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A NEW ANALYTICAL MODEL FOR THE LEAD RUBBER BEARING

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SUMMARY

In this paper a new analytical model is proposed which is a combination of the modified bilinear model and Ramberg-Osgood model. In the proposed model it is easy to simulate all characteristics of lead rubber bearing (LRB), such as equivalent stiffness, equivalent damping ratio and unloading stiffness. The formulation of the model is described first, then compared with the typical hysteresis shear test and earthquake simulator tests. In the typical hysteresis shear test, hysteresis loops and equivalent damping ratio between experimental result and the new proposed model agreed well from small strain to large strain. In earthquake simulator tests, displacement response time history is simulated well by the new model. It is also true for the unloading stiffness and the small displacement after the largest shear strain. Good agreement is observed also from the comparison of total energy time history. Dynamic response analyses on two typical buildings showed that: 1)The maximum response displacement becomes smaller. 2)The maximum response acceleration becomes larger. Shear force becomes smaller at lower part, and bigger at upper part of the building. 3)The response displacement becomes zero gradually after main motion. Consequently, the proposed model may make the design more economical.

INTRODUCTION

The concept of base isolation is gaining widespread acceptance after the 1994 Northridge earthquake in California, USA and the 1995 Hyogoken-Nanbu earthquake in Japan. Three types usage has been widely used, which are natural rubber bearings with extra dampers, high damping rubber bearings and lead rubber bearings. Lead rubber bearings provide an economic solution in that the one unit incorporates the three functions of vertical support, horizontal flexibility and hysteresis damper and are widely used in Japan. Lead rubber bearing type has become the most popular one, since it has good linear property of natural rubber and good damper property of lead. Existing non-linear analytical models (modified bilinear or Ramberg-Osgood) have been applied for the dynamic analyses of base isolated structures to simulate non-linearity and stiffening behavior dependence on share strain. However, in the existing modified bilinear model there will be no energy dissipation in the smaller hysteresis loop after larger deformation, which results in invalid larger response after main shock in some dynamic response analyses. In this paper a new analytical model is proposed which is a combination of the modified bilinear model and Ramberg-Osgood model. In the proposed model it is also easy to simulate all characteristics of LRB, such as equivalent stiffness, equivalent damping ratio and unloading stiffness. The formulation of the model is described first, then compared with test results. Dynamic response analyses on two typical buildings are carried out at last.

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ANALYTICAL MODELS

Modified Bilinear Model for Lead Rubber Bearing

In the bilinear model, a skeleton curve and hysteresis loops are defined separately. In the modified bilinear model, the skeleton curve and hysteresis loops are defined as functions of shear strain. If the detail of a bearing is fixed, they are simply determined by $K_d(\gamma)$, $Q_d(\gamma)$ and $K_u(\gamma)$ in a shear strain of γ shown in equation (1).

$$\begin{aligned} K_d(\gamma) &= C_{kd}(\gamma)K_{d50} \\ Q_d(\gamma) &= C_{qd}(\gamma)Q_{d50} \\ K_u(\gamma) &= \alpha K(\gamma) \end{aligned} \tag{1}$$

where

$$\gamma = \frac{\delta_h}{H_r}; K_{d50} = \beta\frac{GA_r}{Hr}; Q_{d50} = A_l\sigma_l \tag{2}$$

α: pre-selected parameter from the dynamic hysteresis test;

β: strengthening coefficient due to the lead plug;

γ: shear strain; δ_h: relative horizontal displacement between the top and bottom of the bearing;

Hr: total height of rubber in the bearing; *G*: shear modulus of elastomer;

Ar: area of bearing; A_l: area of lead plug; σ_l: yield strength of lead plug.

The equation is very explicit and directly related with property of materials used in the bearing and thus widely used in the engineering society. However in the modified bilinear model there will be no energy dissipation in the smaller hysteresis loop after larger deformation which results in invalid larger response after main shock in some dynamic response analyses.

