_a - {N_a}^*)\end{aligned} \tag{B.6}$$

Choosing

$$c_2 = \frac{p}{\alpha {N_a}^*}$$

we have,

$$\begin{aligned}\frac{\mathrm{dV}}{\mathrm{dt}} &= -(N_a - {N_a}^*)^2(\frac{pM}{N_a {N_a}^*} + \frac{q}{M^*}) \\ &- \frac{p}{\alpha {N_a}^*}(e+\gamma)(M - M^*)^2 \\ &+ [\frac{qN_a}{MM^*}](M - M^*)(N_a - {N_a}^*)\end{aligned} \tag{B.7}$$

Now $\frac{dV}{dt}$ will be negative definite inside the subregion of attraction provided the following inequalities is satisfied:-

$$\frac{4pM^*(e+\gamma)}{q\alpha {N_a}^*} > (\frac{M_i}{M_{min}})^2$$

□

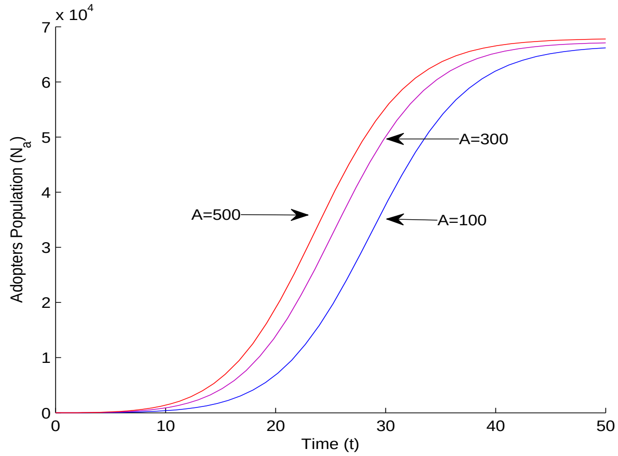

Figure B.1: Case:1:Effect of coefficient of immigration A on adopters' population density

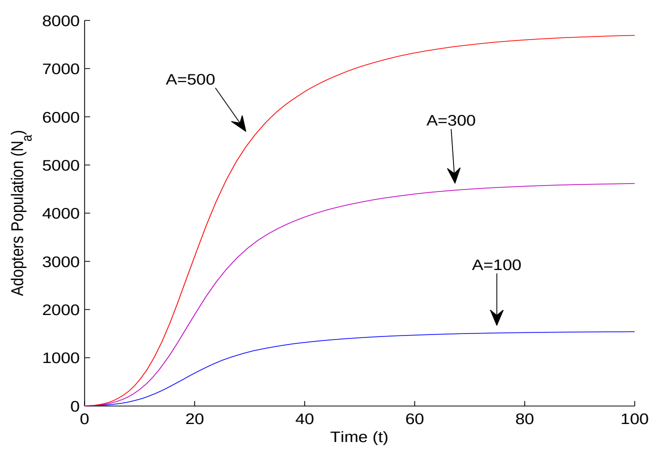

Figure B.2: Case:2:Effect of coefficient of immigration A on adopters' population density when $r_0 = 0$

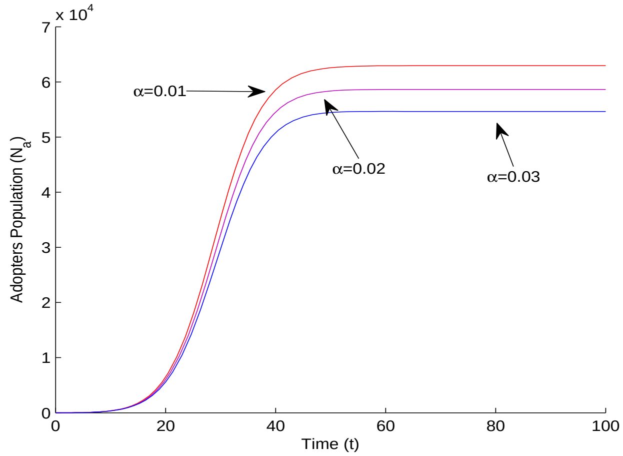

Figure B.3: Case:1:Effect of coefficient of discontinuance α on adopters' population density

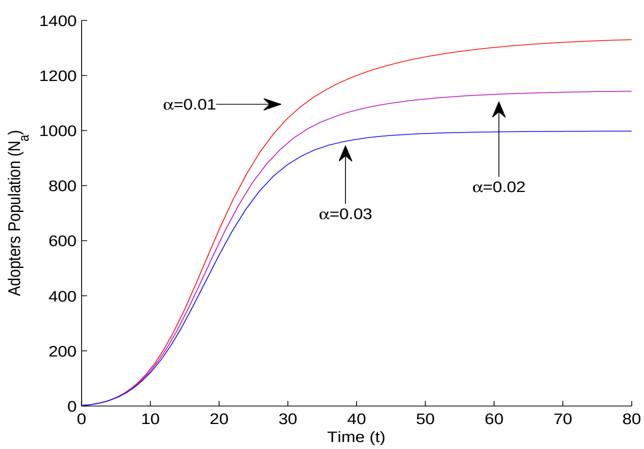

Figure B.4: Case:2:Effect of coefficient of discontinuance α on adopters' population density when $r_0 = 0$

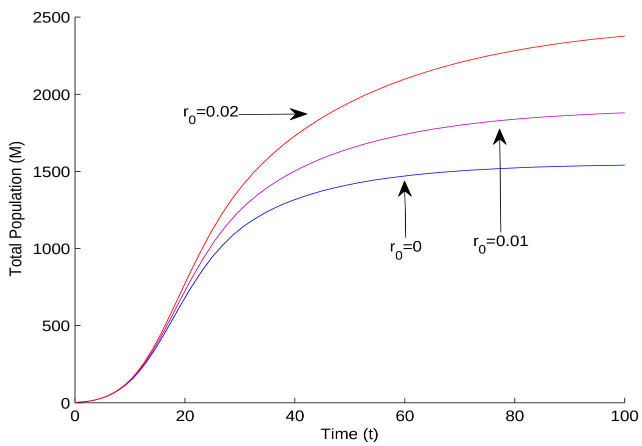

Figure B.5: Effect of coefficient of intrinsic growth rate on density of total population

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Modeling and Analysis Of The Effects of Demographic And Social Factors on Diffusion of Innovation,

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Abstract

In this paper, a non-linear mathematical model is proposed and analyzed to study the effects of demographic factors such as immigration, emigration and intrinsic growth rate of population on diffusion of innovation. In the modeling process, it is assumed that the cumulative density of social influence factors affect favourably the thinking process of non adopters about the innovation.

Thus, the problem of innovation diffusion is governed by three dynamic variables, namely, the density of non-adopters' population, the density of adopters' population and the cumulative density of social factors. The stability theory of differential equations and numerical simulations are used to analyze the model.

The model analysis shows that adopters' population density increases as the density of non-adopters' population increases due to immigration and intrinsic growth of population but it decreases due to emigration.It is also shown that as the cumulative density of social influence factors, increases the density of adopters' population increases.

Keywords: Diffusion of Innovation, Variable Social Influence, Mathematical Model, Dynamic Market Potential, Stability Analysis

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1. Introduction

Innovation Diffusion has been studied extensively using mathematical models by considering external factors such as marketing and internal factors such as word

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of mouth communication by many Researchers. Bass(1969), Mahajan and Robertson(1978, 1990), Sharif et al (1976, 1981), Shukla et al (2012). It is noted here that in these studies the social influence parameter has been assumed to be a constant which is not realistic in a dynamic population particularly when its demography changes due to immigration, intrinsic growth and emigration. Further, as the density of adopters increases in a region, the non-adopters experience a social pressure (influence) to adopt a new technology. Therefore, a model governing diffusion of innovation must consider the cumulative density of social factors as a dynamic variable which should be taken proportional to density of adopters in the region. Since, to our best of knowledge, no one has studied effects of variable social influence factors on innovation in a variable population, in this paper, a non-linear mathematical model is proposed and analyzed to study these aspects using stability theory of differential equations and simulations.

Our focus here is study the effects of cumulative density of social influence factors on innovation diffusion in a dynamic population.

2. Mathematical Model

Consider that the diffusion of innovation (a new technology) in a population with population density M and equilibrium population density M_0 in absence of any change in the population. The following assumptions are made in the modeling process:

1. The population density of non-adopters, N_0, at time t increases due to the factors mentioned below:
    - Immigration rate (A)
    - Intrinsic growth rate (r_0)
2. The population density of non-adopters', N_0, decreases and correspondingly the density of adopters', N_a, increases due to following factors:
    - Emigration rate (e).
    - The coefficient of cumulative density, p, of external influences such as marketing efforts.
    - The coefficients of cumulative density, q, of internal influences, such as word of mouth, mobile communication, etc.
    - The cumulative density, S, of variable social influence factors which is assumed to be proportional to the density of adopters' which grows with rate β and decreases naturally with rate β_0.

3. The population density of adopters, N_a, decreases with a rate ν and these adopters go back to non-adopter population.
4. The population density of adopters decreases due to discontinuance of innovation with a rate α.
5. The parameters used in the model are assumed to be positive constants.

Keeping in view of the above considerations, a model governing the problem is written as follows:-

$$\begin{aligned}
\frac{dN_0}{dt} &= A + r_0 M - r_0 \frac{M^2}{K} - \gamma(M - M_0) - pN_0 - SN_0 - \frac{qN_a}{M} N_0 + \nu N_a - eN_0 \\
\frac{dN_a}{dt} &= pN_0 + sN_0 + \frac{qN_a}{M} N_0 - (e + \nu + \alpha) N_a \\
\frac{dS}{dt} &= \beta N_a - \beta_0, N_a, > 0, S > 0, M > M_0.
\end{aligned} \tag{2.1}$$

where the constant γ is the rate by which the population density M approaches to the non immigrating population M_0, when $A = 0$, r_0, p = 0, q = 0, $\nu = 0$, e = 0 ans S = 0.

Since, the total population density M at any instant of time is the sum of the densities of non adopters' and adopters' population, i.e. $M = N_0 + N_a$, by adding first two equations of model (2.1), we get:

$$\frac{dM}{dt} = A + r_0 M - r_0 \frac{M^2}{K} - \gamma(M - M_0) - eM - \alpha N_a$$

Finally, the above model governing the problem is rewritten as follows,

$$\begin{aligned}
\frac{dN_a}{dt} &= p(M - N_a) + S(M - N_a) + \frac{qN_a}{M}(M - N_a) - e_1 N_a \\
\frac{dS}{dt} &= \beta N_a - \beta_0 S \\
\frac{dM}{dt} &= A_1 + rM - \frac{r_0 M^2}{K} - \alpha N_a
\end{aligned} \tag{2.2}$$

where $N_a(0) \geq 0$, $S(0) \geq 0$ and $M(0) > 0$
and

$$A_1 = A + \gamma M_0 \tag{2.3}$$

$$r = r_0 - e - \gamma \tag{2.4}$$

$$e_1 = e + \nu + \alpha \tag{2.5}$$

From model (2.2) it is noted that if $p \to 0$ and $q \to 0$ the adopters density increases only due to the cumulative density of the social influence factors.

The following lemma is stated here without proof which we will use in the analysis of the model (2.2):

***Lemma* 2.1.** The region of attraction for all solutions initiating in the positive octant is given by the set Ω*Shukla et al (2012).*

$$\Omega = \{(N_a,\ p,\ M):\ 0 \le N_a < M_{max},\ 0 \le S < \frac{\beta M_{max}}{\beta_0},\ M_{min} < M < M_{max}\} \tag{2.6}$$

where

$$M_{min} = \frac{K}{2r_0}[(r-\alpha) + \sqrt{(r-\alpha)^2) + 4r_0\frac{A_1}{K}}];\ M_{max} = \frac{K}{2r_0}[r + \sqrt{r^2 + 4r_0\frac{A_1}{K}}] \tag{2.7}$$

3. Equilibrium Analysis

The model (2.2) has only one non-negative equilibrium $E^*({N_a}^*, S^*, M^*)$.

