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COLLAPSE ANALYSIS OF REINFORCED CONCRETE FRAME STRUCTURES CONSIDERING COLLISION EFFECTS

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SUMMARY

Based on singularity functions, which possess great advantages in describing sharp changes, mixed hinge model is brought forward, and thus the whole collapse process can be simulated. As far as collisions in structural analysis are concerned, current research is mainly focused on the pounding between adjacent buildings. Many models have been built to find out contact forces, whereas the dynamic effects of velocity and acceleration change during impact are completely neglected. In this paper, post-impact conditions of velocity restrictions are imposed on the system so that the dynamic effects can be taken into full consideration in the form of the equilibrium equation of impulse. And the numerical stability of Newmark- β difference formula employed is also studied. In particular, numerical investigations into a frame subjected to strong earthquake are performed to illustrate the good feasibility of the proposed model, as well as the procedure considering collision effects, and the simulation results are rather reasonable.

INTRODUCTION

It has been proved by damage investigations into many devastating earthquakes that, the genesis of collapse of concrete structures and thus great property losses and casualties lies in brittle failures of reinforced concrete members. It is usually characterized by an abrupt decrease of load capacity as well as a violent displacement discontinuity. Hence there comes up an imperative demand on the collapse analysis of reinforced concrete structures to find out how to avoid structural collapse more efficaciously, especially under disastrous ground shakings. Current collapse analysis of structures, however, is usually constrained in the stage of state definition of collapse, which is usually expressed as the stage when a structure makes up a hinged mechanism. But in seismic damage investigation, the collapse process was scarcely excited from a hinged system. Right on the contrary, it was always accompanied by brittle failures and displacement discontinuity. By employing singularity functions, a modeling scheme was proposed for collapse simulation of reinforced concrete buildings, where the concept of mixed hinge was introduced to depict displacement discontinuity. This model is also consistent with conventional procedures and might be reduced to plastic hinge model.

Collision effects are concomitant in the collapse process of structures subjected to disastrous earthquakes. Contact-collide problems have been highlighted in recent years. Research work published on this problem, however, was generally limited to the pounding of adjacent buildings during earthquakes. Earlier solutions to contact problems were generally based on the introduction of special artificial interface elements with tremendous stiffness normal to the contact direction, which is activated after contact confirmed so that no material overlap can occur. In many procedures presented to analyze discontinuous displacements, such as the Discrete Element Method (DEM), the assumption of rigid body with soft surface is generally adapted to obtain contact forces, which is usually obtained by the overlapping distance multiplied by a normal stiffness coefficient. And the accuracy depends greatly on the choice of stiffness coefficients. In this paper, an effective solution method for studying collision effects in the collapse process of structures was presented, which was also valid in studying pounding phenomena of adjacent buildings. By employing a Lagrangian multiplier to modify kinetic energy, constraint conditions imposed by collisions are incorporated into the equations of motion, which will lead to the equilibrium equation of impulse after integration over time interval. Both the impulse of constraint

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forces and contact forces (if any) can be obtained directly through element handling, with no regard of the usual assumption of rigid body with soft surface, nor any identification of normal stiffness coefficients needed.

MIXED HINGE MODEL OF BEAM-COLUMN MEMBER

In this paper all members are assumed to be prismatic with unique stiffness along the member. The mixed hinge is defined as those that can function as moment connector and plastic hinge simultaneously, and can also sustain axial sliding (see Fig.1). Abrupt changes can be transferred to equivalent distributed loading by applying singularity functions, and thus the following equivalent distributed loads can be obtained,

