NONLINEAR DYNAMIC EARTH DAM - FOUNDATION INTERACTION

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ABSTRACT

The nonlinear earth dam - foundation interaction is considered based on a general, rigorous, efficient coupled finite element - boundary element formulation. The dam body and a finite region near the dam, which may consist of materials that behave nonlinearly, are modeled with finite elements. The rest of the halfspace, consisting of a linear elastic material, is modeled with boundary elements. The general formulation is applied to show the effects of nonlinearity and soil-structure interaction on the dam response, using a simple nonlinear model.

KEYWORDS

Earth dams; soil-structure interaction; nonlinear analysis; seismic response; boundary elements.

INTRODUCTION

Earth and rockfill dams built in seismic regions, must be designed safely and economically to withstand potentially strong earthquakes. Extensive earthquake damage or failure of such structures may result into substantial loss of life and property or cause serious environmental problems. This paper presents a general, rigorous, cost-effective method for nonlinear dynamic soil-structure interaction using a mixed finite element-boundary element (FE-BE) formulation. In general, a realistic assessment of the behavior of earth and rockfill dams to strong ground shaking depends significantly on the proper consideration of factors such as the nature of seismic waves, site effects, the dynamic soil-structure interaction, accurate measurement of the material properties, and the nonlinear behavior of the materials, including the development of water pressures and large residual deformations (Gazetas 1987; Gazetas and Dakoulas 1992; Gazetas *et al.* 1994; Finn *et al.* 1995; Dakoulas 1993; Dakoulas *et al.* 1995, 1996; Abouseeda *et al.* 1995). Due to inherent limitations, current methods of dynamic analysis of soil-structure systems may consider inadequately essential factors controlling the response. The FEM allows significant flexibility for problems of finite domain, but it is not perfectly suited for dealing accurately and efficiently with infinite domains. On the other hand, the BEM is superior in handling infinite domain problems, but it can only treat efficiently linear elastic problems. To benefit from the advantages and avoid the limitations of the two methods, a hybrid FE-BE method has been developed for problems of complex geometry, material heterogeneity and nonlinearity in the near field (Zienkiewicz *et al.* 1977; Brebbia *et al.* 1984; Beskos 1987; von Estorff *et al.* 1989).

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NONLINEAR COUPLED FE-BE FORMULATION

The dynamic soil-structure interaction model assumes that the dam and possibly a foundation soil layer (near field) may behave nonlinearly, whereas the rest of the halfspace (far field) behaves as a linear elastic material (Fig. 1). The far field region, Ω^B, is discretized using the BEM. The BE nodes of Ω^B are divided into the outer nodes Γ_o^B and the interface nodes Γ_i^B. The near field region, Ω^F, is discretized using the FEM. The FE region nodes of Ω^F consist of the interface nodes Γ_i^F and the remaining nodes Γ_r^F.

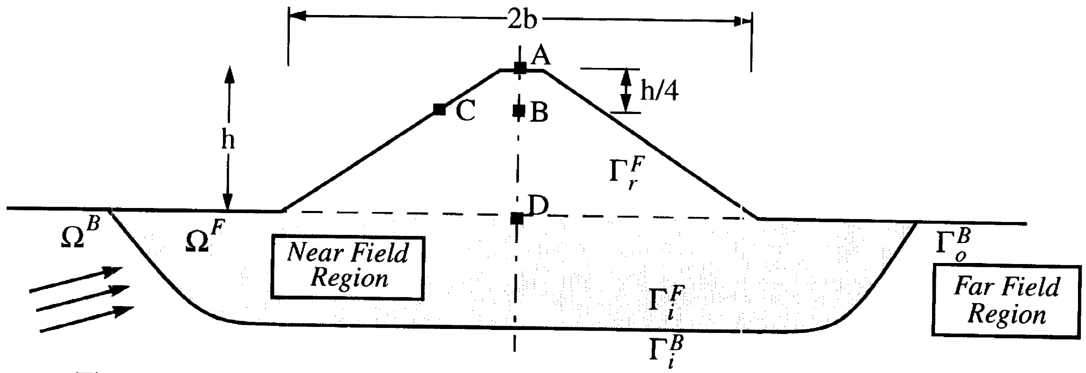

Fig. 1.Earth dam, foundation layer (near field) and elastic halfspace (far field).

