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DETERMINATION OF OPTIMAL TARGET RELIABILITIES FOR DESIGN AND UPGRADING OF STRUCTURES

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ABSTRACT

A systematic approach is proposed for formulating risk-based cost-effective criteria for the design or upgrading of structures with special reference to earthquake protection. Acceptable risks (or target reliabilities) for damage control and life safety are determined on the basis of minimum expected life-cycle cost, and from which risk-consistent criteria for design or upgrading are developed. The approach is illustrated for a specific class of reinforced concrete buildings in Mexico City relative to earthquake protection.

KEYWORDS

Cost-Efective Design; Target Reliability; Seismic Design Criteria; Seismic Upgrading Criteria; Optimal Design.

INTRODUCTION

In the design of new structures or upgrading of existing structures, the underlying economics is of concern and importance. Although this is generally recognized, the significance of economic factors has not been integrated properly or explicitly with the technical issues in the development of criteria for design or upgrading of structures. These issues have been recognized and addressed, however (e.g. Liu, et al, 1972, 1976; Rosenblueth, 1976); Liu, et al (1972) presented a definitive mathematical formulation of aseismic design on the basis of minimum life-cycle cost. The present study extends these efforts to formulate a comprehensive basis for developing cost-effective criteria for design and upgrading.

The purpose of required criteria is to insure adequate level of safety or performance of a structure. In light of unavoidable uncertainties in the prediction of loadings and in the estimation of structural capacities, structural safety or performance may be measured in terms of probability or risk. The first step, therefore, requires the determination of "what constitutes acceptable risk" for design or upgrading, and on the basis of which associated criteria may be formulated. With reference to earthquake protection, the acceptable risk (or target reliability) will depend on the following: (i) the seismicity or seismic hazard at the site of a structure; (ii) the probability of structural damage expected from a given earthquake; (iii) the structure's cost (or cost of upgrading) versus the potential losses from future damage or collapse.

Proposed is an approach to systematically integrate all the above factors quantitatively to obtain the optimal acceptable risks (or target reliabilities) for damage control and collapse prevention against earthquake hazard, and from which the corresponding criteria for design and upgrading may be formulated. The proposed integrative approach is now feasible because of the recent developments in reliability engineering; however, to implement the approach requires the formulation of credible cost functions for specific classes and usage of structures.

Overview of Approach

In the design of new structures, or upgrading of existing structures, the proper level of safety or performance may be based on a trade-off between the cost of protection versus potential future losses caused by earthquakes. This involves the consideration of the expected damage costs from future earthquakes , besides the cost of the structure. The proper level of safety, therefore, may be formulated on the basis of minimizing the expected life-cycle cost as a function of the underlying risk (probability of damage or collapse) or reliability.

The essential risk-based cost functions will consist of the initial cost of the structure (as designed or upgraded) and the potential cost of damage from future earthquakes. The initial (or upgrading) cost will naturally increase with the underlying reliability. Conversely, the expected damage cost will decrease with reliability. As the damage cost pertains to future earthquake events, it must be discounted for its present worth. Combining the initial cost and the expected damage cost yields the expected life-cycle cost function. The expected cost of damage for a given design may be obtained by integrating over all the possible intensities for each earthquake; whereas the reliability for the specified design may be the reliability against the lifetime maximum intensity as defined in the hazard curve. A composite curve is, therefore, generated for all the alternative designs and the *target reliability* will be the reliability corresponding to the minimum expected life-cycle cost. However, if a "design earthquake" is specified, the reliability conditional on that particular intensity corresponding to the minimum expected cost will be the target reliability.

FORMULATION OF COST FUNCTIONS

In the formulation of the cost functions, the different cost items may be classified into three categories as follows:

1. Those that vary explicitly with risk or reliability; i.e. the cost will increase or decrease with the probability of damage underlying a structure- -e.g. the cost of the structure or its upgrading.
2. Those that are consequences of damage or collapse of a structure- -e.g. repair cost and other damage losses, which are conditional on the occurrence of damage.
3. Those that are independent of risk- -e.g., cost of finishings.

Cost items of the first category are directly functions of the underlying risk or reliability, whereas those of the second category depend on the level and occurrence of damage and, therefore, it is the expected cost that is pertinent. Cost items of the third category are constants and may be neglected as they will not influence the determination of the optimal risk.