The existence of $E^*({N_a}^*, S^*, M^*)$ is determined by equating the right hand sides of model (2.2) to zero.

$$p(1 - \frac{N_a}{M}) + S - S\frac{N_a}{M} + \bar{q}\frac{N_a}{M} - q(\frac{N_a}{M})^2 = 0 \tag{3.1}$$

$$S = \frac{\beta N_a}{\beta_0} > 0 \tag{3.2}$$

$$g(M) = \frac{N_a}{M} = \frac{1}{\alpha}(\frac{A_1}{M} + r - r_0\frac{M}{K}) > 0 \tag{3.3}$$

where we assume that:

$$\bar{q} = q - e_1 > 0$$

By differentiating $g(M)$ w.r.t. M, we get

$$g'(M) = -\frac{1}{\alpha}(\frac{A_1}{M^2} + \frac{r_0}{K}) < 0 \tag{3.4}$$

Thus, $g(M)$ has the following properties:

$$g(M_{min}) = 1, \quad \begin{array}{l} g(M_{max}) = 0, \\ \\ g'(M) < 0 \end{array} \tag{3.5}$$

From equations (3.1), (3.2) and (3.3), we define the function $F(M)$ as follows,

$$F(M) = p(1 - g(M)) + (\bar{q} + \frac{\beta M}{\beta_0})g(M) - (q + \frac{\beta M}{\beta_0})g(M)^2 = 0 \tag{3.6}$$

Now we note the following from equation (3.6):-

1. when $M = M_{min}$

$$F(M_{min}) = -(\alpha + e + \nu) < 0 \tag{3.7}$$

2. when $M = M_{max}$

$$F(M_{max}) = p > 0 \tag{3.8}$$

3. $F'(M) > 0$
To prove this, we differentiate $F(M)$ given by eqn.(3.6) with respect to M and get

$$F'(M) = [\bar{q} + \frac{\beta}{\beta_0} + \frac{\beta M}{\beta_0}g'(M)]g(M) - 2(q + \frac{\beta M}{\beta_0})g'(M)g(M) - \frac{\beta}{\beta_0}g(M)^2 \tag{3.9}$$

Using (3.6) in (3.9), we get,

$$F'(M) = -g'(M)[\frac{p}{g(M)} + (q + \frac{\beta M}{\beta_0})g(M)] + \frac{\beta}{\beta_0}g(M)(1 - g(M)) \tag{3.10}$$

By using (3.4) in (3.10), we note that $F'(M) > 0$

Hence, $F(M) = 0$ has one and only one positive root M^* in $M_{min} < M < M_{max}$.
Remarks: From the model (2.2), it can be noted that $\frac{dN_a}{d\beta} > 0$ and $\frac{dN_a}{dp} > 0$, $\frac{dN_a}{dq} > 0$. The first condition implies that adopters' density increases with an increase in cumulative density of social influence factors and the other two conditions suggest that the density of adopters increases with external and internal influences.

4. Stability Analysis

The local stability result of the equilibrium E^* is stated in the following theorem.

***Theorem* 4.1.** *The equilibrium $E^*(N_a{}^*, S^*, M^*)$ is locally stable provided the following conditions are satisfied.*

$$M^* > \frac{rK}{2r_0} \tag{4.1}$$

$$\frac{4r_0 p}{\alpha K}\frac{qN_a{}^*}{M^*}(M^* - \frac{rK}{2r_0}) > [S^* + \frac{qN_a{}^{*2}}{M^{*2}}]^2 \tag{4.2}$$

The proof of the above theorem is given in the **Appendix A**.

For proof of global stability of $E^*(N_a{}^*, S^*, M^*)$ we need the following lemma which we state without proof:

***Lemma* 4.1.** *The region of attraction for all solutions initiating in the positive octant is given by the set, Ω_s, a subset of Ω, where,*

$$\begin{aligned} \Omega_s = \{(N_a, S, M) : 0 < N_a < M_{max},\ 0 \leq S \leq \frac{\beta}{\beta_0} M_{max}, \\ M_l = Max[\frac{rK}{2r_0}, M_{min}] < M < M_{max}\} \end{aligned} \tag{4.3}$$

***Theorem* 4.2.** *The equilibrium $E^*(N_a^*, S^*, M^*)$ is nonlinearly stable in Ω_s, provided the following conditions are satisfied:*

$$(M_l + M^* - \frac{rK}{r_0}) > 0 \quad 2\frac{p}{\alpha N_a{}^*}\frac{r_0}{K}\frac{q}{M^*}(M_l + M^* - \frac{rK}{r_0} > [\frac{S_{max}}{N_a{}^*} + \frac{qM_{max}}{M_l M^*}]^2 \tag{4.4}$$

The proof of this theorem is given in the **Appendix B**.

Remarks: The above discussions imply that under the above conditions, all the variable would attain their corresponding equilibrium values and the number of adopters increases as the growth rate of total population increases but it decreases due to emigration.The density of adopters' increases further if the cumulative density of social influence factors increases.

5. Numerical Simulation

To check the feasibility of our analysis regarding the existence of E^* and the stability conditions, we conduct some numerical computation of the model (2.2) by choosing the following values of parameters:-

$p = 0.01; q = 0.3; e = 0.05; \nu = 0.01; \alpha = 0.002; \beta = 0.0000006; \beta_0 = 0.1; A = 100; r_0 = .3; K = 100000; \gamma = 0.0001; m_0 = 1000;$

It is found that that for the above set of parameters, the stability conditions for $E^*({N_a}^*, S^*, M^*)$ are satisfied.

The model (2.2) is also solved for the above set of parameters w.r.t. N_a, S, M and plotted in Figure-1- Figure-4.

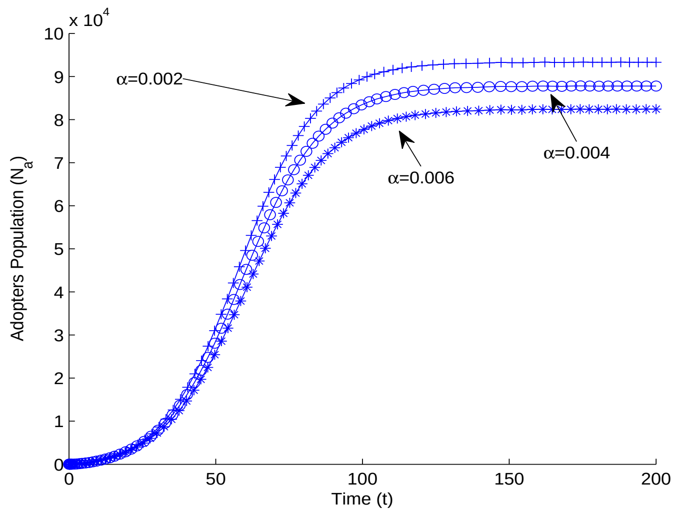

Figure 1: Variation in adopters' population w.r.t different values of coefficient of immigration.

From Figure-1 and Figure-2 it is clear that as coefficient representing social influence factors increases, the adopters' population density and the cumulative density of social influence increases.

From Figure-3 it is evident that as the value of the coefficient of immigration increases the adopters' population density also increases.

From Figure-4 it is clear that the densities of adopters' and total population increase as the intrinsic growth rate increases.

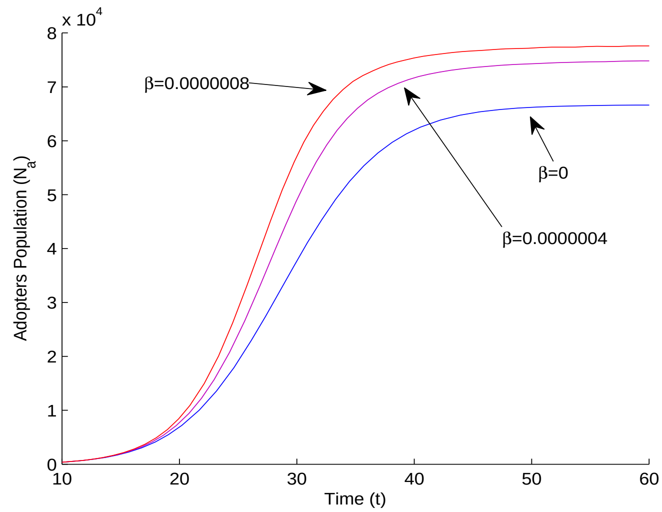

Figure 2: Variation in adopters' population w.r.t. different values of coefficient of social influence.

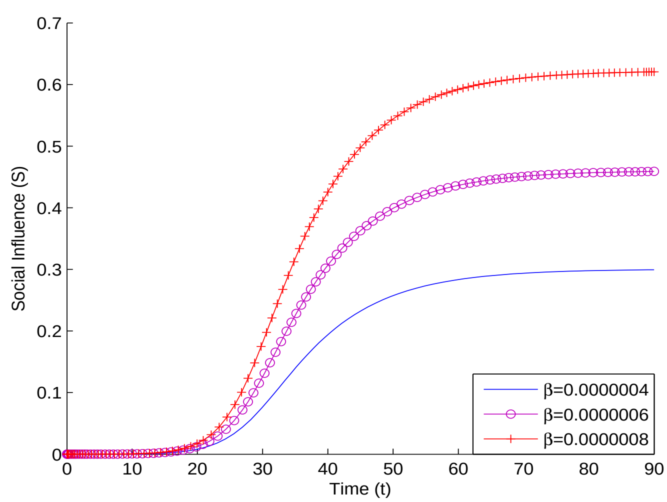

Figure 3: Variation in social influence w.r.t. different values of coefficient of social influence.

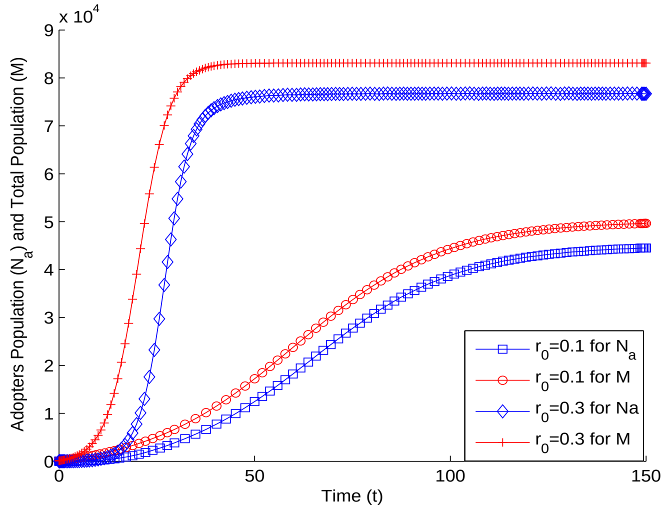

Figure 4: Variation in adopters' and total population w.r.t different values of Intrinsic growth rate.

6. Conclusion

For many decades the diffusion of new technology has been studied by using Bass model which takes into account the effect of external and internal influences but these influences and the total population are assumed to be constants. However, in the realistic scenario of the market the total population and the cumulative density of social influence factors vary and affect the innovation diffusion process in a population. The change in the population occurs due to immigration and intrinsic logistic growth rate, emigration etc.. Therefore, the model of diffusion process of a new technology must consider variable total population as well as variable social influence factors in a population. Thus, in this paper the problem has been assumed to be governed by three dynamic variables namely, the density of non adopters', the density of adopters' and the cumulative density of social influence factors.The proposed models have been studied and analyzed by stability theory of differential equations and simulation, the following results have been obtained,

1. The density of adopters increases due to increase in intrinsic growth of population but it decreases due to emigration of total population and discontinuance rate of the adopters.
2. The density of adopters increases as the growth rate of cumulative density of social influence factors increases.

3. In absence of external influences, the density of adopters may reach to zero due to detrimental factors such as emigration of total population and discontinuance rate of adopters. If the internal influences such as social influence and word of mouth communications are sufficient enough an innovation can successfully diffuse in the region even in the absence of marketing.
4. If hte density of the non immigrating population increases, the rate of immigrating increases.

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Appendix A.