Boundary Integral Formulation

For zero initial conditions and body forces, Love's integral representation becomes (Eringen *et al.* 1975)

$$c_{ij}u_i(\xi, t) = \int_\Gamma \{G_{ij} \times t_i(x, t) - F_{ij} \times u_i(x, t)\} \tag{1}$$

where $u_i(x, t)$ and $t_i(x, t)$ are the displacement and traction components i at point x and time t; $G_{ij} = G_{ij}(x, t, \xi, \tau)$ and $F_{ij} = F_{ij}(x, t, \xi, \tau)$ are the displacement and traction components i at point x due to a concentrated pulse in direction j at point ξ in a homogeneous linear elastic solid of infinite extent; and c_{ij} is a constant. This system of equations can be solved for the displacements and tractions for any boundary and interior point. The numerical solution requires approximation of the temporal and spatial variation of $u_i(x, t)$ and $t_i(x, t)$, which are assumed constant within each time step. At time step N, (1) is written as

$$c_{ij}u_i^N(\xi, t) = \sum_{n=1}^{N} \int_\Gamma \{G_{ij}^n(x, t, \xi, \tau)t_i^{N-n+1}(x) - F_{ij}^n(x, t, \xi, \tau)u_i^{N-n+1}(x)\} \tag{2}$$

where $\{u^N\}$ and $\{t^N\}$ are the displacement and traction vectors for all boundary nodes; G_{ij}^n and F_{ij}^n are the temporal integrals of the fundamental solutions G_{ij} and F_{ij} at time step n. The geometry of the boundary is modeled using isoparametric linear elements, for which the coordinates, displacements and tractions are computed as $\{x_i\} = [N^B]^T\{X_i\}$ where X_i are the corresponding nodal values and N^B is the shape function of a standard element with coordinates (ζ, η). Thus, (2) becomes

$$c_{ij}u_i^k(\xi, t) = \sum_{n=1}^{N}\sum_{m=1}^{M}\left[T_{ik}^{N-n+1}\int_{-1}^{1}\int_{-1}^{1} G_{ij}^n(x, t, \xi, \tau)N^B(\zeta, \eta)d\zeta d\eta \right.$$
$$\left. -U_{ik}^{N-n+1}\int_{-1}^{1}\int_{-1}^{1} F_{ij}^n(x, t, \xi, \tau)N^B(\zeta, \eta)d\zeta d\eta\right] \tag{3}$$

where U_{ik}^N, T_{ik}^N are the nodal displacements and tractions and M is the number of boundary elements. After integration, (3) is expressed in a matrix form as

$$[F^1]\{u^N\} = [G^1]\{t^N\} + \sum_{n=2}^{N}([G^n]\{t^{N-n+1}\} - [F^n]\{u^{N-n+1}\}) \tag{4}$$

where $[G^n]$ and $[F^n]$ are the coefficient matrices of the system at time $n\Delta t$. For the current time step N, all traction vectors for $n=1$ to N (representing the seismic excitation) and displacement vectors for $n=1$ to $N-1$

are known. Truncation of the boundary at a certain distance from the structure has given very good results (Abouseeda and Dakoulas 1995; Dakoulas and Abouseeda 1996).

Finite Element Formulation

The displacements within each FE are approximated by $\{u_i(x,t)\} = [N^F]^T\{\hat{u}_i(t)\}$, where N^F is the shape function and $\hat{u}_i(t)$ are the nodal displacements. The Galerkin weighted residual formulation is used, with a weighting function identical to the shape function. This discretization scheme yields the equation

$$[M]\ddot{u} + [C]\dot{u} + [K]u + f = 0 \tag{5}$$

where $[M]$, $[C]$, and $[K]$ are the global mass, damping and stiffness matrices. The involved integrations are evaluated using the Gauss quadrature. The time integration is carried out by the Newmark β scheme, which approximates the displacement derivatives at time $(m+1)\Delta t$ from the displacement and its derivatives at time $m\Delta t$. The nonlinear inelastic behavior of the material is incorporated by using a time lagging procedure. The system matrices are computed for each time step.

Coupling of Boundary and Finite Elements

Equation (4) can now be written as

$$\{t^N\} = [V^1]\{u^N\} + \sum_{n=2}^{N}([W^n]\{t^{N-n+1}\} - [V^n]\{u^{N-n+1}\}) \tag{6}$$

where $[V^n] = [G^1]^{-1}[F^n]$ and $[W^n] = [G^1]^{-1}[G^n]$. Equation (6) can be further modified as:

$$\begin{bmatrix} t_i^N \\ t_o^N \end{bmatrix} = \begin{bmatrix} V_{ii}^1 & V_{io}^1 \\ V_{oi}^1 & V_{oo}^1 \end{bmatrix}\begin{bmatrix} u_i^N \\ u_o^N \end{bmatrix} + \sum_{n=2}^{N}\left\{\begin{bmatrix} W_{ii}^n & W_{io}^n \\ W_{oi}^n & W_{oo}^n \end{bmatrix}\begin{bmatrix} t_i^{N-n+1} \\ t_o^{N-n+1} \end{bmatrix} - \begin{bmatrix} V_{ii}^n & V_{io}^n \\ V_{oi}^n & V_{oo}^n \end{bmatrix}\begin{bmatrix} u_i^{N-n+1} \\ u_o^{N-n+1} \end{bmatrix}\right\} \tag{7}$$

where the i subscript denotes the interface nodes and the o subscript denotes the outer nodes. Equation (7) represents two linear systems with unknowns t_i^N and t_o^N, respectively. By eliminating u_o^N, one can write the following expression:

$$\{t_i^N\} = [K_{ii}^B]\{u_i^N\} + \{R\} \tag{8}$$

where

$$[K_{ii}^B] = [V_{ii}^1] - [Y][V_{oi}^1] \tag{9}$$

$$[Y] = [V_{io}^1][V_{oo}^1]^{-1} \tag{10}$$

$$\{R\} = [Y][t_o^N] + \sum_{n=2}^{N}\begin{pmatrix} [V_I^n]\{u_i^{N-n+1}\} + [V_{II}^n]\{u_o^{N-n+1}\} \\ -[W_I^n]\{t_i^{N-n+1}\} - [W_{II}^n]\{t_o^{N-n+1}\} \end{pmatrix} \tag{11}$$

$$[V_I^n] = [V_{ii}^n] - [Y][V_{oi}^n] \tag{12}$$

$$[V_{II}^n] = [V_{io}^n] - [Y][V_{oo}^n] \tag{13}$$

$$[W_I^n] = [W_{ii}^n] - [Y][W_{oi}^n] \tag{14}$$

$$[W_{II}^n] = [W_{io}^n] - [Y][W_{oo}^n] \tag{15}$$

Since the tractions on the outer surface are known, $\{R\}$ is also known, and (8) can be used to compute the displacements $\{u_i^N\}$ and tractions $\{t_i^N\}$ at the interface nodes Γ_i^B. Similarly, the FE subregion nodes are divided in the interface nodes Γ_i^F and the remaining nodes Γ_r^F. Equation (5) can be written as

$$\begin{bmatrix} K_{rr}^F & K_{ri}^F \\ K_{ir}^F & K_{ii}^F \end{bmatrix} \begin{bmatrix} \hat{u}_r^N \\ \hat{u}_i^N \end{bmatrix} + \begin{bmatrix} f_r \\ f_i \end{bmatrix} = \{0\} \tag{16}$$

Note that $\{t\}$ represents the tractions along the boundary, whereas $\{f\}$ represents the forces at the nodes. By expressing the BE tractions as a function of nodal values

$$\{t_i^N\} = [N^B]^T \{t_i^N\} \tag{17}$$

the nodal forces are computed using the transformation matrix

$$A = \int_{\Gamma_i} [N^F]^T [N^B] \, d\Gamma_i \tag{18}$$

From (8), (16) and (18), the following system of equations is obtained

$$\begin{bmatrix} K_{rr}^F & K_{ri}^F & 0 \\ K_{ir}^F & K_{ii}^F & A \\ 0 & K_{ii}^B & I \end{bmatrix} \begin{bmatrix} \hat{u}_r^N \\ \hat{u}_i^N \\ \hat{t}_i^N \end{bmatrix} = \begin{bmatrix} -f_r \\ -f_i \\ R \end{bmatrix} \tag{19}$$

which is solved for the displacements $\hat{u}_r^N$, $\hat{u}_i^N$ and the tractions $\hat{t}_i^N$ at the time step N. Once these values are known, the displacements at the outer nodes of the BE subregion Γ_o^B can also be calculated. Further condensation of (19) can be achieved, if only the near field response is of interest.

Nonlinear Stress-Strain Model

Although the final version of the formulation will incorporate a rigorous elasto-plastic model, the current version uses the simple Ramberg-Osgood model with the Masing criterion. The τ–γ curve is defined by

$$\frac{\gamma}{\gamma_y} = \frac{\tau}{\tau_y} + \alpha \left| \frac{\tau}{\tau_y} \right|^r \qquad \text{for virgin loading} \tag{20}$$

$$\frac{\gamma - \gamma_0}{2\gamma_y} = \frac{\tau - \tau_y}{2\tau_y} + \alpha \left| \frac{\tau - \tau_y}{2\tau_y} \right|^r \qquad \text{for unloading/reloading} \tag{21}$$

where γ_y and τ_y are the yielding strain and stress, respectively. For $\alpha = 1.5$ and $r = 2$, the modulus reduction and damping ratio versus cyclic shear strain are in good agreement with the experimental data.