The important cost functions therefore, are those of the initial (or upgrading) cost and the expected damage cost. Each of these can be formulated as follows.

Initial and Upgrading Cost

To develop the initial cost C_i of a new structure, or the upgrading cost of an existing structure, the structure is designed or upgraded using a current code procedure (e.g. 1994 UBC) but with varying base shear coefficients yielding a series of designs (or of varying degrees of upgrading) of the same structure, from which the cost for each of the designs (or upgradings) may be estimated. Then, under an earthquake of specified intensity (with a prescribed spectrum appropriate for the site), the probability of

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damage (or reliability) of each of the designs can be assessed, yielding therefore the initial (or upgrading) cost as a function of reliability under the given earthquake intensity. By varying the earthquake intensity, a family of such cost functions is generated as shown conceptually in Fig. 1.

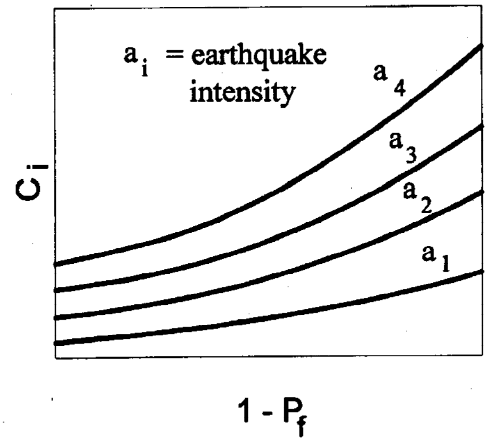

Fig. 1 Initial cost functions

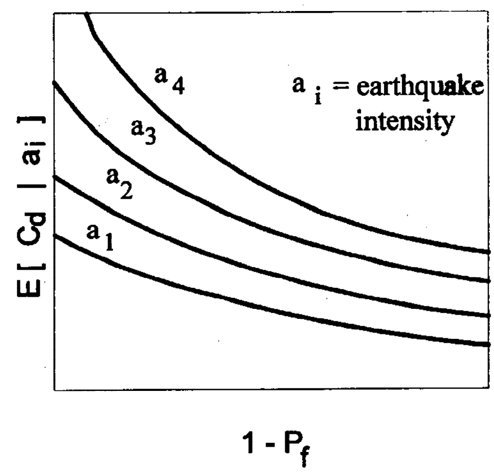

Fig. 2 Damage cost functions

Damage Cost

For each of the designs (or upgradings), the potential cost of damage from future earthquakes must also be estimated. This will include the cost of repair, the loss of contents, the cost of injuries, the loss associated with fatality and the economic impact losses; each of the cost/loss component will be a function of the damage level. Thus, the total damage cost would be

$$C_d = C_r + C_c + C_e + C_{in} + C_f \tag{1}$$

where: C_r = the cost of repair or replacement; C_c = the loss of contents; C_e = the economic loss due to business interruption; C_{in} = the cost of injury; C_f = the cost of life loss.

Each of the damage cost components in eq. (1) will depend on the global damage level x, i.e.

$$C_j = C_j(x) \tag{2}$$

where $j = r, c, e, in, f$.

As there are probabilities associated with damage levels and earthquake occurrences, it is the expected damage cost that is pertinent. Moreover, as this cost pertains to future occurrences of earthquakes, it must be discounted for present worth. Therefore, if the damage level X is defined with PDF, $f_X(x)$, each of the corresponding expected damage cost items in eq. (1) would be:

$$E(C_j) = \int_0^\infty c_j(x) f_X(x) dx \tag{3}$$

Conceptually, the expected damage cost function would be as shown in Fig. 2, in which each curve corresponds to a given earthquake intensity. The initial (or upgrading) cost, and the repair cost and loss of contents, are largely technical issues; however, the other damage cost items in eq. (1) may involve socio-economic considerations.