Differential Equation Model

Ozdemir(1976), Wen (1976), Fujita *et al.*(1990) and Kikuchi and Aiken(1997) proposed and modified differential equation model shown in equations (3)-(5).

$$F = F_1 + F_2 \tag{3}$$

$$F_1 = \frac{1}{2}(1-u)F_m\left\{x + \mathrm{sgn}(X)|x|^n\right\} \tag{4}$$

$$\begin{aligned} F_2 &= uF_m\left\{1 - 2e^{-a(1+x)} + b(1+x)e^{-c(1+x)}\right\} \quad (\dot{X} > 0) \\ F_2 &= -uF_m\left\{1 - 2e^{-a(1-x)} + b(1-x)e^{-c(1-x)}\right\} \quad (\dot{X} < 0) \end{aligned} \tag{5}$$

where F_m is the peak shear force on the skeleton curve, x is the normalized shear displacement ($x=X/X_m$) and X_m is the peak shear displacement on the skeleton curve. In equation (4), the parameter n specifies the stiffening. In equation (5), u is the ratio of shear force at zero displacement, F_u, to F_m ($u=F_u/F_m$), a is calculated from equation (6) and b is calculated from equation (7). Both equation (6) and (7) are derived assuming that the analytical and experimental hysteresis loop areas are equal:

$$\frac{1-e^{-2a}}{a} = \frac{2u - \pi h_{eq}}{2u} \tag{6}$$

$$b = c^2\left[\frac{\pi h_{eq}}{u} - \left\{2 + \frac{2}{a}(e^{-2a} - 1)\right\}\right] \tag{7}$$

where h_{eq} is the equivalent viscous damping ratio and is evaluated from an empirical formula as a function of shear strain, which is determined from the results of tests of individual bearings. The parameter c is a pre-selected constant that specifies the shape of the hysteresis loop.

The non-linear stiffening behavior at displacement to high shear strains in the elastomer can be expressed well by the above equations. The equations can also be used to different type rubber bearings: both high-damping rubber bearing and lead rubber bearing. However the parameters used in the equations are determined by matching dynamic loading tests of individual bearings. Parameter a cannot be solved in closed form in equation (6) and thus must be solved numerically. It is difficult to use such equations in daily design routine work.

A NEW ANALYTICAL MODEL

A new analytical model illustrated in Figure 1 is proposed which is a combination of the modified bilinear model and Ramberg-Osgood model. The skeleton curve is the same one shown in equation (1). The hysteresis loop is a combination of bilinear model at loading and Ramberg-Osgood model at unloading. In the proposed model it is easy to simulate all characteristics of LRB, such as equivalent stiffness, equivalent damping ratio and unloading stiffness.

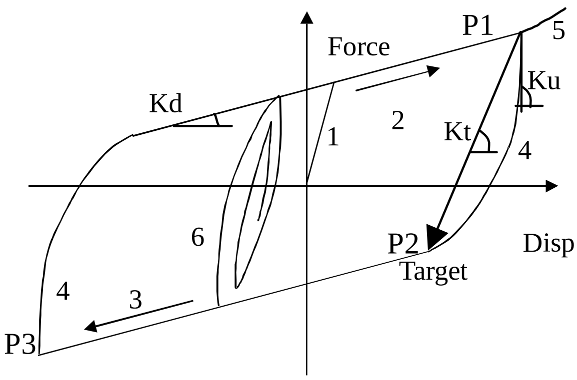


Figure 1: Illustration of the new proposed model

Rule 1: elastic range;

Rule 2(3): increment (decrement) direction of displacement in the bilinear loop determined by maximum displacement up to now;

Rule 4: RO unloading rule;

Rule 5: skeleton curve determined by the properties of material used in the bearing;

Rule 6: repeated cycles of RO loading and unloading rule

The force is unloading by an unloading stiffness of Ku from unloading point P1 to the maximum point between P2 and P3. P2 is determined by the bilinear loop depended on the maximum displacement up to now and an unloading stiffness of Kt. P3 is the maximum displacement up to now.