APPLICATION: NONLINEAR RESPONSE OF A DAM - FOUNDATION SYSTEM

The general formulation presented above is now applied to examine the effects of the soil nonlinearity and soil-structure interaction on the response of an infinitely-long homogeneous earth dam. The dam has a height of H = 100 m, slopes 2:1, shear wave velocity c_d = 400 m/s, mass density ρ_d = 2000 kg/m^3 and Poisson's ratio $\nu_d = 1/3$. The dam body rests directly on the elastic halfspace which assumes two different values of shear wave velocity equal to c_f = 1000 m/s and ∞ (rigid base), respectively, mass density ρ_f = 2400 kg/m^3 and Poisson's ratio $\nu_f = 1/3$. The dam body is discretized with four-node plane-strain isoparametric elements, and the halfspace with two-node boundary elements. The discretized length of the halfspace surface is taken equal to five times the dam base width. The excitation consists of vertically incident SV waves yielding free-field response identical to the El Centro record scaled at a peak ground acceleration equal to 0.6 g. The following four different nonlinear analyses are performed: (a) Rigid base ($c_f = \infty$) and weakly nonlinear soil behavior (γ_y=0.001); (b) Flexible base (c_f = 1000 m/s) and weakly nonlinear soil behavior (γ_y=0.001); (c) Rigid base ($c_f = \infty$) and moderately nonlinear soil behavior (γ_y=0.0003); (d) Flexible base (c_f = 1000 m/s) and moderately nonlinear soil behavior (γ_y=0.0003). Fig. 2 plots the peak accelerations, displacements, shear strains and shear stresses along central axis of the dam for the four analyses. Notice that the level of

