Title: Boundary Hardy type Inequalities.

Boundary Hardy inequality is classical result [1980's] which states that if $1 < p < \infty$ and $\Omega$ is a bounded Lipschitz domain in $\mathbb{R}^d$, then

$$\int_\Omega \frac{|u(x)|^p}{\delta_\Omega^p(x)} dx \leq C \int_\Omega |\nabla u(x)|^p dx, \forall\ u \in C_c^\infty(\Omega),$$

where $\delta_\Omega(x)$ is the distance function from $\partial\Omega$. B. Dyda [ 2004] generalised the above inequality to the fractional setting, which says, for $sp > 1$ and $s \in (0,1)$

$$\int_\Omega \frac{|u(x)|^p}{\delta_\Omega^{sp}(x)} dx \leq C \int_\Omega \int_\Omega \frac{|u(x) - u(y)|^p}{|x-y|^{d+sp}} dx dy,\ \forall\ u \in C_c^\infty(\Omega).$$

The first and the second inequality is not true for $p = 1$ and $sp = 1$ respectively. In this talk, I will present the appropriate inequalities for the critical cases: $p = 1$ for the first and $sp = 1$ for the second inequality. If time, permits I will touch upon another geometric inequality called Michael-Simmon-Sobolev inequality.

Thess are part of joint works with Adimurthi, G. Csato, P. Jana and V. Sahu.