Multi-Sensor Spatio-Temporal Vector Prediction History Tree (V-PHT) Model for Error Correction in Wireless Sensor Networks

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***Abstract*—Wireless Sensor Networks (WSNs) have gained rapid popularity due to their deployment for critical applications such as defense, health care, agriculture, weather and tsunami monitoring etc. However, such sensor networks are fundamentally constrained by the data errors arising due to the harsh power constrained sensing environment. In this paper, we propose a novel multi-sensor vector prediction history tree (V-PHT) decision algorithm for error correction in a wireless sensor network (WSN). This scheme is based on the recently proposed prediction history tree (PHT) algorithm for model based error correction in WSNs. However, unlike the existing PHT model, which exclusively exploits the temporal correlation inherent in the narrowband sensor data, the proposed V-PHT model for sensor data correction exploits the joint spatial and temporal correlation in sensor data arising out of geographical proximity of the sensor nodes. Towards this end, an optimal multi-sensor spatio-temporal AR model is developed for predictive modeling of the sensor data. Further, employing the spatio-temporal correlation structure amongst the sensors, we develop a robust framework for optimal estimation of the multi-sensor AR predictor model. Simulation results obtained employing sensor data models available in literature demonstrate that the proposed spatio-temporal V-PHT model for error correction in a WSN results in a significant reduction in mean-squared error (MSE) compared to the existing PHT scheme which exploits only temporal correlation.**

I. Introduction

Wireless sensor networks (WSNs) [1], [2], [3] are attracting significant research interest due to their employability in a wide spectrum of applications such as environment protection, monitoring, health care, defense applications, disaster warning systems etc. This has been possible to a large extent due to the recent breakthroughs in semiconductor technology combined with the rapid development of sophisticated wireless technologies such as OFDM and MIMO [4], [5], which can combat the adverse fading radio environment. WSNs are often employed in harsh environments, which lack the infrastructure for regular power supply. Hence, the limited battery power of these sensor nodes makes them vulnerable to noise and interference over the fading wireless communication channel, resulting in erroneous reception of the sensor data at the cluster heads. Applying inefficient error correction techniques results in an increase in the processing overheads, in turn increasing the complexity of the system.

In previous related works such as [6] and [7], a data imputation method which uses both spatial and temporal prediction has been described. However, these propositions only consider disruption of data collection due to packet losses and not erroneous data detection, which is the critical adversity in communication over fading wireless channels. The other drawback of the previous approaches is the design of ad-hoc predictors, which choose between the spatial or temporal values rather than building an optimal predictor based on the spatio-temporal correlation of the multi-sensor data. Hence, in this context, we propose a robust vector prediction history tree (V-PHT) scheme for sensor data correction at the cluster heads, based on the prediction history tree (PHT) architecture proposed in [8]. However, while the algorithm proposed therein exclusively employs temporal sensor data correlation, the proposed V-PHT model employs both spatial and temporal correlation, resulting in a significant decrease in the net error of the sensor data. Further, as will be clear from the later sections, the performance of the V-PHT is directly related to the accuracy of the multi-dimensional sensor data prediction model. Hence, we present the associated optimal spatio-temporal auto-regressive (AR) [9] model for multi-sensor data prediction. Subsequently, we derive closed form expressions for estimation of the optimal multi-dimensional AR coefficients, thus comprehensively illustrating the procedure for sensor data correction. The proposed V-PHT approach, which employs the combined spatio-temporal redundancy inherent in the sensor data for error correction, thus results in a significant enhancement over existing schemes that exclusively employ the temporal redundancy. The performance of the presented framework is evaluated in a MATLAB based simulation environment employing realistic sensor data models. Our simulation results clearly demonstrate that employment of the V-PHT model for error correction results in a significant reduction in the mean-squared error (MSE) of the aggregated sensor data at the cluster head compared to the PHT scheme.

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The rest of the paper is structured as follows. In the next section we describe the optimal spatio-temporal WSN data prediction model, followed by the procedure for optimal model estimation. The V-PHT for multi-sensor error correction and the associated decision algorithm is presented in III. The decision algorithm proposed therein is a modified version of the peer algorithm presented in [8]. Section IV describes our experimental setup and simulation results for V-PHT performance validation.