$$\begin{aligned} q_y(x) = & M_1 < x >^{-2} + V_1 < x >^{-1} - EI\phi_1 < x >^{-3} \\ & + M_2 < x-l >^{-2} + V_2 < x-l >^{-1} + EI\phi_2 < x-l >^{-3} \\ & - M_a < x-a^- >^{-2} + V_a < x-a^- >^{-1} - EI\theta_a < x-a >^{-3} + N_a v_a < x-a^- >^{-2}/2 \\ & + M_a < x-a^+ >^{-2} - V_a < x-a^+ >^{-1} - EIv_a < x-a >^{-4} + N_a v_a < x-a^+ >^{-2}/2 \\ & + M_b < x-b^- >^{-2} - V_b < x-b^- >^{-1} + EI\theta_b < x-b >^{-3} + N_b v_b < x-b^- >^{-2}/2 \\ & - M_b < x-b^+ >^{-2} + V_a < x-b^+ >^{-1} + EIv_b < x-b >^{-4} + N_b v_b < x-b^+ >^{-2}/2 \end{aligned} \tag{1}$$

$$\begin{aligned} q_x(x) = & N_1 < x >^{-1} + N_2 < x-l >^{-1} + EA\Delta < x-l >^{-2} \\ & - N_a < x-a^- >^{-1} + N_a < x-a^+ >^{-1} - EAu_a < x-a >^{-2} \\ & + N_b < x-b^- >^{-1} - N_b < x-b^+ >^{-1} + EAu_b < x-b >^{-2} \end{aligned} \tag{2}$$

where EA EI are the member stiffness. Member end rotation ϕ_1 ϕ_2 and axial deformation Δ, plus the relative displacements at mixed hinges (if any), constitute local deformation of the member. All other quantities can be found in Fig.1 with positive directions defined.

Then the displacement functions can be obtained after a series of integration and implement of boundary conditions. It is assumed for convenience that mixed hinge can only occur right adjacent to the member ends, that is to say $a = 0^+$ and $b = l^-$. Therefore after supplements of rigid body movements, one gets

$$\begin{Bmatrix} u \\ v \end{Bmatrix} = \begin{bmatrix} f_1 & 0 & 0 & f_1 & 0 & 0 & f_4 & 0 & 0 & f_4 & 0 & 0 \\ 0 & f_2 & f_3 & 0 & -f_2 & f_3 & 0 & f_5 & f_6 & 0 & -f_5 & f_6 \end{bmatrix} \{\bar{d}\}^e \tag{3}$$

where

$$\begin{cases} f_1 = 1 - x/L \\ f_2 = 1 - 3x^2/L^2 + 2x^3/L^3 \\ f_3 = x + 2x^2/L - x^3/L^2 \end{cases} \qquad \begin{cases} f_4 = x/L \\ f_5 = 3x^2/L^2 - 2x^3/L^3 \\ f_6 = x^2/L - x^3/L^2 \end{cases} \tag{4}$$

And $\{\bar{d}\}^e = \{d_1 \quad d_2 \quad d_3 \,\vdots\, u_a \quad v_a \quad \theta_a \,\vdots\, d_4 \quad d_5 \quad d_6 \,\vdots\, u_b \quad v_b \quad \theta_b\}$ is the displacement vector of element involving relative displacement at mixed hinge (see Fig.1).

For more information, please refer to a paper by Zhang and Liu (1997).

Now the element stiffness and mass matrices can be obtained after considering the plastic constitutive relations at mixed hinges. From the shape function of Eq.3 and Eq.4, it can be seen that the components are the same as a normal beam-column element without mixed hinge, except that their number is increased. Therefore, the stiffness matrix can be obtained by 'fix in place' one component by one component after obtaining the counterpart of a normal element, in order to consider the change of stiffness along the full length of a member. Surely this model can be reduced to plastic hinge model, and the same as to others, such as the one with shearing failure only.

The formal procedure in usual finite element analysis can be followed in global analysis. As for large displacement and large rotation arisen in collapse process, the current unstressed configuration is adopted in global analysis, which is formed by an imaginary removal of stresses and the elastic deformation from the deformed configuration. Since the current configuration is still unknown, the calculation must be performed by iteration starting from the latest balanced configuration.