The equation of RO model is shown in equation (8).

$$(f - f_u)\{a + b|f - f_u|^{\gamma-1}\} = d - d_u \tag{8}$$

where (du,fu): unloading point; a,b: coefficients; γ: Ramberg coefficient.

Thus the varying stiffness in the RO model is:

$$\frac{\partial f}{\partial d} = \frac{1}{a + b\gamma \left| f - f_u \right|^{\gamma - 1}} \tag{9}$$

COMPARISON WITH TESTS

The test specimens, which are widely used products, used in this study are shown in Table 1. The parameters used for equations (7) and (8) are Kt=3.0Kd, Ku=60Kd, γ=4.8.

Table 1. Details of the test specimens

	ϕ500	ϕ900	ϕ1000
rubber(mm)	44@4=176	33@6=198	34@6=204
inner plate (mm)	43@3.1	32@2.7	33@3.1
diameter of lead plug(mm)	130	180	220
Qd50(tf)(test value)	10.15	21.63	29.08
Kd50(tf/cm)	0.738	1.979	1.595
S_1	31.25	37.5	41.7
S_2	2.84	4.55	4.79

Comparison with Typical Hysteresis Shear Tests

Typical sinusoidal shear tests for a shear strain range of 50%, 100%, 150% and 200%, with axial stress of $100 kgf/cm^2$ is conducted to a ϕ1000 specimen. Comparison of hysteresis loops and equivalent damping ratio between experimental result and the new proposed model is shown in Figure 3 and Figure 4, respectively. Since the model has the same skeleton with the modified bilinear model, only the equivalent damping factors are compared with the test results. Good agreement is observed from small strain to large strain.

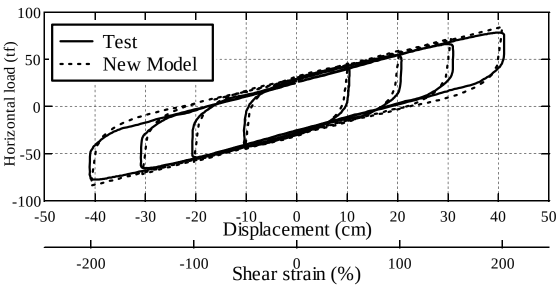


Figure 2: Comparison of hysteresis loops between experimental study and the new proposed model for a ϕ1000 type specimen shown in Table 1.

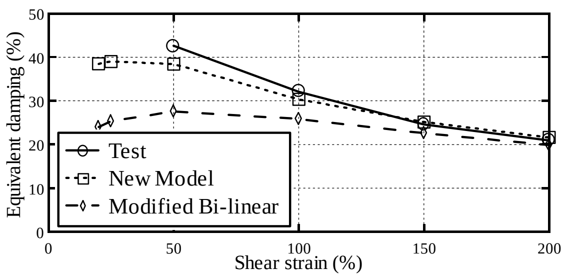


Figure 3: Comparison of equivalent damping ratio between experimental study and the new proposed model for the ϕ1000 type specimen.

Comparison with Earthquake Simulator Tests

Earthquake simulator tests are conducted to both ϕ500 and ϕ900 type specimens. The input displacement is arranged to begin from 25%, 50% to 100%, then after several repeats of small amplitude, to 100%, 50% and 25% in reverse order. Load schedule and comparison of energy spectrum are shown in Figure 4 for the ϕ500 type specimen, in Figure 6 for the ϕ900 type specimen. Comparison of hysteresis loops between experimental study and the new proposed model is shown in Figure 5 for the ϕ500 type specimen, in Figure 7 for the ϕ900 type specimen. In both ϕ500 and ϕ900 type tests, displacement response time history is simulated well by the new model. It is also true for the unloading stiffness and the small displacement after the largest 100% strain. Good agreement is observed also from the comparison of total energy time history.