II. Multisensor Data Model

In this section we describe the model employed for multi-sensor measurement prediction. We employ a multi-sensor vector AR model for data prediction [9]. Such a model accurately captures the spatio-temporal correlation amongst the sensor nodes. Further, the accuracy of the AR model can be readily enhanced by increasing the model order. Thus, one can flexibly trade-off computational complexity for increased prediction accuracy. The proposed multi-sensor AR model, employed to characterize the spatio-temporal data characteristics, integrates naturally the temporal correlation and spatial cross-correlation of the sensed data and can be expressed analytically as,

$$\mathbf{y}\left(n+1\right)=\sum_{k=0}^{M}\mathbf{A}_k\mathbf{y}\left(n-k\right), \tag{1}$$

where $\mathbf{y}\left(n\right)\in\mathbb{R}^{s\times 1}$ is the multi-dimensional sensed data vector at time $t=n$, corresponding to the s sensors of the given cluster head. The AR coefficient matrices $\mathbf{A}_k\in\mathbb{R}^{s\times s}$ contain the coefficient values (estimated from the spatio-temporal correlation structure). This model is characterized by the model order M, which can be adapted to achieve the desired prediction accuracy. A higher value of M results in accurate modeling and a corresponding increase in the modeling complexity. Hence, one needs to carefully choose an appropriate value for the parameter M. Below we describe the procedure for estimation of the matrices $\mathbf{A}_k$.

A. Vector AR Model Estimation

The coefficient matrices $\mathbf{A}_k$, $0\leq k\leq M$ can be evaluated by adapting the standard Yule-Walker procedure for the above multi-dimensional data prediction scenario. For simplicity of illustration, consider the vector AR model of order $M=2$ for $s=2$ sensors. This model can be extended to the general case of $M\geq 2$ and $s\geq 2$ in a straight forward fashion employing the development below. From the expression (1), it can be readily seen that resulting simplified AR model is given as,

$$\mathbf{y}\left(n+1\right)=\mathbf{A}_0\mathbf{y}\left(n\right)+\mathbf{A}_1\mathbf{y}\left(n-1\right).$$

Computing the inner product of the prediction error with $\mathbf{y}^T\left(n\right)$ and $\mathbf{y}^T\left(n-1\right)$ employing the principle of orthogonality, the equation to compute the optimal coefficient matrices can be readily derived as,

$$\begin{aligned}\mathrm{E}\left\{\left(\mathbf{y}\left(n+1\right)-\mathbf{A}_0\mathbf{y}\left(n\right)+\mathbf{A}_1\mathbf{y}\left(n-1\right)\right)^T\mathbf{y}\left(n\right)\right\} &= 0,\\ \mathbf{C}(0)\mathbf{A}_0^T+\mathbf{C}(1)\mathbf{A}_1^T &= \mathbf{C}(1).\end{aligned}$$

where the spatio-temporal correlation coefficient matrix $\mathbf{C}(p)\in\mathbb{R}^{s\times s}$ of the sensor data vectors $\mathbf{y}\left(n\right),\mathbf{y}\left(n+p\right)$ defined as $\mathbf{C}\left(p\right)\triangleq\mathrm{E}\left\{\mathbf{y}^T\left(n\right)\mathbf{y}\left(n+p\right)\right\}$ is given as,

$$\mathbf{C}\left(p\right)=\begin{bmatrix} c_{11}(p) & c_{12}(p) & \dots & c_{1s}(p)\\ c_{21}(p) & c_{22}(p) & \dots & c_{2s}(p)\\ \vdots & \vdots & \ddots & \vdots\\ c_{s1}(p) & c_{s2}(p) & \dots & c_{ss}(p)\end{bmatrix}$$