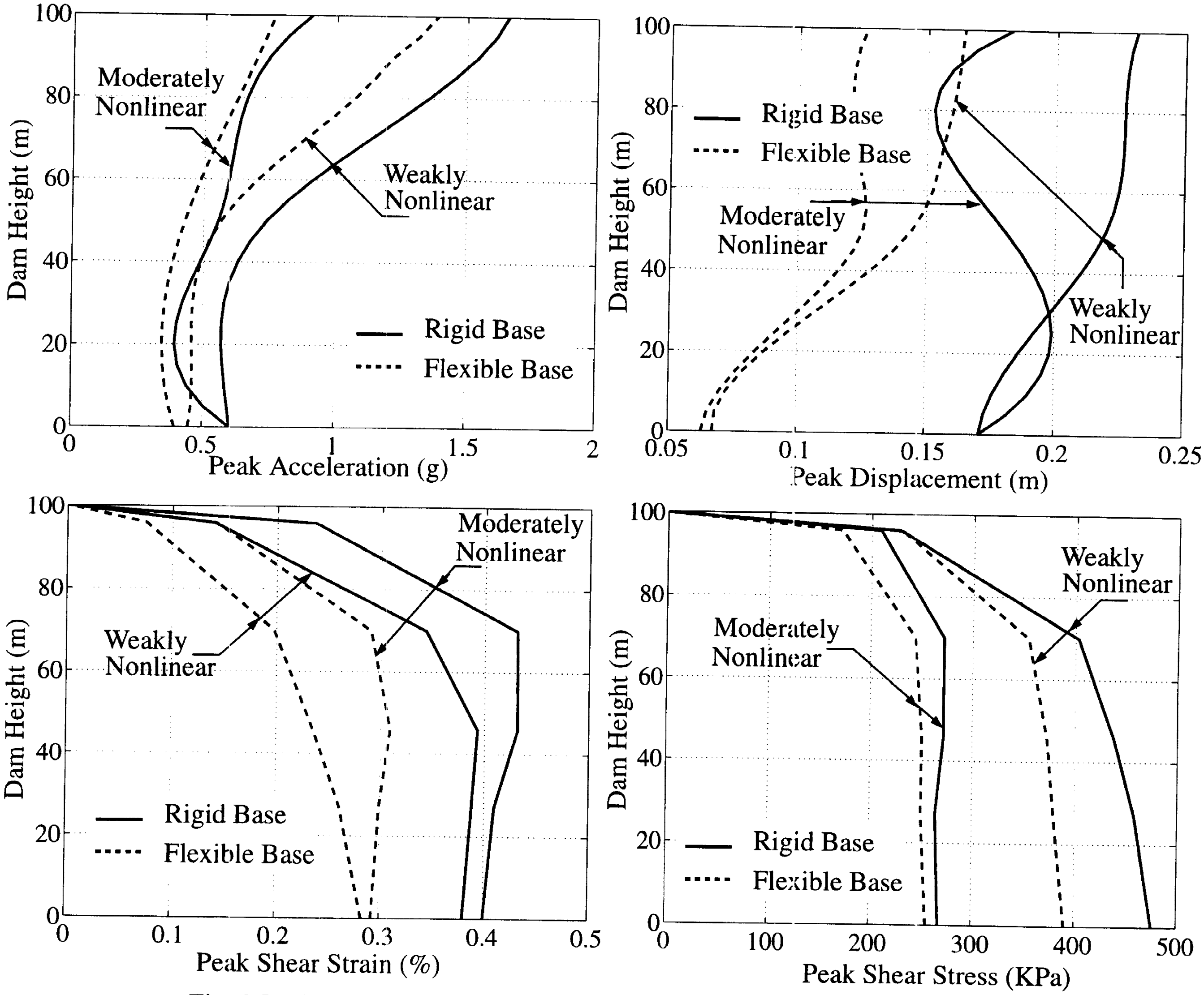

Fig. 2.Peak values of acceleration, displacement, shear strain, and shear stress along the central axis of the dam.

nonlinearity has a significant effect on the value of the acceleration amplification, AF. For the flexible base the crest amplification is only AF = 1.3 for moderate nonlinearity and AF=2.3 for the weak nonlinearity. The corresponding values for the rigid base are AF=1.5 and 2.7, respectively. The peak displacements seem to be significantly lower for the flexible base. This is also true for the peak shear strains which in the upper part of the dam body appear to be up to 50% lower for the flexible base. The peak shear stresses are significantly lower for the more nonlinear behavior due to the higher degradation of the material. Fig. 3 plots the acceleration time histories at four points A (crest), B (slope), C (quarter height from crest) and D (base), as shown schematically in Fig. 1, for the four cases (a), (b), (c) and (d). For the cases (a) and (c) corresponding to rigid base, the acceleration at the dam base (D) is of course equal to the El Centro record. Fig. 3 demonstrates the significant effect of the soil nonlinearity on the amplitude and high-frequency content of the acceleration response. There is an apparent decrease of the high-frequency content from the base to the crest of the dam. Fig. 4 plots the shear strain time histories at the points B, C and D. The accumulation of residual strains is evident in the upper part of the dam and of course is higher for the more nonlinear behavior and the rigid base. Fig. 5 plots the time histories of the nonlinear shear stress - shear strain relationship computed at point B for the four cases examined. As expected, the smaller strains are experienced in the case (b) and the larger strains in case (c). Due to space limitations, only preliminary results are presented here indicating the general trends and the potential of the coupled FE-BE method for nonlinear SSI analysis. More comprehensive results from extensive parametric studies currently under way will be published elsewhere.