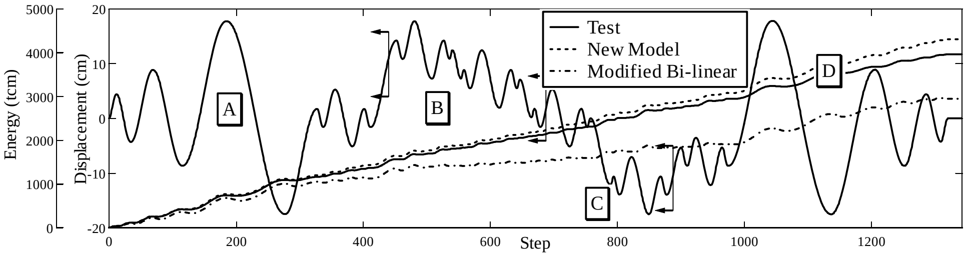


Figure 4: Load schedule and energy spectrum of a ϕ500 type specimen shown in Table 1.

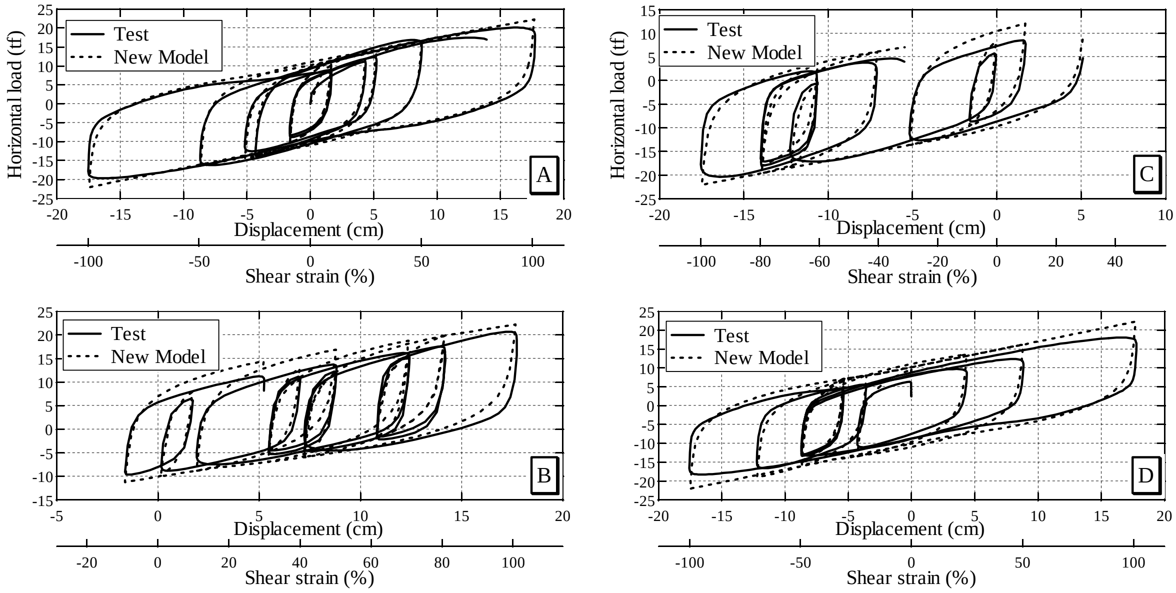


Figure 5: Comparison of hysteresis loops between experimental study and the new proposed model for the ϕ500 type specimen. See Figure 4 for time history of each part.