Proof of Theorem 4.1. Here we consider the local stability of the equilibrium $E^*({N_a}^*, {N_s}^*, M^*)$. For this we linearize equations of model (2.2). To study the local stability we use the following perturbed function by considering:

$$\begin{aligned} N_a &= {N_a}^* + n_a \\ S &= S^* + s \\ M &= M^* + m \end{aligned} \tag{A.1}$$

After using these perturbed function in original differential equation we get:

$$\frac{dn_a}{dt} = -[p - q + \frac{2q{N_a}^*}{M^*} + S^* + e_1]n_a + [M^* - {N_a}^*]s + [p + S^* + \frac{q{N_a}^{*2}}{M^{*2}}]m \tag{A.2}$$

$$\frac{ds}{dt} = \beta n_a - es \tag{A.3}$$

$$\frac{dm}{dt} = -\alpha n_a + [r - \frac{2r_0 M^*}{K}]m \tag{A.4}$$

We consider the following positive definite function as follows:

$$V(n_a, n_s, m) = \frac{1}{2}{n_a}^2 + \frac{c_1}{2}{n_s}^2 + \frac{c_2}{2}m^2 \tag{A.5}$$

By differentiating the above function and after using [A.2-A.4] we have,

$$\begin{aligned} \frac{\mathrm{dV}}{\mathrm{dt}} &= -{n_a}^2[p - q + \frac{2q{N_a}^*}{M^*} + S^* + e_1] \\ &- c_1\beta_0 s^2 + c_2[r - \frac{2r_0M^*}{K}]m^2 \\ &+ [p + S^* + \frac{q{N_a}^{*2}}{M^{*2}} - \alpha c_2]mn_a \\ &+ [c_1\beta + (M^* - {N_a}^*)]sn_a \end{aligned} \tag{A.6}$$

To maximize the negative definiteness of $\frac{\mathrm{dV}}{\mathrm{dt}}$, we choose:

$$c_2 = \frac{p}{\alpha} \tag{A.7}$$

to get,

$$\begin{aligned}\frac{\mathrm{dV}}{\mathrm{dt}} &= -{n_a}^2[p - q + \frac{2q{N_a}^*}{M^*} + S^* + e_1] \\ &- c_1\beta_0 s^2 + \frac{p}{\alpha}[r - \frac{2r_0 M^*}{K}]m^2 \\ &+ [S^* + \frac{q{N_a}^{*2}}{M^{*2}}]mn_a \\ &+ [c_1\beta + (M^* - {N_a}^*)]sn_a\end{aligned}$$

By using the equilibrium equations of model (2.2)we can write $\frac{\mathrm{dV}}{\mathrm{dt}}$ as:

$$\begin{aligned}\frac{\mathrm{dV}}{\mathrm{dt}} &= -{n_a}^2[\frac{(p + S^*)M^*}{{N_a}^*} + \frac{q{N_a}^*}{M^*}] \\ &- c_1\beta_0 s^2 - \frac{2r_0 p}{\alpha K}[M^* - \frac{rK}{2r_0}]m^2 \\ &+ [S^* + \frac{q{N_a}^{*2}}{M^{*2}}]mn_a \\ &+ [c_1\beta + (M^* - {N_a}^*)]sn_a\end{aligned} \tag{A.8}$$

Thus, $\frac{\mathrm{dV}}{\mathrm{dt}}$ will be always negative definite provided the following inequalities are satisfied:-

$$(M^* - \frac{rK}{2r_0}) > 0 \tag{A.9}$$

$$\frac{4r_0 p}{\alpha K}\frac{q{N_a}^*}{M^*}(M^* - \frac{rK}{2r_0}) - [S^* + \frac{q{N_a}^{*2}}{M^{*2}}]^2 > 0 \tag{A.10}$$

$$2c_1\beta_0\frac{q{N_a}^*}{M^*}(M^* - \frac{rK}{2r_0}) - [c_1\beta + (M^* - {N_a}^*)]^2 > 0 \tag{A.11}$$

After solving condition (A.11) we can get the value of constant c_1, thus final conditions for local stability are:

$$(M^* - \frac{rK}{2r_0}) > 0 \tag{A.12}$$

$$\frac{4r_0 p}{\alpha K}\frac{q{N_a}^*}{M^*}(M^* - \frac{rK}{2r_0}) - [S^* + \frac{q{N_a}^{*2}}{M^{*2}}]^2 > 0 \tag{A.13}$$

□

Appendix B.

Proof of Theorem 4.2. To prove this theorem, we consider the following positive definite function about E^*,

$$V(N_a, S, M) = (N_a - {N_a}^* - {N_a}^* ln\frac{N_a}{{N_a}^*}) + \frac{C_1}{2}(S - S^*)^2 + \frac{C_2}{2}(M - M^*)^2 \tag{B.1}$$

where C_1 and C_2 are positive constants, to be chosen appropriately.

Differentiating above equation with respect to t along the solutions of above mentioned system and after a simple algebraic manipulation we get,

$$\begin{aligned}\frac{\mathrm{dV}}{\mathrm{dt}} &= -(N_a - {N_a}^{*2})(\frac{(p+S)M}{N_a {N_a}^*} + \frac{q}{M^*}) \\ &- C_1\beta_0(S - S^*)^2 - C_2\frac{r_0}{K}((M + M^* - \frac{rK}{r_0}))(M - M^*)^2 \\ &+ [(\frac{p+S}{{N_a}^*} + \frac{qN_a}{MM^*}) - C_2\alpha](M - M^*)(N_a - {N_a}^*) \\ &+ (C_1\beta - 1 + \frac{M^*}{{N_a}^*})(S - S^*)(N_a - {N_a}^*)\end{aligned} \tag{B.2}$$

To maximize $\frac{\mathrm{dV}}{\mathrm{dt}}$, we choose:

$$C_2 = \frac{p}{\alpha {N_a}^*}$$

we get,

$$\begin{aligned}\frac{\mathrm{dV}}{\mathrm{dt}} &= -(N_a - {N_a}^{*2})(\frac{(p+S)M}{N_a {N_a}^*} + \frac{q}{M^*}) \\ &- C_1\beta_0(S - S^*)^2 - \frac{p}{\alpha {N_a}^*}\frac{r_0}{K}((M + M^* - \frac{rK}{r_0}))(M - M^*)^2 \\ &+ (\frac{S}{{N_a}^*} + \frac{qN_a}{MM^*})(M - M^*)(N_a - {N_a}^*) \\ &+ (C_1\beta - 1 + \frac{M^*}{{N_a}^*})(S - S^*)(N_a - {N_a}^*)\end{aligned} \tag{B.3}$$

$\frac{\mathrm{dV}}{\mathrm{dt}}$ will be negative definite inside the sub region of attraction ω_s provided the following inequalities are satisfied:-

$$2\frac{p}{\alpha {N_a}^*}\frac{r_0}{K}((M + M^* - \frac{rK}{r_0}))\frac{q}{M^*} - [\frac{S}{{N_a}^*} + \frac{qN_a}{MM^*}]^2 > 0 \tag{B.4}$$

$$2C_1\beta_0\frac{q}{M^*} - [C_1\beta - 1 + \frac{M^*}{{N_a}^*}]^2 > 0 \tag{B.5}$$

After arranging inequality (B.5) we can get the value of C_1. Thus final conditions for global stability will be only the following:

$$2\frac{p}{\alpha {N_a}^*}\frac{r_0}{K}((M_l + M^* - \frac{rK}{r_0}))\frac{q}{M^*} - [\frac{S_{max}}{{N_a}^*} + \frac{qM_{max}}{M_l M^*}]^2 > 0 \tag{B.6}$$

where $M_l + M^* - \frac{rK}{r_0} > 0$

□

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Modeling and Analysis of the Effects of Demography And Switching of Technology Diffussion

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Abstract

In this paper, a non-linear mathematical model is proposed and analyzed to study diffusion of a new technology in a population by considering effects of demographic factors such as immigration, emigration and intrinsic growth of population and by considering switching of primary technology by its adopters to a new substitute technology. The model consists of three variables , namely, the density of non-adopters, the density of adopters and the dnsity of the population who switch from primary technology to a new technology.The model is analysed by using the stability theory of differential equations and simulations. The model analysis shows that the density of adopters of primary technology increases as the density of non-adopter population increases due to immigration and intrinsic growth rate, but it decreases due to emigration. Its density further decreases as the density of adopters of substitute technology increases.

Keywords: Diffusion of innovation, Diffusion, Substitute Technolog, Stability Analysis

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1. Introduction

For several, various models have been proposed to study the diffusion of a new technology (innovation diffusion) in a population by considering the effects of marketing as well as communication between adopters and non-adopters as an internal process, Bass(1969) ,Mahajan et al (1990),Sharif and Kabir(1976), Easingwood et al (1981), Silvennoinen and Vaanananen(1987). In the Bass model, it has been assumed

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that the number of potential adopters (i.e. the total population) remains constant in the diffusion process but this does not match with real market scenario. Therefore, a realistic model of innovation diffusion must consider the effects of demographic factors, causing change in population and affecting the number of adopters' of the new technology. The changes in the demography of the population may be caused by immigration, emigration as well as due to its intrinsic growth (i.e. due to birth and death processes in the population), Centrone et al (2007), Mahajan and Perteerson(1978). Further, the diffusion process of a new technology is also affected if some of its adopters switch over to another new substitute technology.As far as we know, this aspect has never been studied in the modeling of diffusion of a new technology.

In this paper, therefore, we propose a new nonlinear model to study the diffusion of a new technology by considering the effects of demographic factors and the effect of switching of adopters of new technology to a substitute technology.

2. Mathematical Model

Let N_0 be the density of non-adopters population, N_a be the density of adopters population of primary technology, N_s be the density of population who switch from the primary technology and adopt a new substitute technology, M be the total population and M_0 be the density of nn-emigrating population from the market community. Let the constant p be the cumulative coefficient of external influences, q be the cumulative coefficient of internal influences, A be the immigration rate, r_0 be the intrinsic growth rate of population, K be the carrying capacity of the population, e be the emigration rate of the population from the market scenario and ν be the rate of adopters going back to non-adopters class. Let β be the growth rate of adopters adopting substitute technology after leaving the primary technology. Let α be the rate by which adopters permanently left the market of primary technology and would neither go back to non-adopters class, nor use the substitute technology.We assume that all the parameters in the model are positive and constants
In view of these considerations, the model is proposed as follows,

$$\frac{dN_0}{dt} = A + r_0(M - \frac{M^2}{K}) - \gamma(M - M_0) - pN_0 - \frac{qN_a}{M}N_0 + \nu N_a - eN_0 \ , \quad (2.1)$$

$$\frac{dN_a}{dt} = pN_0 + \frac{qN_a}{M}N_0 - eN_a - \nu N_a - \alpha N_a - \beta N_a \ , \quad (2.2)$$

$$\frac{dN_s}{dt} = \beta N_a - eN_s \quad (2.3)$$

By adding Eqs.(2.1), (2.2) and (2.3), the rate of change in total population ($M = N_0 + N_a + N_s$) is given by the following equation,

$$\frac{dM}{dt} = A + r_0(M - \frac{M^2}{K}) - \gamma(M - M_0) - eM - \alpha N_a \quad (2.4)$$

Thus, the problem is finally governed by the following equations:

$$\frac{dN_a}{dt} = p(M - N_a - N_s) + \frac{qN_a}{M}(M - N_a - N_s) - e_1 N_a \quad (2.5)$$

$$\frac{dN_s}{dt} = \beta N_a - eN_s \quad (2.6)$$

$$\frac{dM}{dt} = A_1 + rM - r_0\frac{M^2}{K} - \alpha N_a \quad (2.7)$$

where

$$A_1 = A + \gamma M_0$$

$$r = r_0 - \gamma - e > 0$$

$$e_1 = e + \nu + \alpha + \beta$$

We note here that actual solution M of eq (2.7) which denotes the rate of change in M with the same initial condition is bounded by the upper solution (M_u) and the lower solution (M_l) by the following inequalities:

$$A_1 + (r - \alpha)M_l - \frac{r_0 {M_l}^2}{K} = \frac{dM_l}{dt} \leq \frac{dM}{dt} \leq \frac{dM_u}{dt} = A_1 + rM_l - \frac{r_0 M^2}{K} \quad (2.8)$$