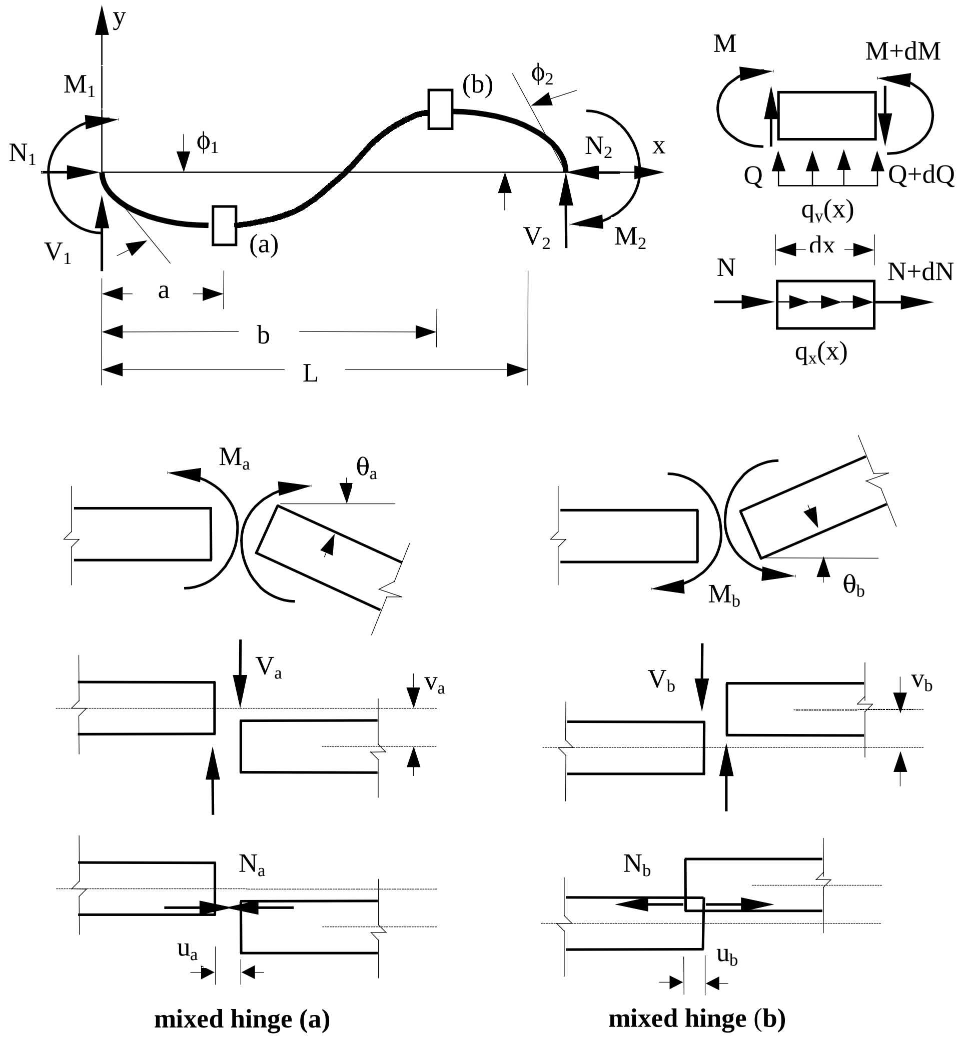

Figure 1: Modeling of beam-column member

COLLISION ANALYSIS

Colliding model

An excited collision may result in additional constraints on generalized coordinates. In the case of a completely plastic impact, for instance, lose of kinetic energy reaches its maximum. In structural analysis, the post-impact conditions on velocity may be written in colliding coordinates (see Fig.2), as follows

$$\{\tilde{v}\}_2' - \{\tilde{v}\}_1' = [e]\left(\{\tilde{v}\}_1^0 - \{\tilde{v}\}_2^0\right) \tag{5}$$

where $[e]$ is defined as restitution coefficient matrix; superscript *0* and prime mean the velocity vectors prior to and after impact respectively, of two points in contact during impact. For perfectly elastic impact, $[e]$ is reduced to an identity matrix, and it vanishes when in the case of a completely plastic impact.

After transporting to global coordinates and assembling all collisions, Eq.5 evolves to

$$[A_s]^T\{\dot{q}\}' = \{P\}_{con} \tag{6}$$

where $[A_s]^T$ = global constraint coefficient matrix; $\{\dot{q}\}'$ = generalized velocity vector after impact; $\{P\}_{con}$ = additional constraint vector.