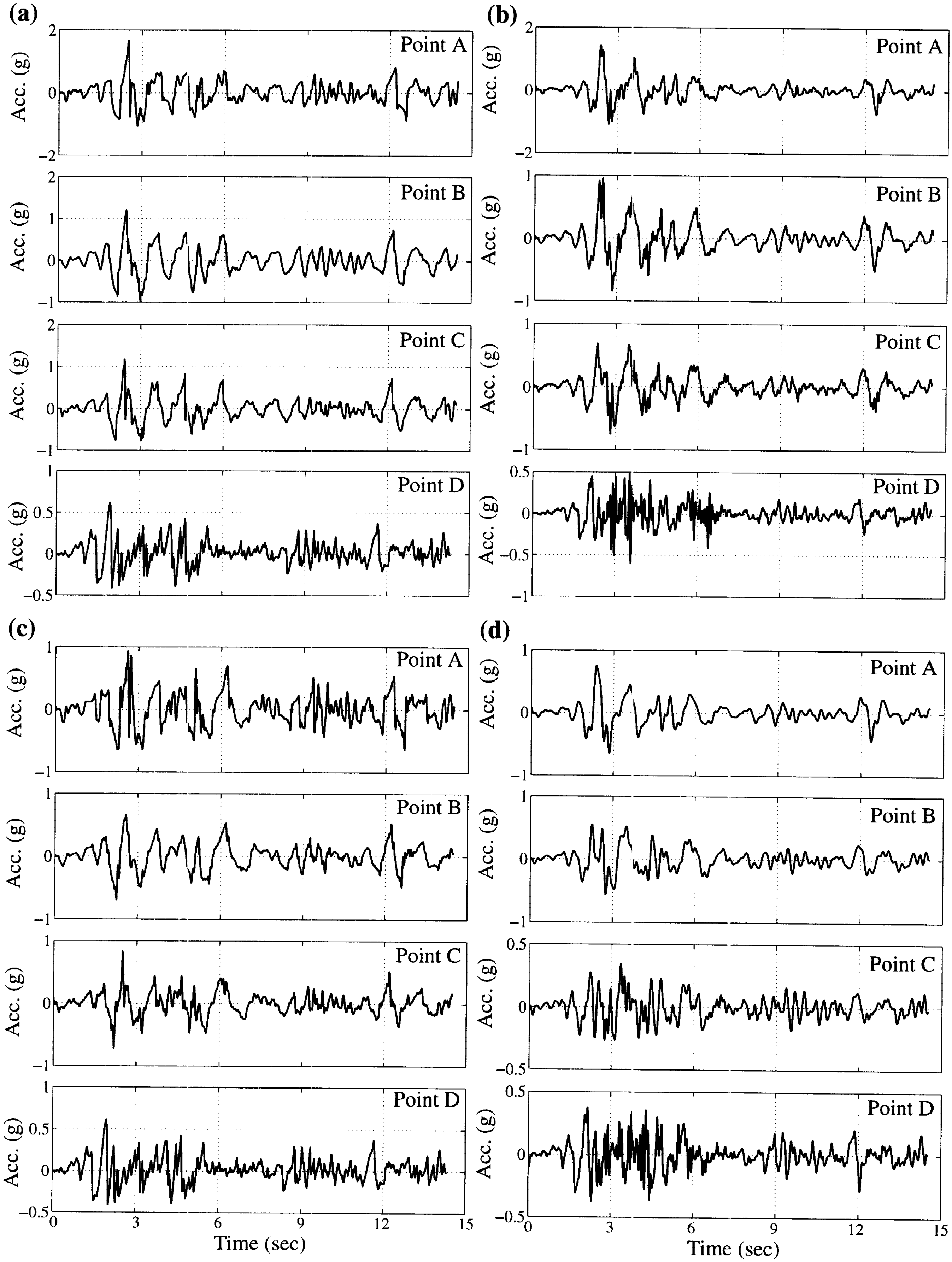

Fig. 3. Acceleration time histories for points A, B, C and D within the dam body.
(a) Rigid base, weakly nonlinear, (b) Flexible base, weakly nonlinear,
(c) Rigid base, moderately nonlinear, (c) Flexible base, moderately nonlinear.