Thus, on solving eq (.8). (2.8)and taking $t \to \infty$, we get

$$\frac{K}{2r_0}[r - \alpha + \sqrt{(r - \alpha)^2 + \frac{4r_0 A_1}{K}}] = M_l \leq M \leq M_u = \frac{K}{2r_0}[r + \sqrt{r^2 + \frac{4r_0 A_1}{K}}] \quad (2.9)$$

3. Equilibrium Analysis

The model given by eqs.(2.5), (2.6) and (2.7) has only one non-negative equilibrium $E^*(N_a^*, N_s^*, M^*)$. This is determined from the following equations obtained by equating the right hand sides of the eqs.(2.5), (2.6) and (2.7) to zero,

$$p + (q - p - e_1)\frac{N_a}{M} - q(\frac{N_a}{M})^2 - (p + q\frac{N_a}{M})\frac{N_s}{M} = 0 \tag{3.1}$$

$$N_s = \frac{\beta N_a}{e} \tag{3.2}$$

$$\frac{N_a}{M} = \frac{1}{\alpha}(\frac{A_1}{M} + r - r_0\frac{M}{K}) > 0 \tag{3.3}$$

Using eq.(3.2) in eq.(3.1), we get

$$p + (q - p(1 + \frac{\beta}{e}) - e_1)\frac{N_a}{M} - q(1 + \frac{\beta}{e})(\frac{N_a}{M})^2 = 0 \tag{3.4}$$

Let

$$q_1 = q - p(1 + \frac{\beta}{e}) - e_1$$

and

$$q_2 = q(1 + \frac{\beta}{e})$$

Then eq.(3.4) can be written as follows:

$$p + q_1\frac{N_a}{M} - q_2(\frac{N_a}{M})^2 = 0 \tag{3.5}$$

From eq.(3.3), substituting the value of N_a in eq.(3.4), we get

$$F(M) = p + \frac{q_1}{\alpha}(\frac{A_1}{M} + r - r_0\frac{M}{K}) - \frac{q_2}{\alpha^2}(\frac{A_1}{M} + r - r_0\frac{M}{K})^2 = 0 \tag{3.6}$$

From eq.(3.6), we note the following:
w

hen $M = M_u$

$$F(M) = p > 0 \tag{3.7}$$

w

hen $M = M_l$

$$F(M) = -\frac{\beta}{e}(p+q) - e_1 < 0 \tag{3.8}$$

O

n differentiating (3.6), we also get,

$$F'(M) = -\frac{q_1}{\alpha}(\frac{A_1}{M^2} + \frac{r_0}{K}) + \frac{2q_2}{\alpha^2}(\frac{A_1}{M} + r - r_0\frac{M}{K})(\frac{A_1}{M^2} + \frac{r_0}{K}) \tag{3.9}$$

Multiplying eq.(3.9) both sides by $(\frac{A_1}{M} + r - r_0\frac{M}{K})$ and using Eq.(3.6) again, we have

$$(\frac{A_1}{M} + r - r_0\frac{M}{K})F'(M) = (\frac{A_1}{M^2} + \frac{r_0}{K})[p + \frac{q_2}{\alpha^2}(\frac{A_1}{M} + r - r_0\frac{M}{K})^2] \tag{3.10}$$

Thus, we get $F'(M) > 0$, by using equation (3.3). Hence, $F(M) = 0$ has one and only one positive root say(M^*) in $M_l \leq M \leq M_u$. Therefore, $E^*(N_a^*, N_s^*, M^*)$ exists and is unique. Differentiating eq.(3.4) with respect to A and using this equation again, we have

$$\frac{dN_a}{dA} = (\frac{N_a}{M})\frac{dM}{dA} \tag{3.11}$$

This shows that when M increases with A, N_a also increases. Again, differentiating (3.6) and eliminating $\frac{dM}{dA}$ using (3.11) we get $\frac{dN_a}{dA} > 0$. Similarly, we can show that $\frac{dN_a}{dr_0} > 0, \frac{dN_a}{d\alpha} < 0$, $\frac{dN_a}{de} < 0$ and $\frac{dN_a}{d\beta} < 0$. These imply that, N_a increases as A and r_0 increase but it decrease as e and α increase. Also N_a decreases as β increases.

4. Stability Analysis

The local stability result of the equilibrium E^* is stated in the following theorem. The equilibrium $E^*({N_a}^*, {N_s}^*, M^*)$ is locally stable provided the following condition is satisfied.

$$\frac{M^*p}{2{N_a}^*} + \frac{q({N_a}^* + {N_s}^*)}{\alpha M^*}\frac{r_0}{K}(2M^* - \frac{rK}{r_0}) > 0 \tag{4.1}$$

The proof of the above theorem is given in the **Appendix A**.

For proof of global stability of $E^*(N_a{}^*, N_s{}^*, M^*)$ we need the following lemma which we state without proof. The region of attraction for all solutions initiating in the positive octant is given by the:

$$\Omega_s = \{(N_a, N_s, M) : 0 < N_a \leq M_u,\ 0 \leq N_s \leq M_u,\ M_l \leq M \leq M_u\} \tag{4.2}$$

The equilibrium $E^*(N_a^*, N_s^*, M^*)$ is nonlinearly stable in Ω_s, provided the following condition is satisfied:

$$\frac{M_l p}{2N_a{}^*} + \frac{q(N_a{}^* + N_s{}^*)}{\alpha M^*}\frac{r_0}{K}(M_l + M^* - \frac{rK}{r_0}) > 0 \tag{4.3}$$

It may be noted from eqs.[4.1] and [4.3] that p, q and r_0 have stabilizing effects on the system but α has destabilizing effect.[see section [**??**]] The proof of this theorem is given in the **Appendix-B**.

Remarks:

The above discussions imply that under the above conditions, the system variables would attain equilibrium values and the number of adopters increases as immigration rate and intrinsic growth rate of total population increase but it decreases due to emigration.It further decreases as the number of adopters' of substitute technology increases.

5. Numerical Simulation

To check the feasibility of our analysis regarding the existence of E^* and the corresponding stability conditions, we conduct some numerical simulation of model (2.3) by choosing the following values of parameters:-

$\gamma = 0.01, M_0 = 10000, K = 100000, r_0 = 0.023, \beta = 0.004, p = 0.001, q = 0.004, A = 200, e = 0.001, \alpha = 0.001, \nu = 0.001$

It is found that for the above set of parameters, the equilibrium values $E^*(N_a^*, N_s^*, M^*)$ of the three dependent variables come out to be $N_a^* = 29220, N_s^* = 14150, M^* = 52730$. It is pointed out here that for above set of parameters, the conditions for linear and nonlinear stability are also satisfied.

To see the effects of various parameters on the dependent variables, we have solved the Eqs. (2.5),(2.6) and (2.7) of model and plotted these in Figures (B1 - B4).

From Figure B1 and B3 it is clear that as the rate of immigration and intrinsic growth of cumulative population density increase, adopters population densities of both primary and substitute technology increase.

From Figure B3 it is evident that as growth rate coefficient of adopting substitute technology increases, the density of adopters population of primary technology decreases but the density of adopters of substitute technology increases.

From Figure B4 it is easily seen that if the rate of discontinuance (α) of the adopters of primary technology increases, the densities of adopters of both primary and substitute technology decrease.

6. Simulation of Stability Condition

In this section we plot, the stability condition w.r.t. various values of data sets to show its feasibility, in figures B5 -B10).

For plotting the graphs the following notation has been used:

$$S_0 = \frac{S_1}{10^5}$$

In Figure (B5) the effect of external marketing influence p shows the stabilizing nature of parameter, as the coefficient p increases the positivity of the stability condition also increases.

In Figure (B6) the effect of internal influence is plotted against the required stability condition of system as internal Influence increases the stability condition becomes more positive showing its stabilizing nature.

In Figure (B7) the stability condition is plotted over various values of adopters' discontinuance rate α.From the monotonically decreasing graph it is clear that as the value of α increases the positivity of stability condition decreases showing destabilizing nature of α.

In Figure (B8) the effect of immigration co-efficient A is plotted.With increasing value of A the stability condition becomes more positive showing its stabilizing nature.

In Figure (B9) the effect of growth rate coefficient r_0 is shown.The graph clearly shows the stabilizing nature of r_0.

In Figure (B10) the effect of interaction coefficient β on stability conditions is plotted.It shows that for increased value of β the stability condition becomes more positive showing the stabilizing nature of β.

7. Conclusions

It has been known for many decades that the diffusion of innovation is governed by Bass model which takes into account the effect of external and internal influences on the diffusion process but the total population has been assumed to be a constant.However, in the real market scenario, the total population changes because of immigration,emigration as well as due to birth and death processes. Further adopters' switching from primary technology to a new substitute technology also affects the diffusion process of primary technology. Thus, a nonlinear model for diffusion process of a new technology must consider demographic and switching effects. In this paper, therefore, we have considered the effects of demographic factors such as immigration,emigration and intrinsic growth of the population in the modeling process. The effect of leaving the primary technology by its adopters and adopting a substitute technology has also been studied.The proposed non linear model has been analyzed by stability theory of differential equations and simulation. The following results have been obtained:

1. The densities of adopters of both technologies increase as immigration of population increases.

2. The densities of adopters of both technologies increase as the intrinsic growth rate of population increases.

3. The densities of adopters of both technologies decrease as the rate of discontinuance of primary technology by its adopters increases.

4. The density of adopters' of primary technology decreases as the growth rate coefficient of adopters' of substitute technology increases.

It may be pointed out that our study does not include variable external and internal influences. This we leave for future research.

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Appendix A.

Proof of Theorem 4.1. Here we consider the local stability of the equilibrium $E^*({N_a}^*, {N_s}^*, M^*)$.For this we linearize Eqs. (2.5), (2.6) and (2.7). To study the local stability we use the following perturbed function by considering:

$$N_a = {N_a}^* + n_a$$
$$N_s = {N_s}^* + n_s$$
$$M = M^* + m$$

$$\frac{dn_a}{dt} = -[\frac{p(M^* - {N_s}^*)}{{N_a}^*} + \frac{q{N_a}^*}{M^*}]n_a - [p + \frac{q{N_a}^*}{M^*}]n_s + [p + \frac{q{N_a}^*({N_a}^* + {N_s}^*)}{{M^*}^2}]m \quad (A.1)$$

$$\frac{dn_s}{dt} = \beta n_a - e n_s \quad (A.2)$$

$$\frac{dm}{dt} = -\alpha n_a + [r - \frac{2r_0 M^*}{K}]m \quad (A.3)$$

We consider lyapunov function as:

$$V(n_a, n_s, m) = \frac{1}{{N_a}^*}{n_a}^2 + \frac{c_1}{2}{n_s}^2 + \frac{c_2}{2M^*}m^2 \quad (A.4)$$

By differentiating the above function and after using [A.1-A.3] we have,

$$\begin{aligned} \frac{\mathrm{dV}}{\mathrm{dt}} &= -{n_a}^2(\frac{p(M^* - {N_s}^*)}{{{N_a}^*}^2} + \frac{q}{M^*}) \\ &- c_1 e(n_s)^2 - c_2\frac{r_0}{M^* K}(2M^* - \frac{rK}{r_0})m^2 \\ &+ [(\frac{p}{{N_a}^*} + \frac{q({N_a}^* + {N_s}^*)}{{M^*}^2}) - c_2\frac{\alpha}{M^*}]mn_a \\ &+ (c_1\beta - \frac{p}{{N_a}^*} - \frac{q}{M^*})n_s n_a \end{aligned} \quad (A.5)$$

To maximize the negative definiteness of $\frac{\mathrm{dV}}{\mathrm{dt}}$, we choose:

$$c_1 = \frac{p}{\beta {N_a}^*} \quad (A.6)$$
$$c_2 = \frac{q({N_a}^* + {N_s}^*)}{\alpha M^*} \quad (A.7)$$

to get,

$$\begin{aligned} \frac{\mathrm{dV}}{\mathrm{dt}} &= -[(\frac{p(M^* - {N_s}^*)}{{{N_a}^*}^2} + \frac{q}{M^*}){n_a}^2 \\ &+ c_1 e{n_s}^2 + c_2\frac{r_0}{M^* K}(2M^* - \frac{rK}{r_0})m^2 \\ &- \frac{p}{{N_a}^*}mn_a + \frac{q}{M^*}n_a n_s] \end{aligned} \quad (A.8)$$