The correlation coefficients $c_{ij}(p)$ are defined as $c_{ij}(p)=E(y_i(n)y_j(n+p))$. For $i=j$, the coefficient $c_{ii}(p)$ is simply the temporal correlation coefficient of the the i^{th} sensor corresponding to a lag of p samples. For $i\neq j$, $c_{ij}(p)$ characterizes the spatial cross-correlation between the i^{th} and j^{th} sensors, for a lag of p. Thus, the matrix $\mathbf{C}\left(p\right)$ comprehensively characterizes the spatio-temporal correlation properties of the WSN. Hence, repeating the procedure above, the complete set of Yule-Walker equations for estimation of the coefficient matrices $\mathbf{A}_k$, $0\leq k\leq 1$ is given as,

$$\begin{aligned}\mathbf{C}(1) &= \mathbf{C}(0)\mathbf{A}_0^T+\mathbf{C}(1)\mathbf{A}_1^T,\\ \mathbf{C}(2) &= \mathbf{C}(1)\mathbf{A}_0^T+\mathbf{C}(0)\mathbf{A}_1^T.\end{aligned}$$

These equations above can be readily recast employing matrix notation as,

$$\begin{bmatrix}\mathbf{C}(1)\\ \mathbf{C}(2)\end{bmatrix}=\begin{bmatrix}\mathbf{C}(0) & \mathbf{C}(1)\\ \mathbf{C}(1) & \mathbf{C}(0)\end{bmatrix}\begin{bmatrix}\mathbf{A}_0^T\\ \mathbf{A}_1^T\end{bmatrix} \tag{2}$$

Similarly, generalizing the above for s sensors and model Order M, the resulting Yule-Walker equations to compute the optimal $s\left(M+1\right)\times s$ AR coefficient matrix $\left[\mathbf{A}_0,\mathbf{A}_1,\dots,\mathbf{A}_M\right]^T$, which minimize the mean-squared prediction error are given as,

$$\begin{bmatrix}\mathbf{C}(1)\\ \mathbf{C}(2)\\ \vdots\\ \mathbf{C}(M)\end{bmatrix}=\mathcal{T}_M\left(\mathbf{C}\right)\begin{bmatrix}\mathbf{A}_0^T\\ \mathbf{A}_1^T\\ \dots\\ \mathbf{A}_M^T\end{bmatrix},$$

where the block *Toeplitz* matrix $\mathcal{T}_M\left(\mathbf{C}\right)$ is defined in terms of the spatio-temporal correlation coefficient matrices $\mathbf{C}(0),\mathbf{C}(1),\dots,\mathbf{C}(M)$ as,

$$\mathcal{T}_M\left(\mathbf{C}\right)\triangleq\begin{bmatrix}\mathbf{C}(0) & \mathbf{C}(1) & \cdot & \mathbf{C}(M-1)\\ \mathbf{C}(1) & \mathbf{C}(0) & \cdot & \mathbf{C}(M-2)\\ \cdot & \cdot & \cdot & \cdot\\ \cdot & \cdot & \cdot & \cdot\\ \mathbf{C}(M-1) & \mathbf{C}(M-2) & \cdot & \mathbf{C}(0)\end{bmatrix}.$$

Hence, the AR coefficient matrices $\mathbf{A}_0,\mathbf{A}_1,\dots,\mathbf{A}_M$ can be computed from the above equations as,

$$\begin{bmatrix}\mathbf{A}_0^T\\ \mathbf{A}_1^T\\ \vdots\\ \mathbf{A}_M^T\end{bmatrix}=\mathcal{T}_M\left(\mathbf{C}\right)^{-1}\begin{bmatrix}\mathbf{C}(1)\\ \mathbf{C}(2)\\ \dots\\ \mathbf{C}(M)\end{bmatrix}.$$

The computed coefficients above can be employed for prediction of the multi-dimensional sensor data sample $\mathbf{y}\left(n+1\right)$

from the past samples $\mathbf{y}(n), \mathbf{y}(n-1), ..., \mathbf{y}(n-M)$ as given by the AR model in (1). In the section below, we describe the multi-sensor algorithm for sensor data correction.