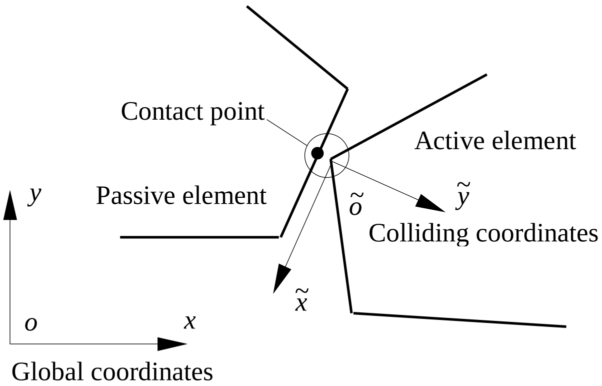

Figure 2: Contact pairs in collision

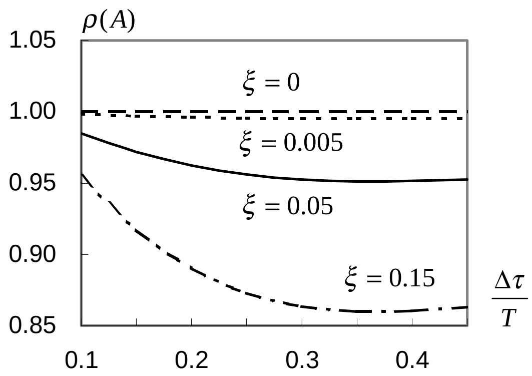

Figure 3: Relations between $\rho(A)$ and $\frac{\Delta\tau}{T}$

GOVERNING EQUATIONS

A Lagrangian multiplier $\{P_c\}$ is introduced to release the constraints on velocity vectors, which gives the modified kinetic energy as follows

$$T_G = T + \{P_c\}^T\left([A_s]^T\{\dot{q}\}' - \{P\}_{con}\right) \tag{7}$$

Substitution of Eq.7 into the Lagrangian equations of motion leads to the equations of motion considering collisions, as follows

$$[M]\{\ddot{q}\} + [C]\{\dot{q}\} + [K]\{q\} + [A_s]\frac{d}{dt}\{P_c\} = \{Q\} \tag{8}$$

where $[M]$= the structural mass matrix; $[C]$= the structural damping matrix; $[K]$= the structural stiffness matrix; $\{Q\}$= the equivalent external loading vector; $\{q\}$= the generalized coordinates, and the super dot represents the derivative with respect to time.

Eq.6 and Eq.8 constitute the governing equations of dynamic analysis considering collision effects. Solutions for Eqs.6 and 8 have to be obtained by direct numerical integration. Integration of Eq.6 from t_0 to $t_0 + \Delta\tau$ yields

$$[M]\{\dot{q}\}^{t_0+\Delta\tau} + [C]\left(\{q\}^{t_0+\Delta\tau} - \{q\}^{t_0}\right) + \int_{t_0}^{t_0+\Delta\tau}[K]\{q\}dt + [A_s]\{P_c\} = \int_{t_0}^{t_0+\Delta\tau}\{Q\}dt + [M]\{\dot{q}\}^{t_0} \tag{9}$$

This is actually the equilibrium equation of impulse for the whole system, from which it can be seen that the multiplier is the constraint impulse vector, and its first derivative with respect to time, $\frac{d}{dt}\{P_c\}$, is the constraint force imposed by impact (see Eq.8).

Herein time interval $\Delta\tau$ has two choices: one is the collision duration when there are new collisions happened, and the other is normal time interval when a step-by-step method is applied. After choosing a numerical integration method, such as Newmark- β , re-arrangement of Eq.9 and Eq.6 will lead to similar integral formulation, as follows

$$\begin{bmatrix} [M]_{eq} & [A_s] \\ [A_s]^T & 0 \end{bmatrix} \begin{Bmatrix} \{\dot{q}\}^{t_0+\Delta\tau} \\ \{P_c\} \end{Bmatrix} = \begin{Bmatrix} \{P\}_{ext} \\ \{P\}_{con} \end{Bmatrix} \tag{10}$$

where $[M]_{eq}$ = the equivalent mass matrix; $\{P\}_{ext}$ = the equivalent external impulse vector.