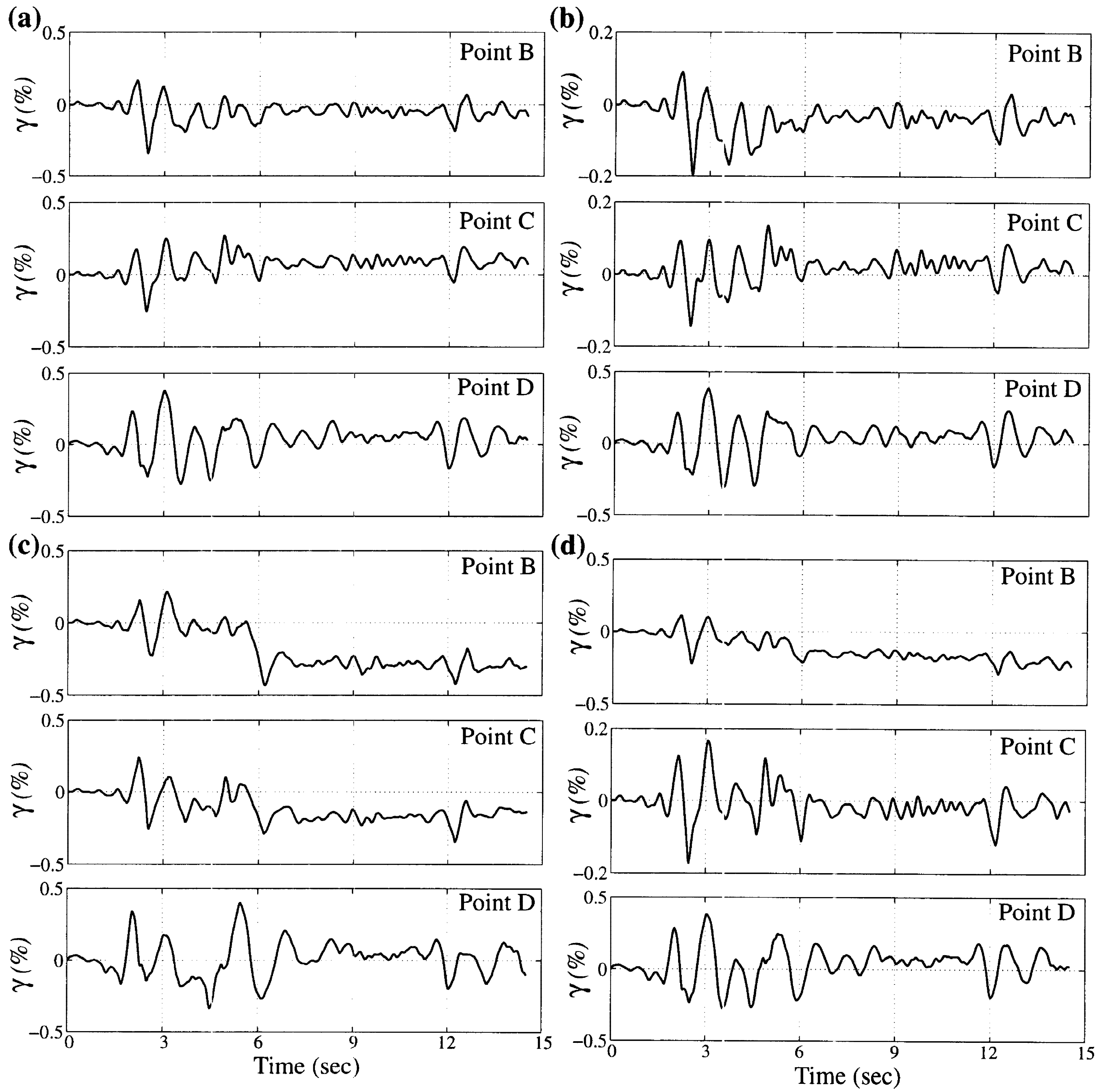

Fig. 4. Shear strain time histories for points B, C and D within the dam body.
(a) Rigid base, weakly nonlinear, (b) Flexible base, weakly nonlinear,
(c) Rigid base, moderately nonlinear, (c) Flexible base, moderately nonlinear.

CONCLUSIONS

A general, rigorous, coupled finite element - boundary element formulation was presented for nonlinear earth dam - foundation interaction. The method proved to be computationally very powerful, allowing efficient nonlinear analysis of 2D soil-structure interaction problems. This formulation can be used to assess the relative importance of the effects of nonlinearity, soil structure-interaction, presence of a soft foundation layer, type of excitation (P, S, Rayleigh waves), and other parameters affecting the response. Preliminary results from few analyses of a dam-foundation system using a simple nonlinear model were presented to show the effects of soil nonlinearity and foundation flexibility. The incorporation of a more rigorous elasto-plastic model for effective-stress analysis considering the development of water pressures is the next step in improving the usefulness of the model.

