

IDEAL FACTORIZATION IN NUMBER FIELDS, CLASS GROUPS, AND INTRODUCTION TO IWASAWA THEORY OF CLASS GROUPS

Given a number field K , there exists a corresponding finite abelian group known as the class group, denoted by $\mathcal{C}l_K$. This group is of great significance, as it captures how far the ring of integers of K is from being a PID. The investigation of class groups of number fields is a frequently explored theme in algebraic number theory. The study of class groups is generally carried out by considering their ℓ -Sylow subgroups, also known as ℓ -class groups. Iwasawa theory helps in studying the growth of the ℓ -class group of the intermediate fields in the $\mathbb{Z}_\ell$ -extension K_∞ of K for a prime ℓ .

In this talk, we will first explore the prime factorization of ideals in number fields and the construction of class groups. Further, we will try to understand Iwasawa modules formed by the class groups over $\mathbb{Z}_p$ -extension of number fields. We will conclude by looking into conjectures by Iwasawa and Greenberg on Iwasawa modules and recent developments in this area.