III. Multisensor Estimation

In this section we describe the vector prediction history tree (V-PHT) algorithm for spatio-temporal WSN data correction. This is adapted from the PHT algorithm proposed for temporal correlation based error correction in [8]. At any time $t = n$, every sensor i, $1 \leq i \leq s$ can be associated with two values, viz. the observed value $y_i^o(n)$ and the predicted value $y_i^p(n)$. When multiple sensors are sensing the same phenomenon, one has to choose the correct value from amongst $y_i^o(n)$ and $y_i^p(n)$ for each sensor. Ideally, the observed value $y_i^o(n)$ is the most accurate level in the absence of errors. However, in the event of erroneous reception of the sensor data, the value $y_i^p(n)$, which is predicted from the past multi-dimensional sensor data samples, employing the spatio-temporal AR model in (1), is potentially closer to the true value. Hence, the V-PHT procedure is aimed at decisively choosing the observed or predicted value for each sensor so as to minimize the error between the chosen and true values. A sub-optimal solution in such a scenario is to choose independently for each sensor from amongst $(y_i^o(n))$ or $(y_i^p(n))$. However, such a procedure does not exploit the valuable information embedded in the spatial correlation amongst the sensors, since a typical sensing process is a narrowband spatial process. Thus, it results in a much higher mean-squared estimation error for the corrected sensor data.

Each V-PHT is characterized with its depth, defined as no. of levels L in the tree, which is also the decision delay. The structure and update rules of a multi-sensor prediction history tree can be illustrated with the help of the simplified example for the $L = 3$ level V-PHT corresponding to the $s = 2$ sensor case shown in Fig.1. At the current time instant n, The V-PHT contains the observed and the predicted sensor values for time instants n through $n-L+2$. The single node at level $n-L+1$ corresponds to the chosen values (observed or predicted) at the previous decision epoch. The total number of nodes T in the tree are is given as,

$$T = \frac{2^{sL} - 1}{2^s - 1}.$$

Hence, the total number of nodes of the tree in Fig.1 is $N = 21$, corresponding to $s = 2$, $L = 3$. Each level $l \in \{1, 2, 3\}$ corresponds to the time instant $t = n - L + l$ and each node splits into $2^s = 4$ possible outcomes listed below:

- OO: Both sensors S_1, S_2 assigned the sensed (observed) values i.e. $(y_1^o(l), y_2^o(l))$.
- OP: Sensor S_1 assigned the sensed (observed) value and sensor S_2 assigned the predicted value i.e. $(y_1^o(l), y_2^p(l))$.
- PO: Similar to OP, with S_1 assigned the predicted value and S_2 assigned the sensed value i.e. $(y_1^p(l), y_2^o(l))$.
- PP: Both the sensors assigned the predicted values i.e. $(y_1^p(l), y_2^p(l))$.

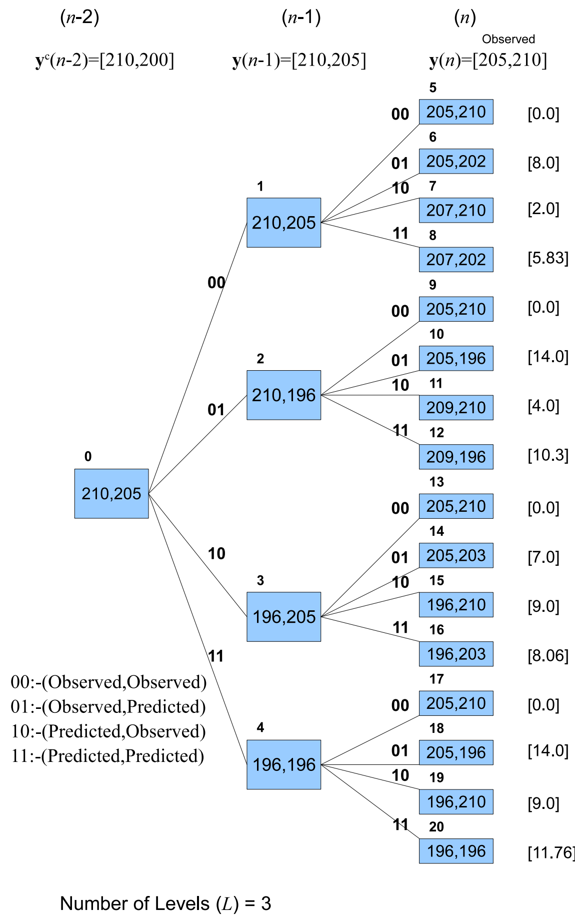

Fig. 1. Vector Prediction History Tree (s=2)