Now with the help of following inequalities:

$$\pm XY \leq \frac{X^2+Y^2}{2} \tag{A.9}$$

$\frac{\mathrm{dV}}{\mathrm{dt}}$ can be written as:

$$\begin{aligned}\frac{\mathrm{dV}}{\mathrm{dt}} &\leq -[\frac{p(M^*-N_s{}^*)}{N_a{}^*}^2 n_a{}^2 + \frac{q}{M^*}n_a^2 \\ &+c_1 e n_s^2 + c_2\frac{r_0}{M^*K}(2M^* - \frac{rK}{r_0})m^2 \\ &+\frac{p}{2N_a{}^*}[m^2 + n_a{}^2] + \frac{q}{2M^*}[n_a{}^2 + n_s{}^2]]\end{aligned} \tag{A.10}$$

After putting the values of c_1 and c_2, we finally get:

$$\begin{aligned}\frac{\mathrm{dV}}{\mathrm{dt}} &\leq -[[\frac{p(M^*-N_s{}^*)}{N_a{}^{*2}} + (\frac{p}{2N_a{}^*} + \frac{3q}{2M^*})](n_a)^2 \\ &+[\frac{pe}{\beta N_a{}^*} + \frac{q}{2M^*}](n_s)^2 \\ &+[\frac{q(N_a{}^*+N_s{}^*)}{\alpha M^*}\frac{r_0}{M^*K}(2M^* - \frac{rK}{r_0}) + \frac{p}{2N_a{}^*}]m^2]\end{aligned} \tag{A.11}$$

Thus, $\frac{\mathrm{dV}}{\mathrm{dt}}$ will be always negative definite provided the following inequality is satisfied:-

$$s_0 = \frac{M^*p}{2N_a{}^*} + \frac{q(N_a{}^*+N_s{}^*)}{\alpha M^*}\frac{r_0}{K}(2M^* - \frac{rK}{r_0}) > 0 \tag{A.12}$$

which is given in Eq.(4.1) □

Appendix B.

Proof of Theorem 4.2. To prove this theorem, we consider the following positive definite function about E^*,

$$V(N_a, N_s, M) = (N_a - N_a{}^* - N_a{}^* ln\frac{N_a}{N_a{}^*}) + \frac{C_1}{2}(N_s - N_s{}^*)^2 + C_2(M - M^* - M^* ln\frac{M}{M^*}) \tag{B.1}$$

where C_1 and C_2 are positive constants, to be chosen appropriately.

Differentiating above equation with respect to t along the solutions of above mentioned system and after a simple algebraic manipulation we get,

$$\begin{aligned}\frac{\mathrm{dV}}{\mathrm{dt}} &= -(N_a - N_a{}^*)^2(\frac{p(M-N_s)}{N_aN_a{}^*} + \frac{q}{M}) \\ &-C_1e(N_s - N_s{}^*)^2 - C_2\frac{r_0}{MK}(M + M^* - \frac{rK}{r_0})(M - M^*)^2 \\ &+[(\frac{p}{N_a{}^*} + \frac{q(N_a{}^*+N_s{}^*)}{MM^*}) - C_2\frac{\alpha}{M}](M - M^*)(N_a - N_a{}^*) \\ &+(C_1\beta - \frac{p}{N_a{}^*} - \frac{q}{M})(N_a - N_a{}^*)(N_s - N_s{}^*)\end{aligned} \tag{B.2}$$

Now maximizing the negative definiteness of $\frac{\mathrm{dV}}{\mathrm{dt}}$ we choose,

$$C_1 = \frac{p}{\beta {N_a}^*} \tag{B.3}$$

$$C_2 = \frac{q({N_a}^* + {N_s}^*)}{\alpha M^*} \tag{B.4}$$

we then have,

$$\begin{aligned} \frac{\mathrm{dV}}{\mathrm{dt}} &= -(\frac{p(M-N_s)}{N_a {N_a}^*} + \frac{q}{M})(N_a - {N_a}^*)^2 \\ &-C_1 e(N_s - {N_s}^*)^2 - C_2 \frac{r_0}{MK}(M + M^* - \frac{rK}{r_0})(M - M^*)^2 \\ &+\frac{p}{{N_a}^*}(M - M^*)(N_a - {N_a}^*) - \frac{q}{M}(N_a - {N_a}^*)(N_s - {N_s}^*) \end{aligned} \tag{B.5}$$

Now with the help of the following inequalities:

$$\pm XY \leq \frac{X^2 + Y^2}{2} \tag{B.6}$$

$\frac{\mathrm{dV}}{\mathrm{dt}}$ can be written as:

$$\begin{aligned} \frac{\mathrm{dV}}{\mathrm{dt}} &\leq -[\frac{p(M-N_s)}{N_a {N_a}^*}(N_a - {N_a}^*)^2 + \frac{q}{M}(N_a - {N_a}^*)^2 \\ &+C_1 e(N_s - {N_s}^*)^2 + C_2 \frac{r_0}{MK}(M + M^* - \frac{rK}{r_0})(M - M^*)^2 \\ &+\frac{p}{2{N_a}^*}[(M - M^*)^2 + (N_a - {N_a}^*)^2] + \frac{q}{2M}[(N_a - {N_a}^*)^2 + (N_s - {N_s}^*)^2]] \end{aligned} \tag{B.7}$$

After putting the values of C_1 and C_2:

$$\begin{aligned} \frac{\mathrm{dV}}{\mathrm{dt}} &\leq -[\frac{p(M-N_s)}{N_a {N_a}^*}(N_a - {N_a}^*)^2 + (\frac{p}{2{N_a}^*} + \frac{3q}{2M})(N_a - {N_a}^*)^2 \\ &+(\frac{pe}{\beta {N_a}^*} + \frac{q}{2M})(N_s - {N_s}^*)^2 \\ &+(\frac{q({N_a}^* + {N_s}^*)}{\alpha M^*}\frac{r_0}{MK}(M + M^* - \frac{rK}{r_0}) + \frac{p}{2{N_a}^*})(M - M^*)^2] \end{aligned} \tag{B.8}$$

$\frac{\mathrm{dV}}{\mathrm{dt}}$ will be negative definite inside the region of attraction provided the following inequality is satisfied:-

$$S_0 = \frac{M_l p}{2{N_a}^*} + \frac{q({N_a}^* + {N_s}^*)}{\alpha M^*}\frac{r_0}{K}(M_l + M^* - \frac{rK}{r_0}) > 0 \tag{B.9}$$

□

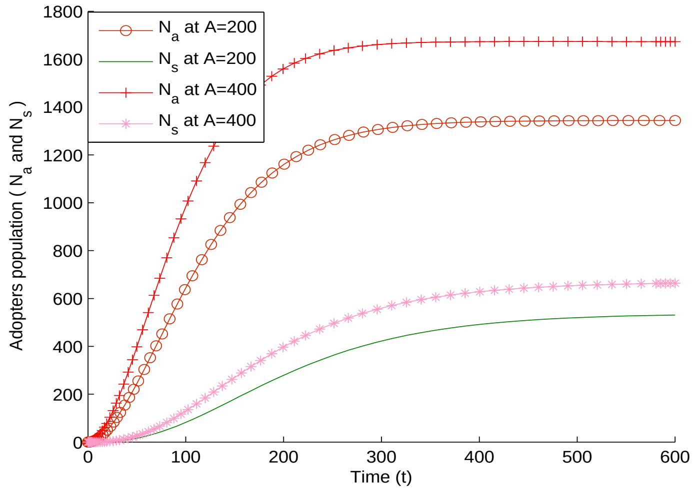

Figure B.1: Effect of Immigration rate (A) on Adopters Population Densities of Primary and Substitute Technology(N_a and N_s).

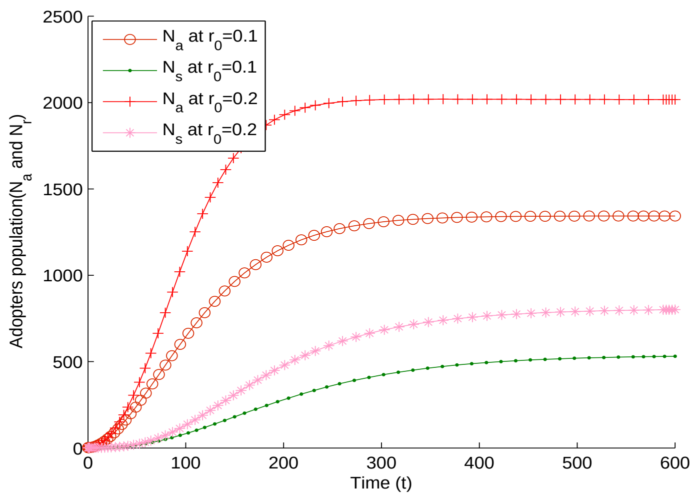

Figure B.2: Effect of Intrinsic Growth Rate (r_0) on Adopters Population Densities of Primary and Substitute Technology(N_a and N_s).

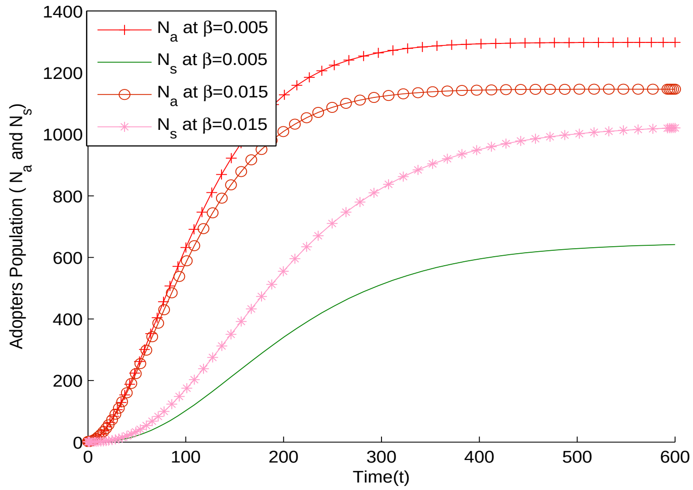

Figure B.3: Effect of growth rate coefficient of adopting substitute technology (β) on Adopters Population Densities of Primary and Substitute Technology(N_a and N_s).

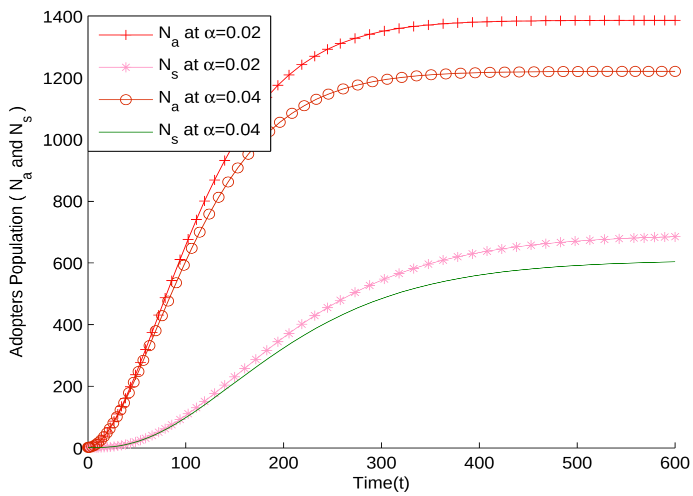

Figure B.4: Effect of discontinuance rate coefficient of Primary Technology (α) on Adopters Population Densities of Primary and Substitute Technology (N_a and N_s).

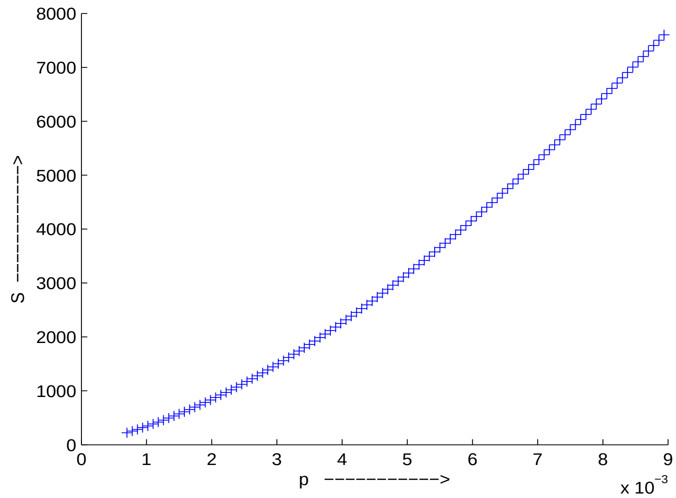

Figure B.5: Effect of external influence p on stability conditions.