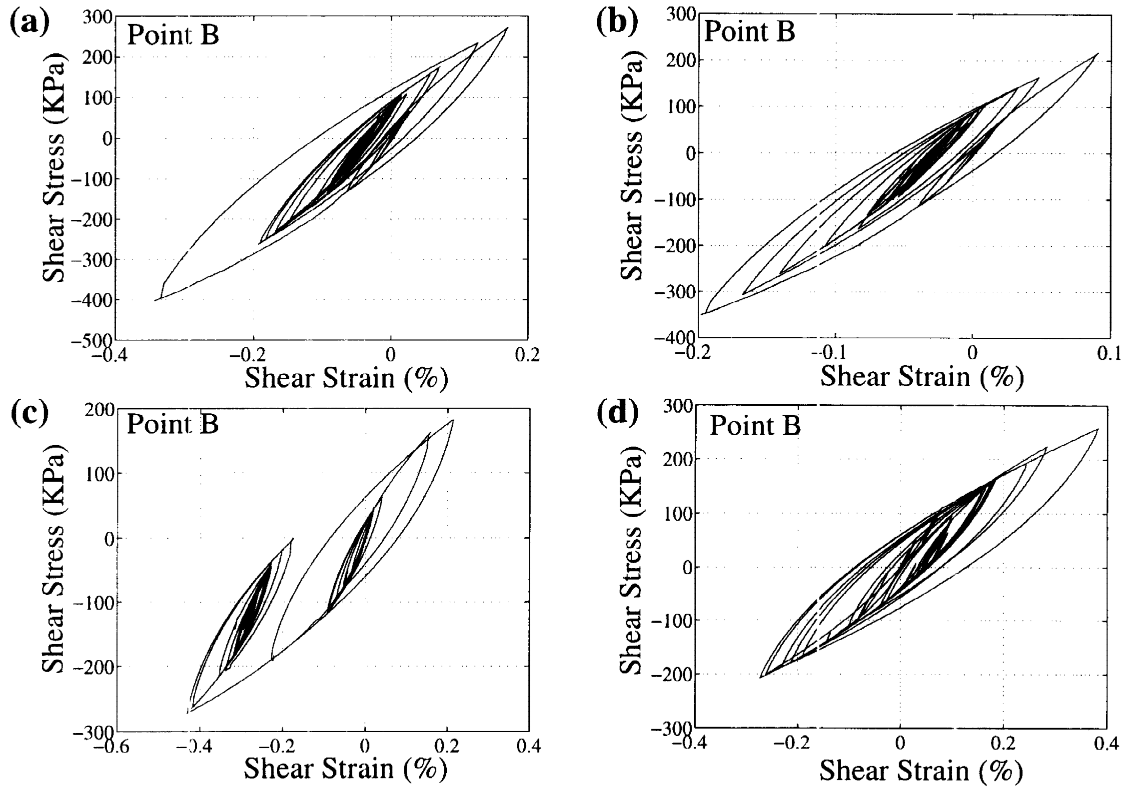

Fig. 5. Stress Strain curve for point B within the dam body.
(a) Rigid base, weakly nonlinear, (b) Flexible base, weakly nonlinear,
(c) Rigid base, moderately nonlinear, (c) Flexible base, moderately nonlinear.

REFERENCES

Abouseeda, H. and Dakoulas P. (1995). Response of earth dams to P and SV waves using coupled FE-BE formulation. Submitted for publication in the *J. of Earthquake Engineering and Structural Dynamics.*

Beskos, D.E. (1987). Boundary element methods in dynamic analysis. *Appl. Mech. Rev.* **40**, 1-23.

Brebbia, C.A., Telles, J.C.F. and Wrobel, L.C. (1984), *Boundary Element Techniques*, Springer, Berlin, Heidelberg, New York.

Dakoulas, P. (1993). Earth dam - canyon interaction effects for obliquely incident SH waves. *Journal of Geotechnical Engineering Div.*, ASCE, **119** (11), 1696-1716.

Dakoulas, P. and Hsu (1995). Response of earth dams in semielliptical flexible canyons to oblique SH waves. *Journal of Engineering Mechanics,* ASCE, **121** (3), 379-391.

Dakoulas P. and Abouseeda, H. (1996). Response of earth dams to Rayleigh waves using coupled FE-BE formulation. Submitted for publication in the *J. of Engineering Mechanics*, ASCE.

Finn, L. W. D., Ledbetter, R. H. and Marcuson III, W. F. (1995). The evolution of Geotechnical Earthquake Engineering Practice in North America: 1954-1994. Proceedings *3rd Int. Conf. on Recent Advances in Geotech. Earthquake Engineering and Soil Dynamics*, St. Louis, Mo, **2**, 881-909.

Eringen, A.C. and Suhubi, E.S. (1975). *Elastodynamics*, Academic Press, New York.

Gazetas, G. (1987). Seismic Response of Earth Dams: Some Recent Developments. *J. of Soil Dynamics and Earthquake Engineering,* **6**(1), 1- 47.

Gazetas, G. and Dakoulas, P. (1992). Seismic Analysis and Design of Rockfill Dams: State-of-the Art. *J. of Soil Dynamics and Earthquake Engineering*, **11**(1), 27-61.

Gazetas, G. and Uddin, N. (1994). Permanent deformation on preexisting sliding surfaces in dams. *J. of Geotechnical Engineering*, ASCE, **120**(11), 2041-2061.

von Estorff, O. and Kausel, E. (1989). Coupling of boundary and finite elements for soil-structure interaction problems. *Earthquake Eng. Struct. Dyn.* **18**, 1065-1075.

Zienkiewicz, O.C., Kelly, D.M. and Bettes, P. (1977). The coupling of the finite element method and the boundary solution procedures. *Int. J. Num. Meth. Eng.* **11**, 355-37.