Hence, for the i^{th} node in the V-PHT above, the corresponding child nodes can be labeled as $4i+k$, $1 \leq k \leq 4$. The 0^{th} or the root node contains the most recent corrected sensor data vector i.e. $\mathbf{y}^c(n-2) = (210, 205)$ in the above example. At the time instance n, when the V-PHT encounters a sensor data sample $\mathbf{y}(n)$, the V-PHT is populated by computing all the values at the predicted nodes i.e. the child nodes $4i+k$, $2 \leq k \leq 4$ (The node $4i+1$ corresponds to replicating the observed sensor data values at each node i). The decision algorithm described below is run on this V-PHT to choose the correct value $\mathbf{y}^c(n-1)$ amongst the nodes $1 \leq i \leq 4$.

Several algorithms can be employed for the selection of the appropriate node as mentioned in [8], and we adapt the *Peer* algorithm proposed therein to frame a modified *Peer* algorithm

Peer Set	min Error Node	Subtree of min Error Node
$\{6, 10, 14, 18\}$	14	PO
$\{7, 11, 15, 19\}$	7	OO
$\{8, 12, 16, 20\}$	8	OO

TABLE I
Table listing MPA Comparisons for V-PHT in Fig.1

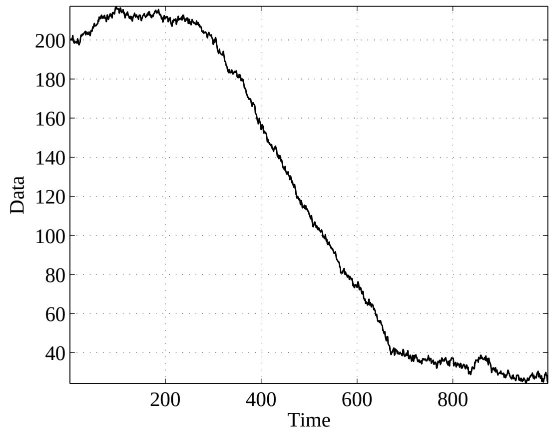

Fig. 2. Multisensor Model: Original Data for first sensor

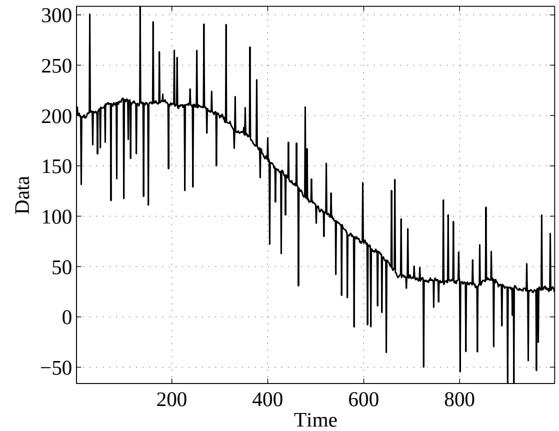

Fig. 3. Multisensor Model: Observed Data for first sensor

i.e. multi-sensor *Peer* algorithm (MPA). MPA compares peer sets of nodes in each subtree originating from the OO,OP,PO and PP nodes at level $n-L+2$ as the root nodes. For instance, nodes $6, 10, 14, 18$ in the V-PHT in Fig.1 form a peer set. The comparisons are made in terms of the prediction error, indicated next to each node. Consider node 10 in the V-PHT example being considered. This corresponds to the OP node at level n belonging to the subtree originating from the OP node at level $n-1$. Hence, the multi-dimensional value assigned to this node is $(205, 196)$, corresponding to the observed value for S_1 and predicted value for S_2. Hence, the associated error e_{10} is $210.0 - 196.0 = 14.0$ with respect to the reported sensor data $(205, 210)$ at level n. Similarly, since the node 12 corresponds to both sensors assigned predicted values $(209, 196)$, the associated error e_{12} is given as, $e_{12} = \sqrt{\frac{1}{2}\left(4^2 + 14^2\right)} = 10.3$. After comparisons of all the peer sets, the root node of the subtree comprising of the maximum peers with the lowest estimation error is chosen as the decision node $\mathbf{y}^c\left(n-L+2\right)$ ($\mathbf{y}^c\left(n-1\right)$ in the example) at level $n-L+2$. The results of MPA for each peer set in the V-PHT shown are listed in table I from which it is clear that the OO subtree has a greater number of peers with lower prediction error. This is in turn made the root node at level $n-1$ and the procedure above is repeated for successive observations $\mathbf{y}\left(n+1\right)$ and so on.