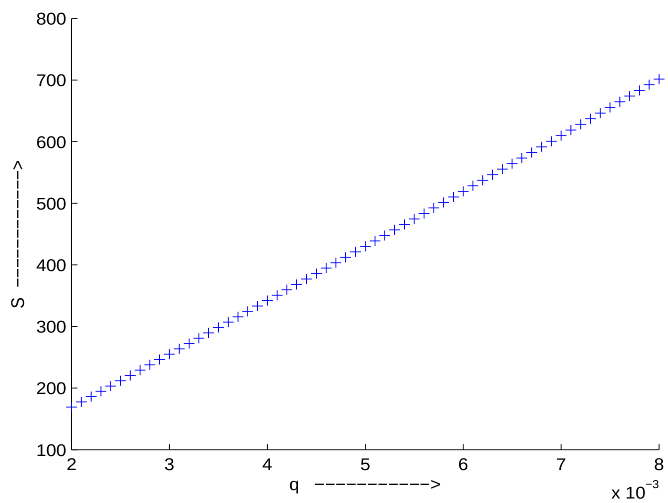

Figure B.6: Effect of internal influence q on stability conditions.

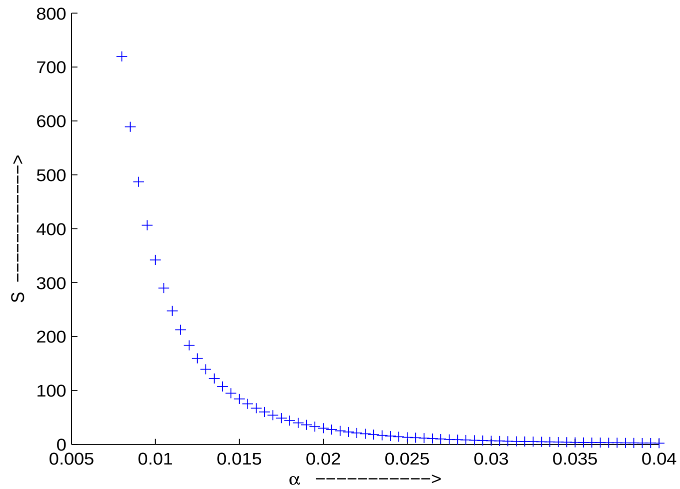

Figure B.7: Stability Conditions for various values of discontinuance rate (α).

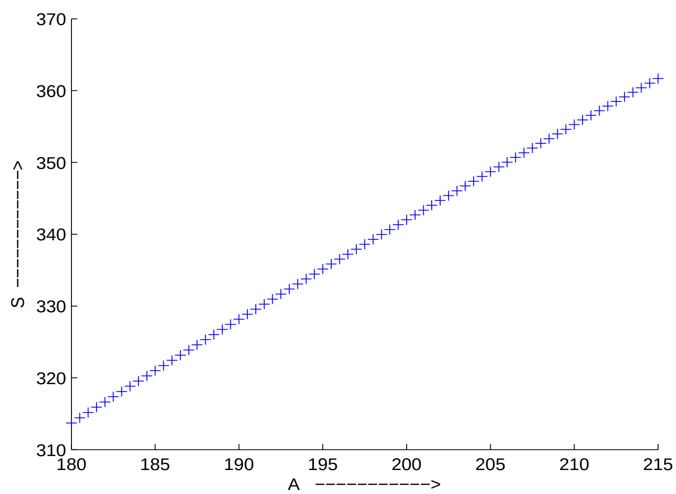

Figure B.8: Effect of immigration A on stability conditions.

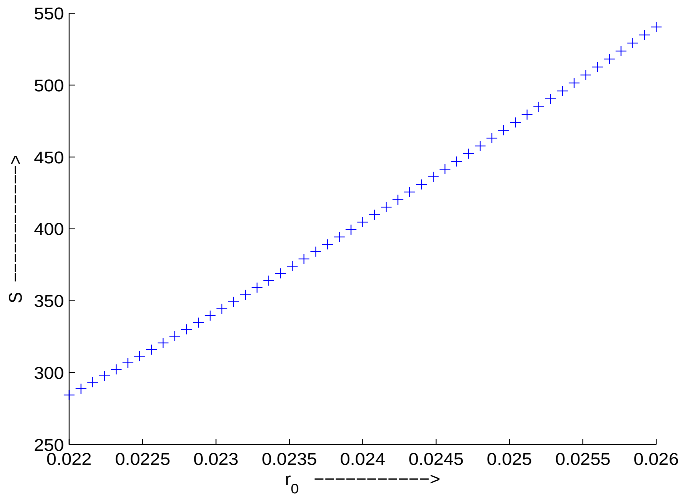

Figure B.9: Effect of growth rate coefficient r_0 on stability conditions.

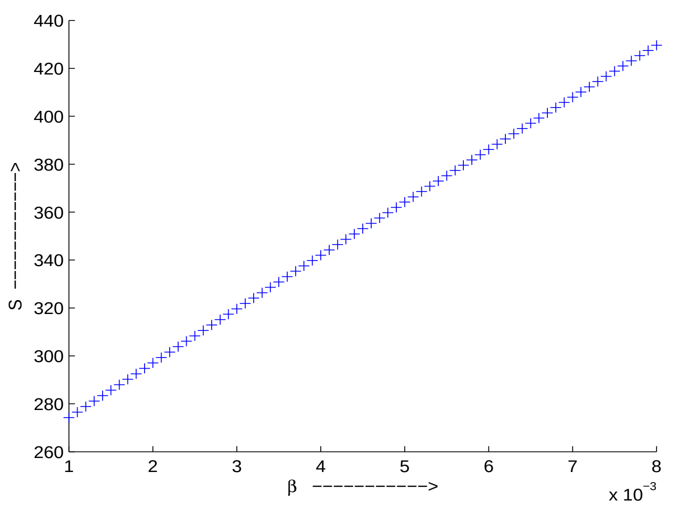

Figure B.10: Effect of interaction coefficient β on stability Conditions.

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Modeling the effect of Media on Diffusion of Innovation in a Population

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Abstract

In this paper, a non-linear mathematical model is proposed and analyzed to study the effect of media on the diffusion (spread) of innovation in a population. The model consists of three dependent variables, namely, the number of non-adopters, the number of adopters and the variable governing the cumulative density of media involved in the diffusion process. In the modelling process, it is assumed that the coefficient of external influence is a variable and is a linear function of the media variable dedicated to diffusion of innovation under consideration while the coefficient of internal influence is assumed to be a constant. The nonlinear model is analyzed by stability theory of differential equations and numerical simulations. The analysis shows that the number of adopters increases swiftly in the population as the external influence caused by media increases. This result is confirmed by numerical simulations.

Keywords: Diffusion models, spread of innovation, mathematical model, stability analysis, communication channels.

1. Introduction

For the successful spread of any innovation (new technology) in a population it must be communicated properly to the people. Media is a very useful method of propagation, for example, loudspeaker, TV, Radio, YouTube, Facebook, etc. This aspect has not been considered in the study of innovation diffusion, where the core concept of diffusion process has been based on the framework proposed by Bass

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(1969). Since then many diffusion models have been proposed and studied to understand and predict the diffusion trend by highlighting the role of transmission between adopters and non adopters [Bass (1969), Centrone et al. (2007), Easingwood et al. (1981), Floyd (1968), Mahajan et al. (1978, 1990), Sharif and Kabir (1976), Shukla et al. (2012), Wang et al. (2003), Yu and Wang (2003, 2006, 2007)]. It is mentioned here that in most of these models, apart from Shukla et al. (2012), the external influence factors have been assumed constants, which is not realistic in dynamic market population as the demography changes because of immigration, emigration, discontinuation of use of an innovation, etc. Mathematical modeling of effect of media as a separate variable has also not been discussed in most of the existing relevant literature. Therefore, in this paper, we propose a non-linear mathematical model by considering the effect of variable external influence involving media governed by the following assumptions-

i The coefficient of external influence is assumed to be linearly related to the variable governing the cumulative density of media.
ii The media variable is assumed to be directly proportional to number of non-adopters of the innovation at any time.
iii A fraction of the total population is not migrating from the population.
iv A certain fractions of the non-adopters and adopters populations migrate out of the main population.

2. Mathematical Model

Let N_o, N_a represent numbers of non adopters and adopters population respectively and N is the total variable population. Suppose M is the variable governing media promoting the spread of innovation. Let A be the immigration rate, $\overline{N}$ be the number of nonmigrating population. p be the variable cumulative coefficient of external influence such that $p = p_o + p_1 M$ where p_o is the constant coefficient of external influence in absence of media and p_1 is the coefficient of external influence due to media.

Using above mentioned considerations and assumptions the following mathematical model is proposed:-

$$
\begin{aligned}
\frac{dN_o}{dt} &= A - \gamma(N - \overline{N}) - pN_o - q_o\left(\frac{N_a N_o}{N}\right) - eN_o + \nu N_a \\
\frac{dN_a}{dt} &= pN_o + q_o\left(\frac{N_a N_o}{N}\right) - eN_a - \alpha N_a - \nu N_a \\
\frac{dM}{dt} &= \mu N_o - \mu_o M
\end{aligned}
\tag{2.1}
$$

$N_o(0) > 0, N_a(0) \geq 0, M(0) \geq 0$

Here q_o is the constant internal influence. The constant e is the emigration rate of the population from potential market, ν is the rate of adopters moving back to non adopters, γ is a constant of proportionality and α is the rate by which the adopters discontinue the use of innovation and would neither go back to non adopters class nor follow any other innovation. All these variables and parameters are positive constants.

Using $p = p_o + p_1 M$ and $N = N_o + N_a$, we can rewrite the model (2.1) as follows,

$$\begin{aligned} \frac{dN_a}{dt} &= p_o(N - N_a) + p_1 M(N - N_a) + q_o\left(\frac{N_a(N - N_a)}{N}\right) - e_1 N_a \\ \frac{dN}{dt} &= A_1 - (e + \gamma)N - \alpha N_a \\ \frac{dM}{dt} &= \mu(N - N_a) - \mu_o M \end{aligned} \tag{2.2}$$

where

$$e_1 = e + \alpha + \nu, A_1 = A + \gamma\overline{N} \tag{2.3}$$

and

$$N(0) > 0, N_a(0) \geq 0, M(0) \geq 0 \tag{2.4}$$

The region of attraction for model system (2.2) is given in the following lemma, which is needed later in the analysis.