So explosive loading can be conveniently taken into account by applying the equilibrium equation of impulse. If no contact remains in the time step, the aforementioned procedure is valid too.

NUMERICAL STABILITY

The difference formula of Newmark- β method is adopted in this paper. Collision may cause abrupt change of acceleration, so the acceleration vector at the beginning of the time interval should be avoided. Thus in the difference formula of Newmark- β method

$$\begin{aligned} \{\dot{q}\}^{t_0+\Delta\tau} &= \{\dot{q}\}^{t_0} + \left[(1-\alpha)\{\ddot{q}\}^{t_0} + \alpha\{\ddot{q}\}^{t_0+\Delta\tau}\right]\Delta\tau \\ \{q\}^{t_0+\Delta\tau} &= \{q\}^{t_0} + \{\dot{q}\}^{t_0}\Delta\tau + \left[(\frac{1}{2}-\beta)\{\ddot{q}\}^{t_0} + \beta\{\ddot{q}\}^{t_0+\Delta\tau}\right](\Delta\tau)^2 \end{aligned} \tag{11}$$

α is assumed to be 1 and β be 0.5.

Since the equilibrium equation of impulse is applied in dynamic analysis of structures, numerical stability of direct integral method should be discussed. For convenience, a linear elastic system of singe degree of freedom without collision is taken as an exemplification.

From Eq.11 the acceleration and displacement can be expressed in terms of velocity. After substituting them into Eq.10, the velocity after time interval $\Delta\tau$ can be obtained. So one has

$$\begin{Bmatrix} \ddot{q} \\ \dot{q} \\ q \end{Bmatrix}^{t_0+\Delta\tau} = [A] \begin{Bmatrix} \ddot{q} \\ \dot{q} \\ q \end{Bmatrix}^{t_0} + [L]\frac{Q_{ext}}{m} \tag{12}$$

where $[A]$= the transfering matrix; $[L]$= the loading operator; Q_{ext} = equivalent external load; m = equivalent mass.

Relations between the radius of spectrum of $[A]$ and $\frac{\Delta\tau}{T}$ is shown in Fig.3, from which one can see that the Newmark- β method is unconditionally stable, when $\alpha = 1$ and $\beta = 0.5$. Herein T is the period of the system, and ξ is the ratio of damping.

NUMERICAL EXAMPLE

The proposed model has been incorporated into a program. A frame collapsed during the 1976 Tangshan Earthquake is taken as an example. Sketches of the structure are shown in Fig.4 and the input earthquake records is shown in Fig.5. The peak horizontal acceleration (PHA) is 149gal.

The normal collapse process is shown in Fig.6. It is recorded that columns of the first and second floor were seriously damaged. After disjoint of one beam at the first story at about 3.31sec, the lateral displacement was increased strikingly, and failures were strongly centralized on the first story. And thus the collapse was unavoidable.

Collapse of the frame assumed to have some weak components is shown in Fig.7~9. It can be seen that, the duration of collapse process for frames with weak components was greatly reduced. This is because of the weak components that are broken through too early. It should also be noted that the collisions during collapse were recorded rather well.

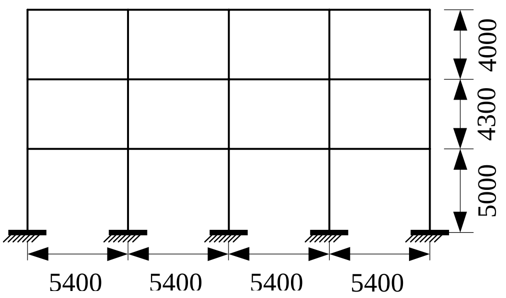

Figure 4: Sketch of frame (unit mm)