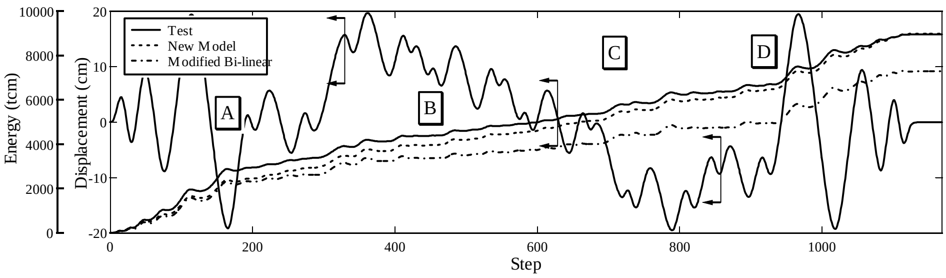
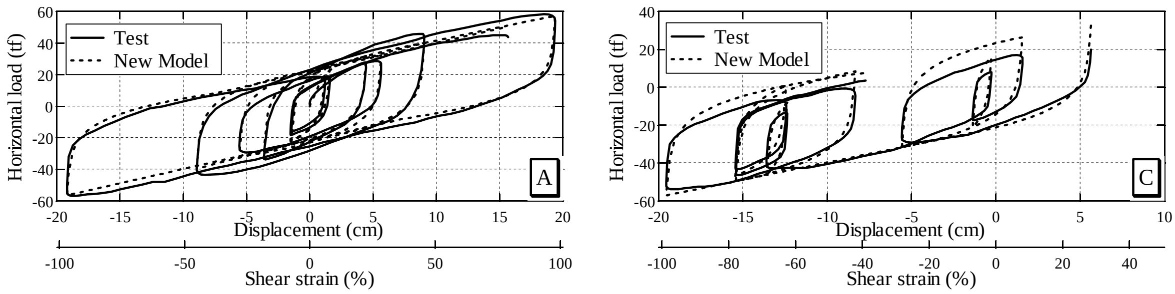


Figure 6: Load schedule and energy spectrum of a ϕ900 type specimen shown in Table 1



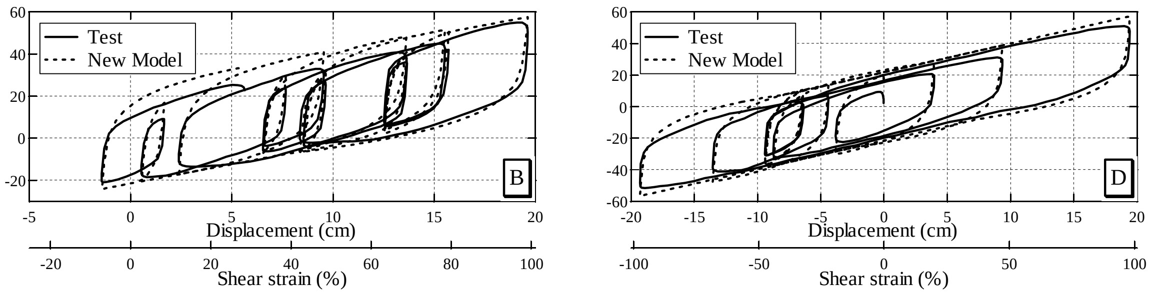


Figure 7: Comparison of hysteresis loops between experimental study and the new proposed model for the ϕ900 type specimen. See Figure 6 for time history of each part.

DYNAMIC RESPONSE ANALYSIS

Dynamic Analysis Model

Dynamic response analyses by the proposed new analytical model and the modified bilinear model are compared in this section.

Dynamic response analyses on a 14 story and an 8 story building are conducted using the new model and the modified bilinear model. Both buildings are residential RC frame structure. The super-structure is modeled as a shear type multiple-degree-of-freedom system, where tri-linear model is used for the column members. The base isolation floor is modeled as a Sway-Rocking system, where the new proposed model and modified bilinear model are used in the sway component, respectively. The energy proportional type damping is assumed, where the ratio is 3% for super-structure, 0% for sway and 1% for rocking, respectively. El Centro NS and Hachinohe NS, EW recommended by the Building Center of Japan (BCJ), which are normalized to 50cm/sec, are used as input motions.