2.1. **Lemma**

The region of attraction for all solutions initiating in the positive octant is given by:

$$\Omega = \{(N_a, N, M) : 0 \leq N_a \leq \frac{A_1}{e+\gamma}, \frac{A_1}{e+\alpha+\gamma} \leq N \leq \frac{A_1}{e+\gamma}, 0 \leq M \leq \frac{\mu A_1}{\mu_0(e+\gamma)}\} \tag{2.5}$$

Proof . Since $0 \leq N_a \leq N$, we have

$A_1 - (e+\alpha+\gamma)N \leq \frac{dN}{dt} \leq A_1 - (e+\gamma)N$, and hence

$$0 \leq N_a \leq \frac{A_1}{e+\gamma} and \frac{A_1}{e+\alpha+\gamma} \leq N \leq \frac{A_1}{e+\gamma} \tag{2.6}$$

Also since

$$\frac{dM}{dt} \leq \mu N_{max} - \mu_o M \tag{2.7}$$

we have

$$0 \leq M \leq \frac{\mu A_1}{\mu_0(e+\gamma)} \tag{2.8}$$

□

3. Equilibrium Analysis

The model (2.2) has only one non negative equilibrium $E^*(N_a^*, N^*, M^*)$, which can be calculated by the following equations,

$$p_o(N-N_a) + p_1 M(N-N_a) + q_o \frac{N_a(N-N_a)}{N} - e_1 N_a = 0 \tag{3.1}$$

$$A_1 - (e+\gamma)N - \alpha N_a = 0 \tag{3.2}$$

$$\mu(N-N_a) - \mu_o M = 0 \tag{3.3}$$

From equations (3.2) and (3.3), we get

$$M = \frac{\mu}{\mu_o}(N-N_a),\ N_a = \frac{A_1-(e+\gamma)N}{\alpha},\ N-N_a = \frac{(e+\gamma+\alpha)N - A_1}{\alpha} \tag{3.4}$$

Using (3.4) in (3.1), we get

$$\begin{aligned} F(N) = &\left[p_o + \frac{p_1\mu}{\mu_0}\left(\frac{(e+\gamma+\alpha)N - A_1}{\alpha}\right)\right]\left(\frac{(e+\gamma+\alpha)N - A_1}{\alpha}\right) \\ &+ q_0\left(\frac{A_1-(e+\gamma)N}{\alpha N}\right)\left(\frac{(e+\gamma+\alpha)N - A_1}{\alpha}\right) \\ &- e_1\left(\frac{A_1-(e+\gamma)N}{\alpha}\right) = 0 \end{aligned} \tag{3.5}$$

From (3.5), we get the following:

$F\left(\frac{A_1}{e+\gamma+\alpha}\right) = -\frac{e_1 A_1}{e+\gamma+\alpha} < 0$ and

$F\left(\frac{A_1}{e+\gamma}\right) = \left(p_0 + \frac{p_1 \mu A_1}{\mu_0(e+\gamma)}\right)\frac{A_1}{e+\gamma} > 0$

Hence, there exists at least one root of N in the interval $\frac{A_1}{e+\alpha+\gamma} < N < \frac{A_1}{e+\gamma}$. For uniqueness of this root, we need to show that $F^{'}(N) > 0$ in $\frac{A_1}{e+\alpha+\gamma} < N < \frac{A_1}{e+\gamma}$. We can rewrite F(N) as

$$F(N) = \left(\frac{(e+\gamma+\alpha)N - A_1}{\alpha}\right) G(N) - \frac{e_1}{\alpha}(A_1 - (e+\gamma)N) = 0 \tag{3.6}$$

where

$$G(N) = p_0 + \frac{p_1 \mu}{\mu_0}\left(\frac{(e+\gamma+\alpha)N - A_1}{\alpha}\right) + q_0\left(\frac{A_1 - (e+\gamma)N}{\alpha N}\right) \tag{3.7}$$

Now differentiating (3.6), we get

$$F^{'}(N) = \left(\frac{(e+\alpha+\gamma)}{\alpha}G(N)\right) + \left(\frac{(e+\alpha+\gamma)N - A_1}{\alpha}\right)G^{'}(N) + \frac{e_1(e+\gamma)}{\alpha} \tag{3.8}$$

or

$$\begin{aligned}(A_1 - (e+\gamma)N)F^{'}(N) = &\frac{(e+\alpha+\gamma)}{\alpha}G(N)(A_1 - (e+\gamma)N) \\ &+ (A_1 - (e+\gamma)N)\left(\frac{(e+\alpha+\gamma)N - A_1}{\alpha}\right)G^{'}(N) \\ &+ \frac{e_1}{\alpha}(e+\gamma)(A_1 - (e+\gamma)N)\end{aligned} \tag{3.9}$$

Using equation (3.6) again,

$$\begin{aligned}(A_1 - (e+\gamma)N)F^{'}(N) = &\frac{(e+\alpha+\gamma)}{\alpha}G(N)(A_1 - (e+\gamma)N) \\ &+ (A_1 - (e+\gamma)N)\left(\frac{(e+\alpha+\gamma)N - A_1}{\alpha}\right)G^{'}(N) \\ &+ \frac{(e+\gamma)}{\alpha}((e+\alpha+\gamma)N - A_1)G(N)\end{aligned} \tag{3.10}$$

Putting the values of $G(N)$ and $G'(N)$ from equation (3.7) and simplifying, we get

$$
\begin{aligned}
(A_1-(e+\gamma)N)F'(N) = & \frac{(e+\alpha+\gamma)}{\alpha}(A_1-(e+\gamma)N)\left(p_0+\frac{p_1\mu}{\mu_0}\left(\frac{(e+\gamma+\alpha)N-A_1}{\alpha}\right)\right) \\
& + \frac{(A_1-(e+\gamma)N}{\alpha}((e+\gamma+\alpha)N-A_1)\frac{p_1\mu}{\mu_0}\frac{(e+\alpha+\gamma)}{\alpha} \\
& + \left(\frac{(A_1-(e+\gamma)N}{\alpha}\right)^2\frac{A_1q_0}{N^2} \\
& + \frac{(e+\gamma)}{\alpha}((e+\alpha+\gamma)N-A_1)\left(p_0+\frac{p_1\mu}{\mu_0}\left(\frac{(e+\gamma+\alpha)N-A_1}{\alpha}\right)\right) > 0.
\end{aligned}
\tag{3.11}
$$

Thus, there exists a unique non trivial equilibrium $E^*(N_a^*, N^*, M^*)$in the region bounded by $0 \le N_a \le \frac{A_1}{(e+\gamma)}$ and $\frac{A_1}{(e+\alpha+\gamma)} \le N \le \frac{A_1}{e+\gamma}$.

4. Stability Analysis

***Theorem* 4.1.** *The equilibrium E^* is locally asymptotically stable without any conditions. For proof, see Appendix A.*

***Theorem* 4.2.** *The equilibrium E^* is non linearly stable inside the region of attraction Ω, if the following condition are satisfied:*

$$p_1\left(N^*-N_a^*\right)\mu\alpha < 2p_0\mu_0\left(e+\gamma\right) \tag{4.1}$$

$$\left(\frac{p_1\mu N_{max}}{\mu_0 N_a^*}+\frac{q_0 N_{max}}{N_{min}N^*}\right)^2 \le 2\frac{q_0}{N^*}\frac{p_0\left(e+\gamma\right)}{N_a^*\alpha} \tag{4.2}$$

For proof, see Appendix B.

We note that when $p_1 = 0$ the inequality (4.1) is automatically satisfied and the chance of satisfying (4.2) increases, this implies that media has stabilizing effect on the system.

5. Numerical Simulation

Now we conduct some numerical simulations by choosing following set of values of parameters:

$A = 1000, e = 0.04, p_0 = 0.09, q_0 = 0.1, \mu_0 = 0.05, \alpha = 0.004, \gamma = 0.02, \nu = 0.001, p_1 = 0.00000875, \mu = 0.04, \overline{N} = 1000.$

With this set of values of parameters, conditions (4.1) and (4.2) are satisfied and the value of equilibrium point is obtained as $N_a^* = 13071.9, N^* = 16128.54, M^* = 2445.31$. Now we plot graphs in figures 1 - 8 to verify results obtained in sections 3 and 4, using the values of parameters mentioned above.

In Fig.1 and Fig.2 it is shown that curves with different initial conditions tend to equilibrium point E^* as time increases, thereby showing the nonlinear stability of E^*. In Fig. 3, we plot the effect of immigration rate on adopters population taking three different values of parameter of immigration rate A as 1000, 1500 and 2000, keeping other parameters fixed to the values mentioned above. This shows that as immigration increases adopter population increases. In Fig.4, we see that adopters population decreases as μ_0 increases. Similarly in figures 5, 6, 7 and 8 we see that number of adopters increases with increase in μ, p_1, p_0 and q_o.

6. Conclusion

In this paper, a nonlinear model has been proposed and analyzed to study the effect of media on diffusion of innovation under consideration. In the modelling process, the external influence is considered as a linear function of the variable governing media. The model has been considered to consist of three variables, namely, the number of non-adopters and the number of adopters and the variable governing the cumulative density of media. The following assumptions have been made with respect to Bass Model, Bass (1969)

i The coefficient of external influence is a linear function of the cumulative density of media.

ii A fraction of the total population in the market scenario is not migrating from the region under consideration.

iii A certain fraction of the non-adopters from adopters population migrates out of the population under consideration.

iv The coefficient of internal influence is a constant.

The mathematical model is analyzed by using the stability theory of differential equations and numerical simulation. The analysis has shown that the number of adopters increases due to the variability of external influence. This result is confirmed by numerical simulations.

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Appendix A.

Proof of Theorem 4.1. Here we examine the local stability of the equilibrium $E^*(N_a^*, N^*, M^*)$. The Jocabian matrix at E^* for the linearised form of (2.2) is as follows,

$$J=\begin{bmatrix} -p_0 - p_1M^* + q_0 - \frac{2q_0N_a^*}{N^*} - e_1 & p_0 + p_1M^* + \frac{q_0N_a^{*2}}{N^{*2}} & 0 \\ -\alpha & -e-\gamma & 0 \\ -\mu & \mu & -\mu_0 \end{bmatrix}$$

Using equations (3.1) and (3.2) at the equilibrium point $E^*(N_a^*, N^*, M^*)$, we can express

$$-p_0 - p_1M^* + q_0 - \frac{2q_0N_a^*}{N^*} - e_1 = -\frac{p_0N^*}{N_a^*} - \frac{p_1N^*M^*}{N_a^*} - \frac{q_0N_a^*}{N^*} \tag{A.1}$$

and thus we rewrite the Jacobian matrix as

$$J=\begin{bmatrix} -\frac{p_0N^*}{N_a^*} - \frac{p_1N^*M^*}{N_a^*} - \frac{q_0N_a^*}{N^*} & p_0 + p_1M^* + \frac{q_0N_a^{*2}}{N^{*2}} & 0 \\ -\alpha & -e-\gamma & 0 \\ -\mu & \mu & -\mu_0 \end{bmatrix}$$

The characteristic equation is of the form

$$(\mu_0 + \lambda)(\lambda^2 - (a_{11} + a_{22})\,\lambda + (a_{11}a_{22} - a_{12}a_{21})) = 0 \tag{A.2}$$

where,

$$a_{11} = -\frac{p_0N^*}{N_a^*} - \frac{p_1N^*M^*}{N_a^*} - \frac{q_0N_a^*}{N^*} < 0$$

$$a_{22} = -e - \gamma < 0$$

$$a_{12} = p_0 + p_1M^* + \frac{q_0N_a^{*2}}{N^{*2}} > 0$$

and

$$a_{21} = -\alpha < 0.$$

Since $a_{11} + a_{22} < 0, (a_{11}a_{22} - a_{12}a_{21}) > 0$, all eigen values lie in the negative, left half plane. Thus, the equilibrium $E^*(N_a^*, N^*, M^*)$ is locally asymptotically stable without any condition.

Appendix B.