IV. Simulation Results

Our WSN simulation setup for spatio-temporal error correction was implemented in MATLAB. We considered an $s = 2$ sensor WSN and a three stage sensing phenomenon corresponding to the time intervals $n <= 300$, $301 \leq n \leq 700$, $n > 700$ ($N = 1000$ sensed samples), shown in Fig.2. The sensor data for each state is generated by the AR model,

$$\mathbf{y}\left(n+1\right) = \mathbf{A}_0^u \mathbf{y}\left(n\right) + \mathbf{A}_1^u \mathbf{y}\left(n-1\right) + g\mathbf{1} + \mathbf{w}\left(n+1\right), \quad (3)$$

where $u \in \{1, 2, 3\}$ denotes the stage and $\mathbf{A}_0^u, \mathbf{A}_1^u$ are the corresponding spatio-temporal AR coefficient matrices. The vector $\mathbf{1} \in \mathbb{R}^{s\times 1} \triangleq \left[1, 1, ..., 1\right]^T$ and serves as the bias vector.

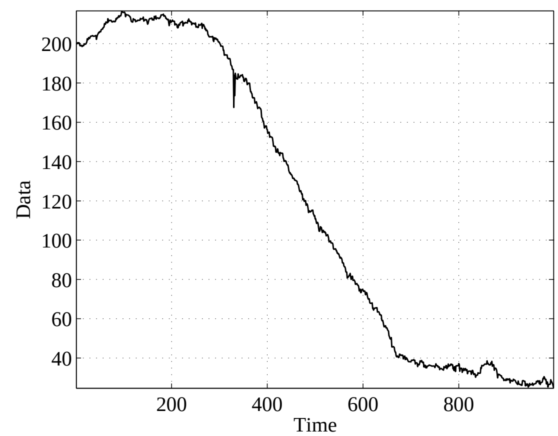

Fig. 4. Multisensor Model:Corrected data for first sensor

The noise $\mathbf{w}\left(n+1\right)$ is additive spatio-temporally white Gaussian noise with covariance matrix $\sigma_n^2 \mathbf{I}_s$. The noise variance $\sigma_n^2 = 1$. The matrices corresponding to the 3 stage generation model are given in table II. The erroneous observed data sets for both the sensors are simulated by randomly adding errors at intervals of $\Delta n = \frac{1}{\text{BER}}$ to the sensor data. The error magnitudes are distributed uniform randomly in the interval $[-100, 100]$. Fig.3 shows the observed data for $\text{BER} = 10^{-1}$. This erroneous observed data is shown in Fig.3. The prediction coefficients are computed using the procedure described in section II-A. The AR channel coefficient matrices for each stage are calculated employing a training window consisting of $n_t = 70$ samples. It is assumed that we receive error free data during the training period. This is possible by allocating a larger power budget to the critical training phase to ensure accuracy of data detection. The computed AR coefficient matrices for the 3 stage model described above are given in table II. Following the estimation of the AR coefficient matrices, the MPA algorithm described in section III is applied to the V-PHT and the corrected values of the sensor data are

Stage	$\mathbf{A}_0^u$	$\mathbf{A}_1^u$	g	$\hat{\mathbf{A}}_0^u$	$\hat{\mathbf{A}}_1^u$
Stage I	$\begin{bmatrix} 0.7 & 0.125 \\ 0.125 & 0.7 \end{bmatrix}$	$\begin{bmatrix} 0.1 & 0.075 \\ 0.075 & 0.1 \end{bmatrix}$	0	$\begin{bmatrix} 0.69 & 0.24 \\ 0.06 & 0.62 \end{bmatrix}$	$\begin{bmatrix} 0.11 & -0.05 \\ 0.18 & 0.14 \end{bmatrix}$
Stage II	$\begin{bmatrix} 0.7 & 0.125 \\ 0.125 & 0.7 \end{bmatrix}$	$\begin{bmatrix} 0.1 & 0.075 \\ 0.075 & 0.1 \end{bmatrix}$	-0.441	$\begin{bmatrix} 0.66 & 0.10 \\ 0.04 & 0.69 \end{bmatrix}$	$\begin{bmatrix} 0.09 & 0.14 \\ 0.16 & 0.09 \end{bmatrix}$
Stage III	$\begin{bmatrix} 0.7 & 0.125 \\ 0.125 & 0.7 \end{bmatrix}$	$\begin{bmatrix} 0.1 & 0.075 \\ 0.075 & 0.1 \end{bmatrix}$	0	$\begin{bmatrix} 0.53 & 0.35 \\ 0.32 & 0.36 \end{bmatrix}$	$\begin{bmatrix} 0.10 & 0.02 \\ 0.09 & 0.21 \end{bmatrix}$