Present Worth of Future Losses. For consistency, all the cost items in eq. (4) must be expressed on a common basis; e.g. in terms of the present worth. The initial cost (or the upgrading cost) would normally be in present value. All the expected damage costs are associated with structural damage or

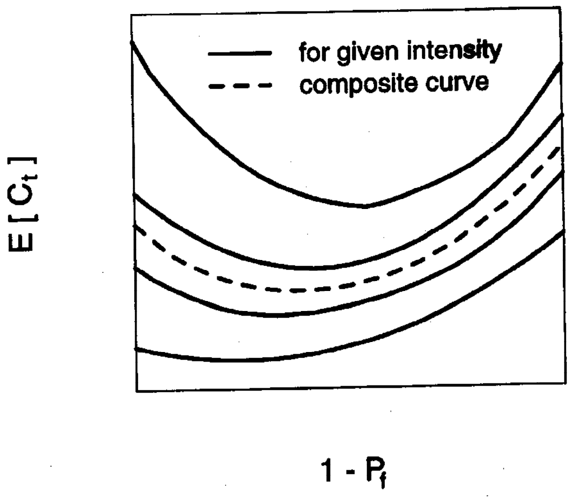

Fig. 3 Expected life-cycle cost functions

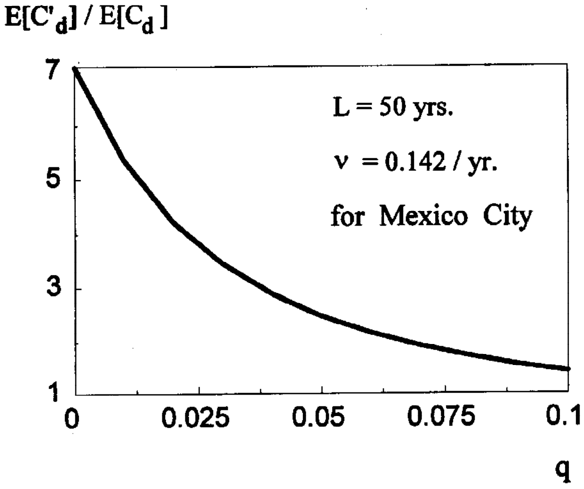

Fig. 4 Present worth factor

collapse caused by future earthquakes , and thus the present worth of the respective costs will depend on the times of occurrence of these earthquakes. As the occurrences of the earthquakes are unpredictible, the corresponding occurrence times may be described as random variables. In this regard, assuming that the occurrences of potentially damaging earthquakes at a site constitute a Poisson process, the occurrence time of each earthquake is defined by the gamma distribution (Ang and De Leon, 1995). Over the life L, the present worth of the expected damage cost due to future earthquakes can then be shown to be (Ang and De Leon, 1995):

$$E[C_d'] = E(C_d) \sum_{n=1}^{\infty} [\sum_{k=1}^{n} \frac{\Gamma(k, \alpha L)}{\Gamma(k, \nu L)} (\frac{\nu}{\alpha})^k] \frac{(\nu L)^n}{n!} e^{-\nu L} \tag{4}$$

where:

$$E[C_d] = \int_{a_{min}}^{a_{max}} E[C_d(a)] f_A(a) da \tag{5}$$

$C_d(a)$ = current cost of damage under intensity a; $f_A(a)$ = PDF of the intensity of one earthquake, which may be derived from the annual seismic hazard curve; q = annual discount rate; ν = annual mean occurrence rate of significant earthquake intensities; $\alpha = \nu + ln(1+q)$; and $\Gamma(k, \alpha L)$ = the incomplete gamma function. A plot of the present worth factor, $E[C_d']/E[C_d]$, as a function of the discount rate, q, is shown in Fig. 4 appropriate for Mexico City. Implicit in eq. (4) is the assumption that whenever damage occurs, the structure is restored/repaired to its original condition.

STRUCTURAL DAMAGE AND RELIABILITY ASSESSMENTS

Extensive assessments of structural damage and reliability are obviously needed in the formulation of the risk-based cost functions. For these purposes, the required probabilities of damage for each of the designed or upgraded structures may be obtained using the damage model of Park and Ang (1985) and reliability method that are well established. For a structure, its global damage is a function of the damages of its constituent elements or components, particularly of the critical components.

In assessing the global damage of a given structure, the damages of its constituent components are, therefore, required. For this purpose, the structure must be modeled and analyzed for its response to a given earthquake; the process involves nonlinear and hysteretic response analysis of the structure from which the response statistics (namely, the maximum deformation and dissipated hysteretic energy) of the constituent elements can be calculated and the respective damage indices, global damage and associated probabilities, can be evaluated.

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OPTIMAL TARGET RELIABILITY

Expected Life-Cycle Cost

Combining the initial (or upgrading) cost with the expected damage cost should yield the expected life-cycle cost function; thus,

$$E(C_t) = C_i + E(C_d') \tag{6}$$

in which $E(C_d')$ is obtained from eqs. (1), (3), (5) and (6).