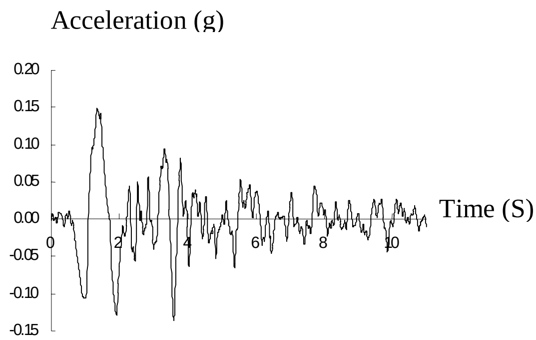

Figure 5: Ground motion records

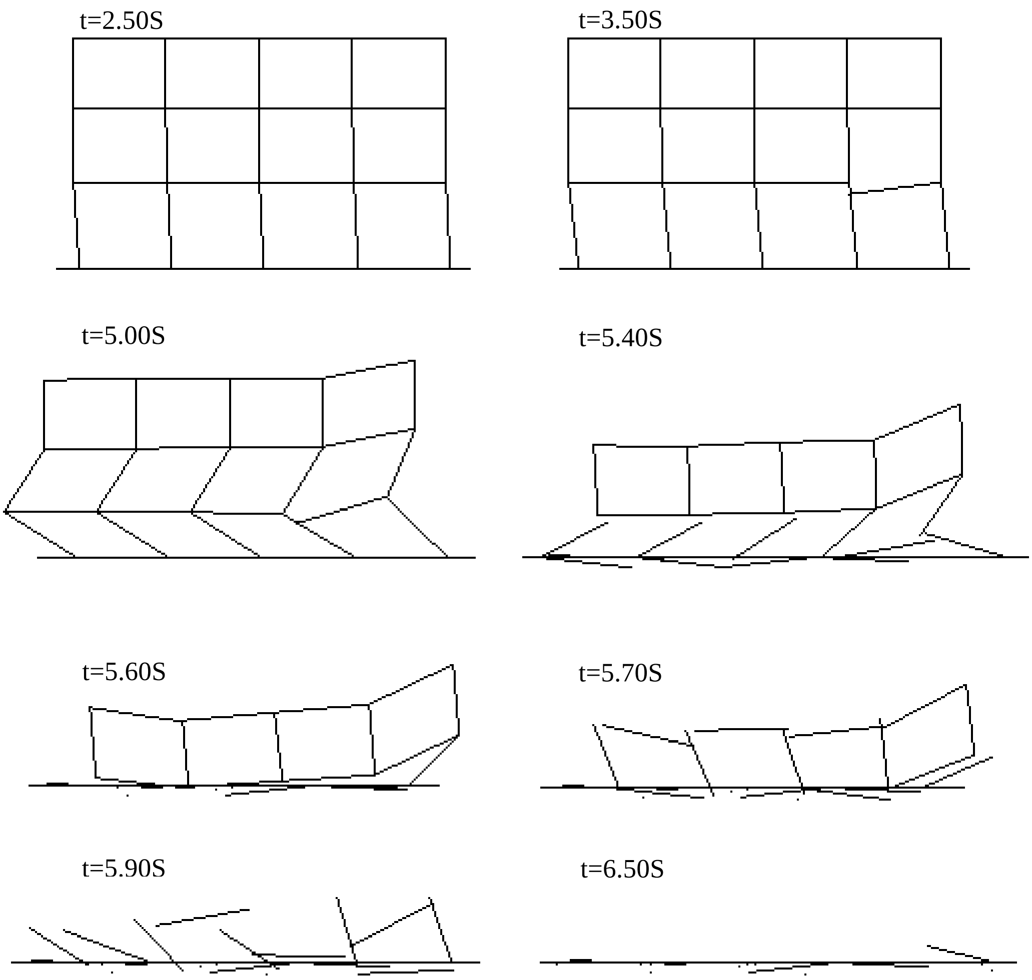

Figure 6: Collapse of normal frame

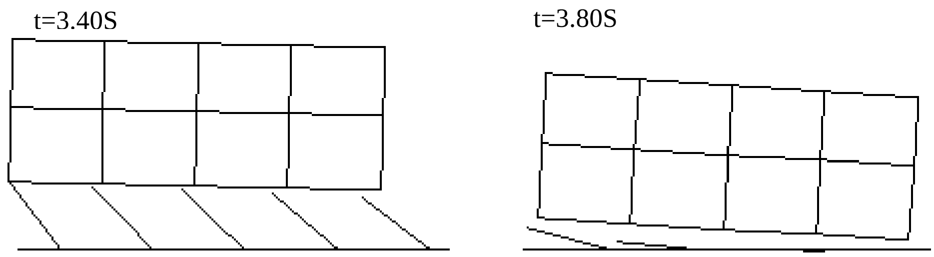

Figure 7: Collapse with soft foot

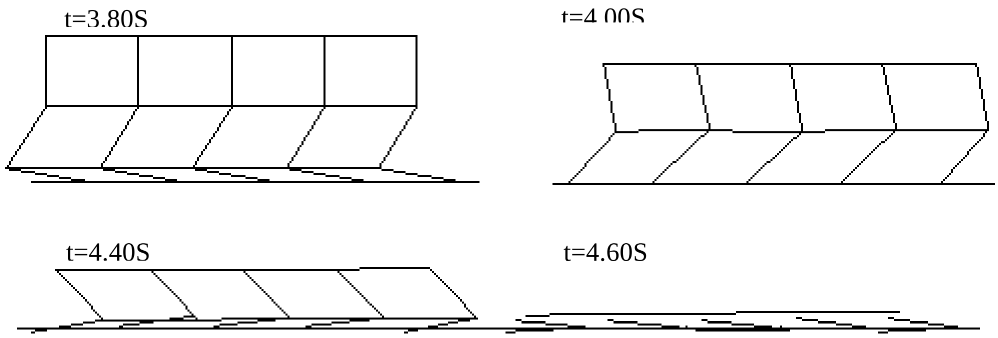

Figure 8: Collapse of Pan-cake

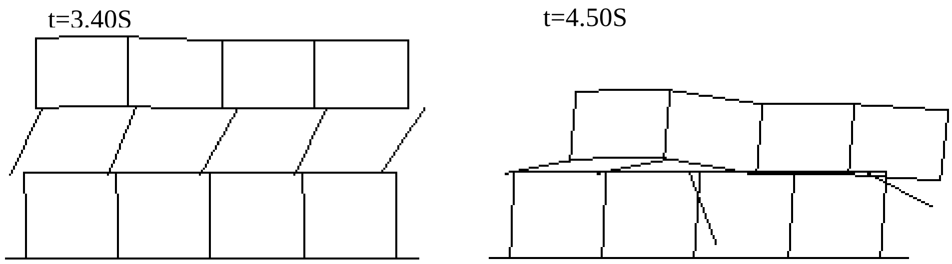

Figure 9: Collapse with weak story

CONCLUSIONS

A modeling scheme is developed to simulate the collapse process of reinforced concrete frame structures under dramatic loading. Based on singularity functions, the deflection discontinuity is submerged within element and conventional procedures are still valid in global analysis.

By introducing a Lagrangian multiplier to modify the kinetic energy, dynamic effects of impact can be simulated fairly well in the collapse analysis of structures. With the help of high-speed computers, the tedium of numerical computations may be greatly alleviated. By proper division of network, the colliding model proposed herein is also suitable for complex types of geometry.

The numerical example illustrates the bright prospect of the mixed hinge model and colliding model. It is observed that brittle failure of one part of the structure will cause abrupt acceleration changes, which in turn might bring on notable re-distribution of internal forces and hence more failures of other portions. As a result, brittle failure may damage adjacent portions of a structure or overload other members, and consequently a local failure may spread to the whole or a significant portion of the structure, only to create a progressive collapse. Moreover, the duration of collapse will be rather shorter than a normal structure. So brittle failure must be considered in seismic analysis of reinforced concrete frames especially under severe ground motions, in order to find out vulnerable components for aseismic strengthening.

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