TABLE II
TABLE LISTING ACTUAL AND COMPUTED AR COEFFICIENT MATRICES

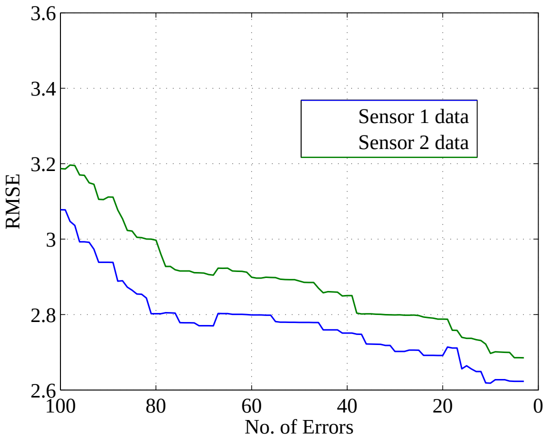

Fig. 5. Multi-Sensor RMSE vs. No.of errors

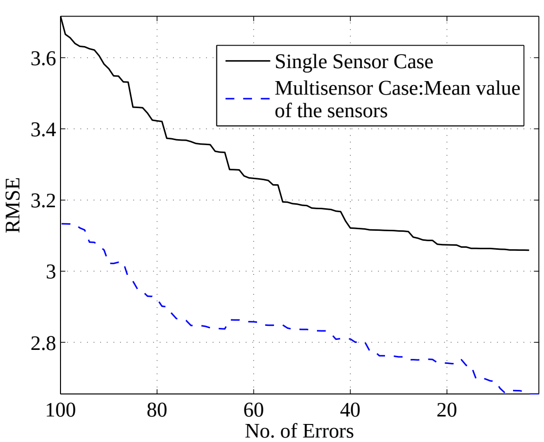

Fig. 6. RMSE comparison between MPA and Scalar Peer Algorithm

plotted in Fig.4. From a visual comparison of this plot with the erroneous observed data in Fig.3, it can be readily seen that application of the V-PHT based MPA algorithm results in a significant decrease in the MSE of the corrected sensor data. We plot the root mean-squared error (RMSE) vs number of errors for each of the $s = 2$ sensors in Fig. 5. The number of errors is related to the BER as BER $\times N$. The plot therein shows that the output MSE decreases as the frequency of transient errors or essentially the BER decreases. In Fig.6 we plot the RMSE vs number of errors for the existing temporal correlation based Peer algorithm in [8] and the average sensor RMSE for MPA. We clearly observe from the plot that RMSE values are lower in the case of the MPA algorithm as compared to the scalar PEER algorithm for PHT. Thus, the V-PHT, which employs the spatio-temporal correlation, results in a significant reduction in sensor error compared to the existing PHT.

V. CONCLUSION

In this work we have proposed a novel approach for multi-sensor error correction in WSNs based on the rich spatio-temporal correlation structure inherent in such a setup. We developed the optimal multi-dimensional sensor data AR prediction model, which naturally captures the spatio-temporal structure of the sensor data. The procedure for the estimation of the optimal vector AR model coefficient matrices has been derived. Further, we described a novel vector prediction history tree (V-PHT) algorithm and the associated MPA decision algorithm for correction of erroneous reported multi-dimensional sensor values. Simulation results demonstrate the superior performance of the proposed multi-sensor error correction scheme compared to the single sensor temporal correlation based scheme existing in literature.

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