Composite Cost Function

For each of the designs, the expected damage cost under a given earthquake can be obtained through eq. (6) in which $E[C_d(a)]$ is evaluated through eq. (3). Also, for each design, the corresponding expected reliability against the lifetime maximum earthquake intensity, defined by the lifetime seismic hazard curve, is evaluated. These results yield the composite expected life-cycle cost function.

Determination of Optimal Target Reliability

As shown conceptually in Fig. 3, for each of the earthquake intensities, there is a reliability that corresponds to the minimum expected life-cycle cost. Therefore , if a "design earthquake" is specified, the optimal target reliability would be defined as the reliability that corresponds to the minimum life-cycle cost for the specified design earthquake intensity. However, in order to take account of all possible earthquake intensities at the site of the structure, as defined by a seismic hazard curve, the expected optimal reliability may be adopted as the target reliability for both design and upgrading. In this latter case, the corresponding expected acceptable risk would be risk associated with the minimum expected life-cycle cost from the composite cost function.

ILLUSTRATIVE APPLICATION

The approach described above is illustrated with the development of criteria for the design and upgrading of a 7-story R/C building in Mexico City. The plan and elevation of the building (which is a regular R/C framed structure) is shown in Fig. 5.

Initial and Upgrading Cost Functions

The initial cost for the 7-story building are shown in Fig. 6 as a function of the reliability $(1-p_f)$ under several intensities of the 1985 Mexico City earthquake. The building was designed following the 1987 Mexican seismic code, but using several base shear coefficients, from which the initial cost is estimated and the reliability is calculated for each of the designs. Observe that each of the points in Fig. 6 corresponds to a particular design of the building. In the case of upgrading, the cost will depend on the method of upgrading and will increase with the reliability of the upgraded structure above that of the original structure. With this consideration, the upgrading cost function may be expressed as follows:

$$\frac{C_u}{C_i} = e^{k_1[1-(\frac{p_f}{p_{f_0}})^{k_2}]} - 1 \tag{7}$$

where: C_u = cost of upgrade, C_i = cost of original structure, k_1, k_2 = constants, p_f = probability of damage (failure) of the upgraded structure, p_{f_0} = probability of damage (failure) of the original structure.

The upgrading cost for the 7-story building are shown in Fig. 7 as a function of the reliability $(1-p_f)$ under several intensities of the 1985 Mexico City earthquake. The constants, k_1 and k_2, were evaluated empirically based on repair cost data for Mexico City (Guerrero, 1990).