Proof of Theorem 4.2. To establish the conditions of global stability stated in theorem(4.2), we consider the following positive definite function about E^*

$$V(N_a, N, M) = c_1\left(N_a - N_a^* - N_a^* ln\frac{N_a}{N_a^*}\right) + \frac{c_2}{2}(N - N^*)^2 + \frac{c_3}{2}(M - M^*)^2 \quad \text{(B.1)}$$

Differentiating with respect to t along the system (2.2) and applying some simple algebraic manipulations, we get,

$$\begin{aligned}\frac{dV}{dt} =& c_1\left(-\frac{p_0 N}{N_a N_a^*} - \frac{p_1 M N}{N_a N_a^*} - \frac{q_0}{N^*}\right)(N_a - {N_a}^*)^2 \\ &+ \left(c_1\left(\frac{p_1 M}{N_a^*} + \frac{p_0}{N_a}^* + \frac{q_0 N_a}{N N^*}\right) - c_2\alpha\right)(N - N^*)(N_a - N_a^*) \\ &+ \left(c_1\frac{p_1}{N_a^*}(N_a - N_a^*) - c_3\mu\right)(M - M^*)(N_a - N_a^*) \\ &+ c_3\mu(N - N^*)(M - M^*) \\ &- c_2(e+\gamma)(N - N^*)^2 - c_3\mu_0(M - M^*)^2\end{aligned} \quad \text{(B.2)}$$

Further, taking

$$c_1 = \mu, c_2 = \frac{p_0\mu}{N_a^*\alpha}, c_3 = \frac{p_1(N^* - N_a^*)}{N_a^*} \quad \text{(B.3)}$$

We obtain,

$$\begin{aligned}\frac{dV}{dt} =& -\mu\frac{q_0}{N^*}(N_a - N_a^*)^2 + \mu\left(\frac{p_1 M}{N_a^*} + \frac{q_0 N_a}{N N^*}\right)(N - N^*)(N_a - N_a^*) \\ &- \frac{1}{2}\frac{p_0\mu}{N_a^*\alpha}(e+\gamma)(N - N^*)^2 - \frac{1}{2}\frac{p_0\mu}{N_a^*\alpha}(e+\gamma)(N - N^*)^2 \\ &+ \frac{p_1(N^* - N_a^*)}{N_a^*}\mu(N - N^*)(M - M^*) - \frac{p_1(N^* - N_a^*)}{N_a^*}\mu_0(M - M^*)^2 \\ &- \mu(\frac{p_0 N}{N_a N_a^*} + \frac{p_1 M N}{N_a N_a^*})(N_a - N_a^*)^2\end{aligned} \quad \text{(B.4)}$$

$\frac{dV}{dt}$ is negative definite if the following conditions are satisfied-

$$\left(\frac{p_1 M}{N_a^*} + \frac{q_0 N_a}{N N^*}\right)^2 < 2\frac{q_0}{N^*}\frac{p_0(e+\gamma)}{N_a^*\alpha} \quad \text{(B.5)}$$

$$p_1 \left(N^* - N_a^*\right) \mu\alpha < 2 p_0 \mu_0 \left(e + \gamma\right) \tag{B.6}$$

Taking supremums over N_a, M and infimum over N in inequality (B.5), the inequality (B.5) holds strictly if

$$\left(\frac{p_1 \mu N_{max}}{\mu_0 N_a^*} + \frac{q_0 N_{max}}{N_{min} N^*} \right)^2 \leq 2 \frac{q_0}{N^*} \frac{p_0 \left(e + \gamma\right)}{N_a^* \alpha} \tag{B.7}$$

The inequalities (B.6) and (B.7) are the two conditions as stated in theorem 4.2. □

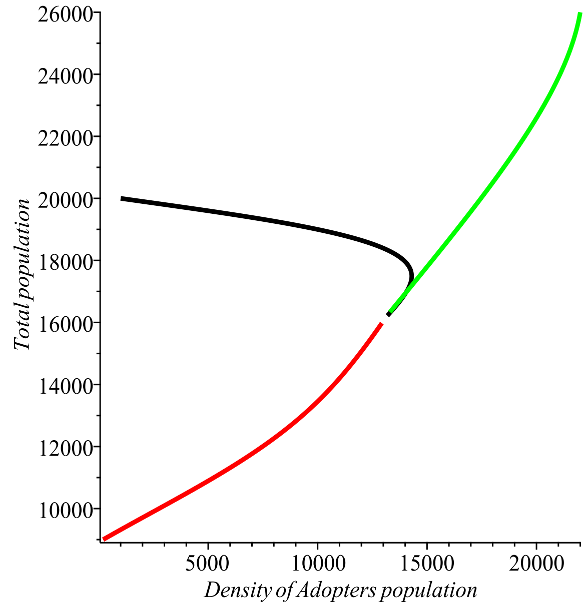

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Figure B.1: Figure1: Total population with adopters population taking different initial conditions.

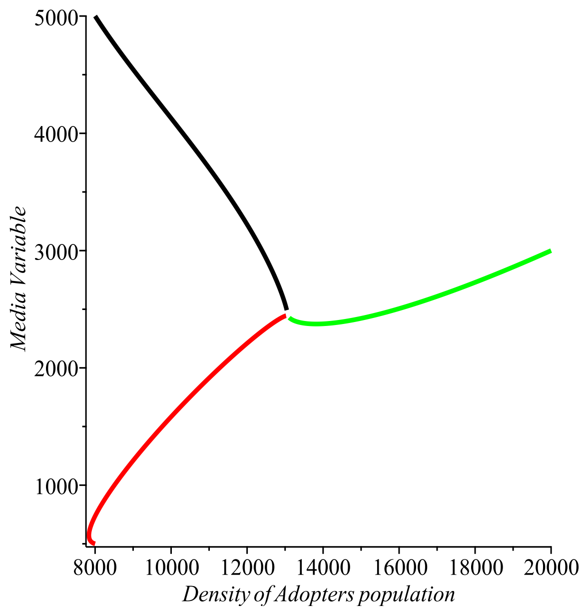

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Figure B.2: Figure2: Media with adopters population taking different initial conditions.

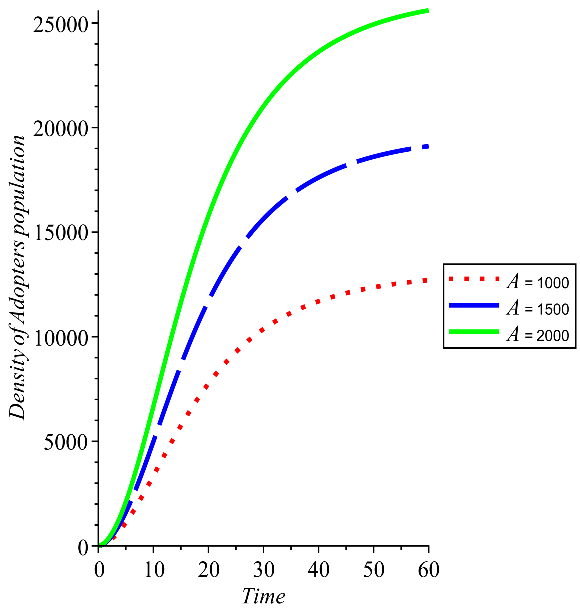

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Figure B.3: Figure3: Density of adopters population with time taking different values of A .

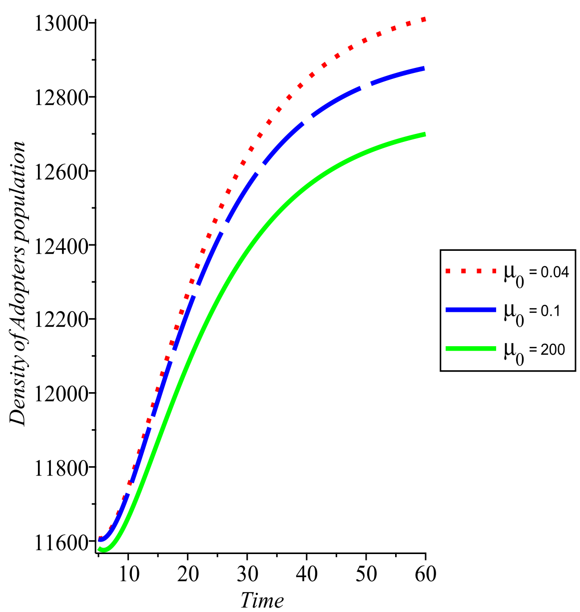

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Figure B.4: Figure4: Density of adopters population with time taking different values of μ_0.

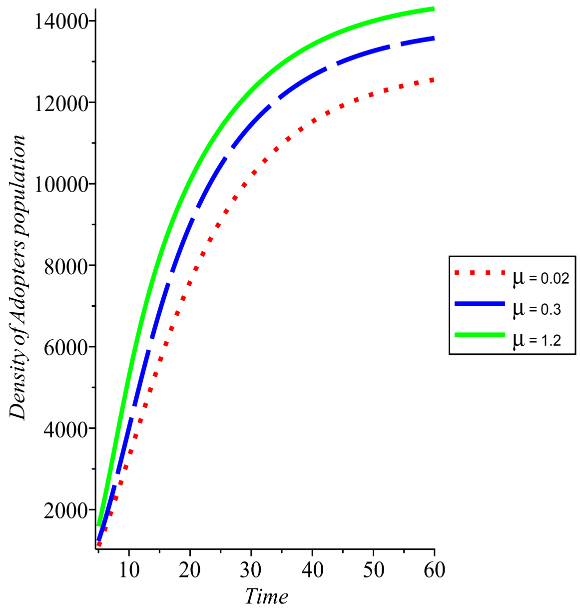

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Figure B.5: Figure5: Density of adopters population with time taking different values of μ.

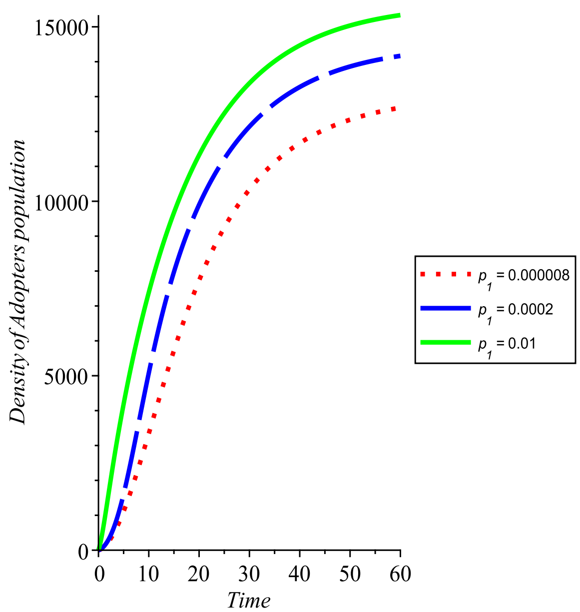

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Figure B.6: Figure6: Density of adopters population with time taking different values of p_1.

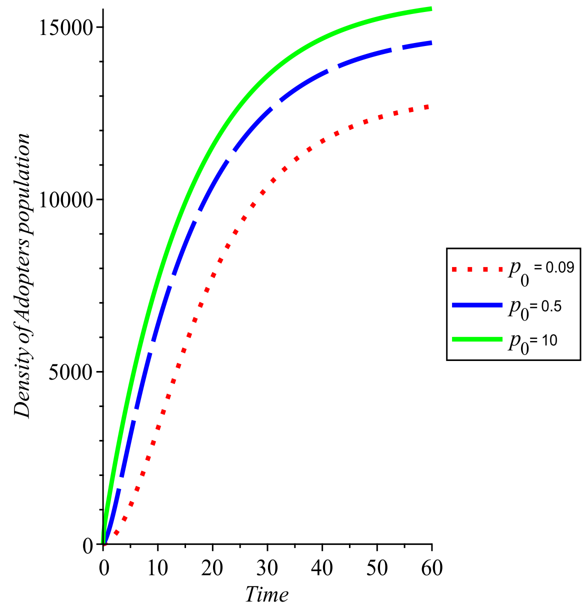

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Figure B.7: Figure7: Density of adopters population with time taking different values of p_0.

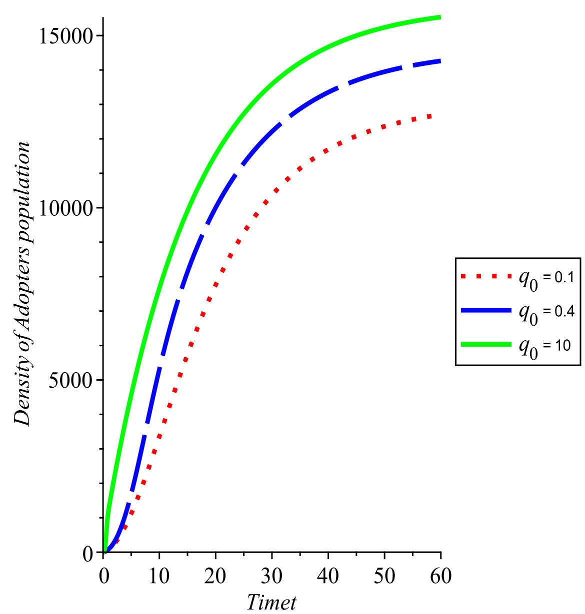

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Figure B.8: Figure8: Density of adopters population with time taking different values of q_0.

AIMS AND SCOPE:

Real phenomena are complex and non-linear. To study such a system, mathematical modeling plays an important role in predicting its future behavior both qualitatively and quantitatively by using Mathematical Methods, Analysis and Simulation. This new international journal is a multidisciplinary periodical that covers various aspects of modeling of complex phenomena with non-linear interactions, which arise in natural and human environments.

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SECTION 1

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