

Title: L^p -asymptotic behaviour of solutions to certain evolution equations

Abstract: In this talk, we investigate the long-time behaviour of solutions to evolution equations in different settings. We first prove a general result for homogeneous groups (e.g., the Heisenberg group), showing that for any complex Borel measure μ and a suitable approximate identity $\{\psi_t\}_{t>0}$, the normalized difference

$$\|\psi_t\|_{L^p(G)}^{-1} \|\mu * \psi_t - \mu(G)\psi_t\|_{L^p(G)}$$

converges to zero as $t \rightarrow \infty$, for all $p \in [1, \infty]$. This provides a unified approach for describing the L^p -asymptotic properties of solutions to many evolution equations across a variety of spaces.

Next, we focus on Riemannian symmetric spaces of noncompact type $X = G/K$, where for all left K -invariant functions $f \in L^1(X)$, the normalized difference

$$\|h_t\|_{L^p(X)}^{-1} \|f * h_t - M_p(f)h_t\|_{L^p(X)}$$

converges to zero as $t \rightarrow \infty$, where h_t is the heat kernel on X . We further extend this result to fractional heat kernels h_t^α with $\alpha \in (0, 1)$.

Finally, we present our recent work on similar asymptotic phenomena for non-local evolution equations on Riemannian symmetric spaces of noncompact type.