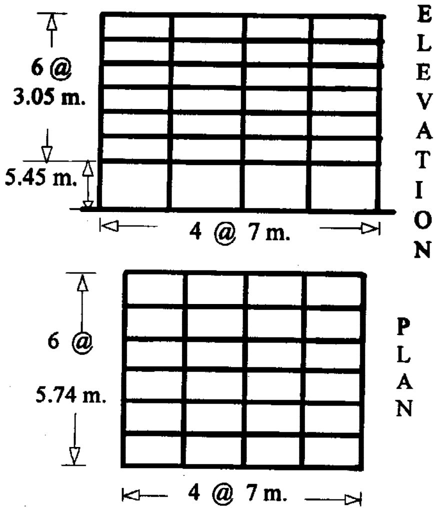

Fig. 5 Building plan and elevation

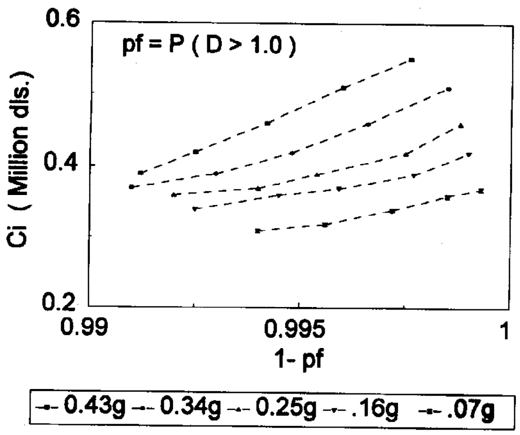

Fig. 6 Initial cost for design

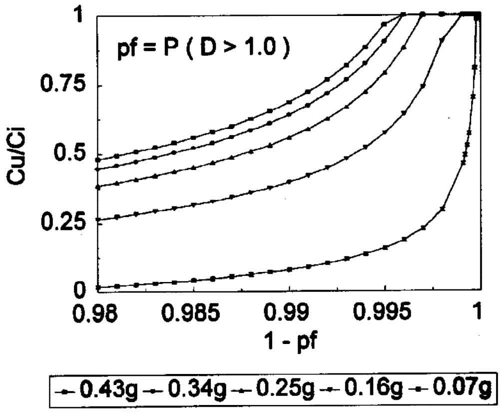

Fig. 7 Upgrading cost

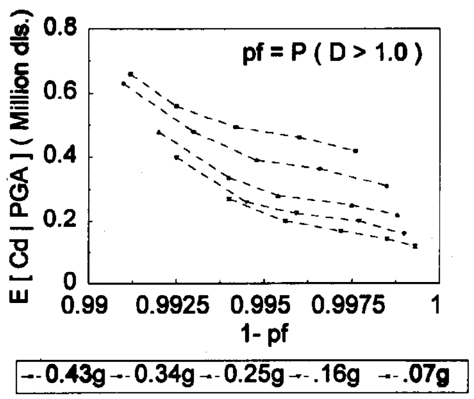

Fig. 8 Expected damage cost

Expected Damage Cost Functions

For either design of new structures, or upgrading of existing structures, the expected damage cost from future earthquakes will be the same for a structure of given reliability. Based on the damage and cost data reported for the 1985 Mexico City earthquake (Guerrero, 1990), the damage cost components have been developed (Ang and De Leon, 1995) as follows:
As the limit of repairable damage in Mexico City is $D_m = 0.5$ (De Leon and Ang, 1994), the repair cost function developed on the basis of available repair cost data is:

$$C_r = 1.64\, C_R\, D_m \ , \ 0 \leq D_m < 0.5 \ ; \ C_r = C_R \ , \ D_m > 0.5 \tag{8}$$

where C_R = replacement cost of the original structure = $1.15C_i$.

The total loss of contents is assumed to be 50% of the replacement cost, and will vary linearly with D_m for intermediate damage; thus:

$$C_c = 0.5C_iD_m \ , \ 0 \leq D_m \leq 1.0 \ ; \ C_c = 0.5\, C_i \ , \ D_m > 1.0 \tag{9}$$

The direct economic loss from loss of rentals (assuming the building is an apartment dwelling) has been determined (Ang and De Leon, 1995) to be:

$$C_e = 480A(D_m^2) \ , \ 0 \leq D_m < 1.0 \ ; \ C_e = 480A \ , \ D_m \geq 1.0 \tag{10}$$

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where A = floor area of the building. The cost of injuries is (Ang and De Leon, 1995):

$$C_{in} = 672A(D_m^2) \ , \ 0 \leq D_m < 1.0 \quad ; \quad C_{in} = 672A \ , \ D_m \geq 1.0 \tag{11}$$

And the cost of fatalities, determined on the basis of the human capital approach or loss to the national GNP, was determined for Mexico City (Ang and De Leon, 1995) as:

$$C_f = 1572AD_m^4 \ , \ 0 \leq D_m < 1.0 \quad ; \quad C_f = 1572A \ , \ D_m \geq 1.0 \tag{12}$$

eqs. (8) through (12) are used (with $D_m = x$) in eq. (3) to obtain the respective expected damage costs. All the cost are in terms of 1985 US dollars. With the present worth factor from Fig. 4 for $q = 8\%$, the expected damage costs for the 7-story building are generated as functions of the reliability $(1 - p_f)$ under varying intensities of the 1985 Mexico City earthquake. The results are summarized in Fig. 8, for five PGA's.

Expected Life-Cycle Cost Functions

Combining the initial cost and the expected damage cost under each of the various earthquake intensities yields the expected life-cycle cost as a function of the reliability $(1 - p_f)$. As can be seen in Fig. 9, for a given intensity there is a reliability that corresponds to the minimum life-cycle cost.