Dynamic Analysis Result

For a typical case of the 14F building due to the El Centro NS input, the displacement responses of the base isolation floor are shown in Figure 8. Comparing that by the modified bilinear model, the maximum response displacement by the new proposed model became smaller due to the consideration of energy dissipation in the RO part. The displacement also converged to zero, which accords with observation fact. On the other hand, the response acceleration in the roof became 15% larger, while there was little change in the LRB floor. The increase of acceleration in the roof is considered to be aroused by the response from higher order mode of the building. The simply averaged maximum responses of shear force coefficient are shown in Figure 9. Similar to the acceleration response, the shear force coefficient became larger at roof, while the displacement response became smaller by the new proposed model. Since the super-structure in the base isolated building usually have enough strength, we can use smaller bearing for the same displacement response. In Figure 10 the dynamic response hysteresis loops are compared. In the new model, there was also energy dissipation in the smaller hysteresis loop after the largest deformation.

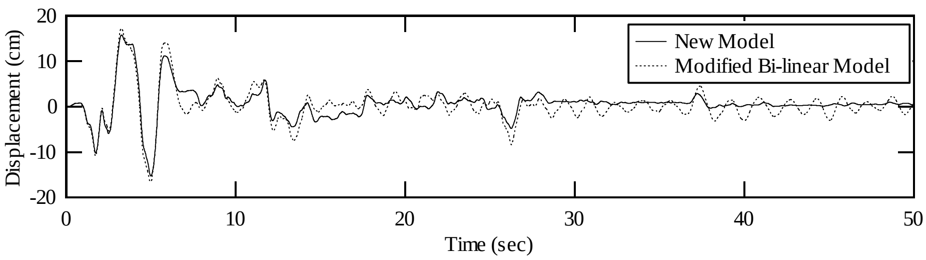


Figure 8: Time history of the response displacement (isolation floor of 14F building by El Centro NS)

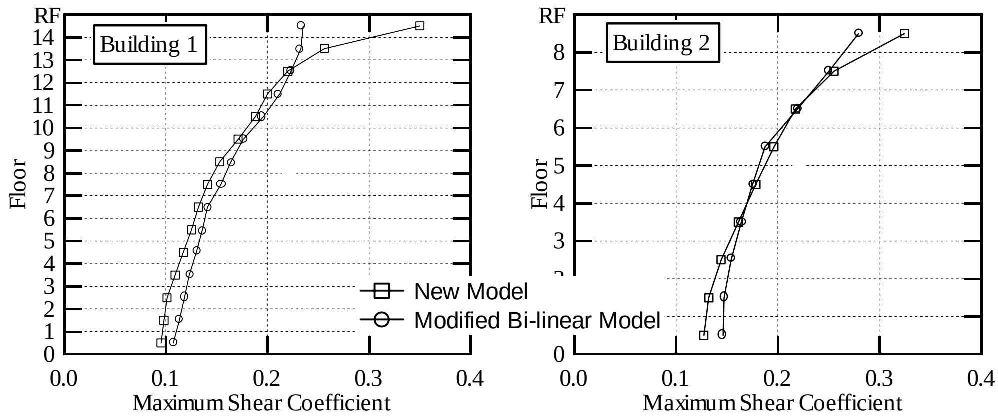


Figure 9: Maximum response of shear force coefficient.

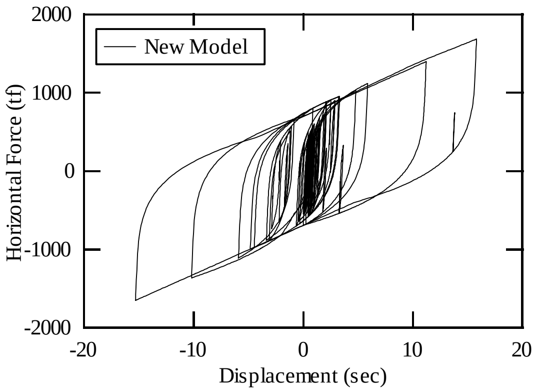


Figure 10: Response hysteresis loops.

CONCLUSIONS

The proposed model is found to simulate well the results of lead rubber bearing in both the typical hysteresis shear tests and earthquake simulator tests. Comparison of the dynamic response analyses between the modified bilinear model and the proposed model showed that the proposed model may make the design more economical.

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