Optimal Target Reliability and Associated Criteria

In Fig. 9 the optimal reliability is indicated for each earthquake intensity. Therefore, if a "design earthquake" is specified, the indicated optimal reliability can be used as the target reliability for design. For example if a PGA of 0.24g is designated as the design earthquake (as suggested by Rosenblueth, 1989) the optimal target reliability against collapse would be $4.5x10^{-3}$, and the corresponding base shear coefficient would be 0.48. Whereas, with the seismic hazard curve of Mexico City (according to Esteva and Ruiz, 1989), the expected optimal reliability would be $6x10^{-3}$ and the associated base shear coefficient is 0.42. The target reliabilities and associated base shear coefficients for the design of this class of R/C buildings in Mexico City are summarized in Table 1. Similar results are also obtained for upgrading an existing 7-story building, assumed to be originally designed and built with a base shear coefficient of 0.15. The upgrading is to be accomplished through jacketing of the columns. The corresponding family of expected life-cycle cost functions for upgrading is shown in Fig. 10, for five PGA's. Also, the target reliabilities and associated base shear coefficients are summarized in Table 2. In this case, the optimal upgrading costs are also indicated, for all possible earthquakes, as well as for specified "design" earthquakes.

CONCLUSIONS

Cost-effective criteria for design and upgrading of structures can be developed systematically by integrating all the important technical factors with the economics of earthquake hazard mitigation. The approach described herein offers an effective way to accomplish this objective. Optimal target reliabilities, or acceptable risks, are determined for damage control and collapse prevention of designed and upgraded structures for earthquake protection, and on the basis of which the corresponding criteria (e.g. required base shear coefficients) for design and upgrading can be developed. The methodology provides a means for implementing cost optimal criteria for structural design and upgrading. The approach may also be used to appraise the cost-effectiveness of existing engineering criteria.

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Table 1. Summary of results for design

Limit state $D =$	Optimal acceptable risk	Base shear coeff.	Prescribed earthquake intensity
0.2	0.13	0.42	all possible
0.5	0.03	0.42	"
1.0	$6.0x10^{-3}$	0.42	"
0.5	0.0190	0.60	$0.36g^*$
1.0	$5.3x10^{-3}$	0.60	$0.36g^*$
1.0	$4.5x10^{-3}$	0.48	$0.24g^{**}$

* Corresponds to return period of 475 yrs.
** Corresponds to return period of 143 yrs.

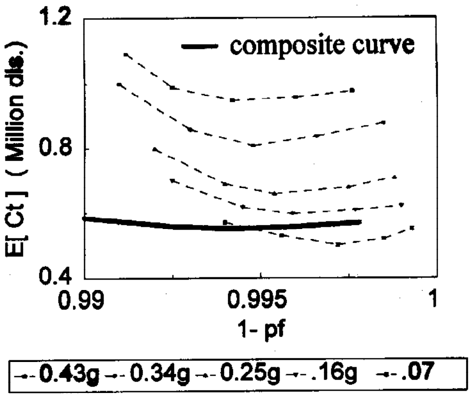

Fig. 9 Expected life-cycle cost for design

Table 2. Summary of results for upgrading

Limit state $D =$	Optimal acceptable risk	Base shear coeff.	Optimal upgrade cost	Prescribed earthquake intensity
0.2	0.15	0.27	0.35 C_0	all possible
0.5	0.060	0.27	0.36 C_0	"
1.0	$2.7x10^{-3}$	0.27	0.37 C_0	"
0.2	0.055	0.21	0.28 C_0	$0.07g^{***}$
1.0	0.014	0.36	0.45 C_0	$0.24g^{**}$
1.0	0.020	0.58	0.80 C_0	$0.36g^*$

*** Corresponds to a return period of 10 yrs.

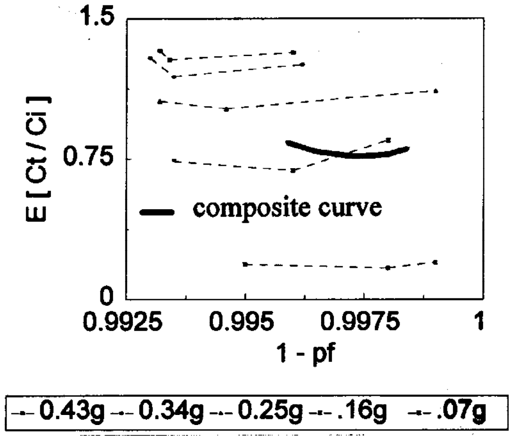

Fig. 10 Expected life-cycle cost for upgrading

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