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THE INFLUENCE OF REINFORCEMENT BUCKLING IN THE ULTIMATE RESPONSE OF R.C. ELEMENTS: A COMPARISON WITH SEISMIC CODES

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ABSTRACT

In order to analyze the ultimate behavior of r.c. members subjected to seismic loads it is fundamental to consider the interaction between longitudinal and transverse reinforcement, which can control longitudinal buckling and prevent ultimate strain failure in the confined concrete. In fact, in a correct seismic design, the flexural elements should fail at the tensile reinforcement with a residual deformation capacity for the confined concrete and compressive bars, while the combined compressive and flexural elements should exhibit considerable ductility up to failure. The proper detailing in order to obtain this behavior must be investigated.

A condition of maximum ductility is obtained by defining the balanced condition as that for which ultimate strain in tensile bars and buckling strain in compressive well-confined bars occur; this balanced condition leads to a lower limit in terms of reinforcement ratio. It is also possible to consider lower ductility levels and hence to obtain the corresponding design curves in terms of the amount of tensile rebars. Moreover other balanced conditions are investigated considering lower compressive concrete strain in order to limit section strength decay. These situations are assumed as ductility boundary conditions and are compared to Seismic Codes.

A simple procedure is used to control the failure mechanism. Balanced diagrams in terms of the ratio of compressive and tensile reinforcement against the ratio of tensile reinforcement are shown. These diagrams are compared to Seismic Code requirements (ACI 318-83 Revised 1989, Eurocode No. 8).

On the basis of a buckling criterion that appears conservative with respect to structural safety, some design charts are obtained considering actual steel characteristics; experimental results of recent Italian steel production are considered. The importance of limiting the flexural reinforcement ratio in earthquake-resistant design in order to enhance structural ductility is pointed out; furthermore it is observed that a correct ductility ratio should be defined when structural elements are designed according to Seismic Codes.

KEYWORDS

Reinforcement buckling; critical compressive strain; ultimate tensile strain; balanced section; sectional ductility; flexural element; combined flexural and compressive element; compressive and tensile reinforcement ratio.

INTRODUCTION

Seismic Codes for r.c. frames prescribe that plastic hinge formation should be guaranteed in beams rather than in columns (strong-column, weak-beam) and define various rules in terms of external loads and reinforcement arrangement in order to obtain low strength decay and high inelastic deformation capacity. A balanced section in terms of failure strains is defined and an acceptable range is prescribed for reinforcement ratio.

Considering that the actual ultimate tensile strain is much greater than the design steel strain, while the design underestimation in terms of compressive concrete strain is lower, the reinforcement ratio necessary to obtain an actual balanced failure has to be particularly low. Such reinforcement ratios are lower than the minimum values prescribed in the seismic code and are also less than the quantity of reinforcement generally used in normal practice; it is therefore necessary to determine the ductility level connected to these quantities in order to evaluate correctly the post-elastic behavior of structural elements and r.c. frames.

In order to point out the influence of steel ductility upon section ductility and to obtain results on the safe side regarding the prediction of actual strength various critical compressive strains in the concrete are considered up to the global buckling of the compressive reinforcement, assumed as the condition for member failure. The Italian steel production is considered and the results obtained are compared to seismic code requirements.

BEHAVIOR OF CONCRETE MEMBERS UNDER SEISMIC LOAD

In order to define the critical condition for a r.c. element the scheme of Fig. 1. is assumed considering either a flexural or a low combined compressive and flexural load. According to this failure condition the top concrete cover is spalled, the concrete core is at ultimate compressive strain, ε_{cu}, and the compressive bars are at critical buckling strain, ε_{scr}, while the ultimate strain in tensile bars, ε_{su}, is assumed; it is therefore possible to investigate sectional ductility considering the yielding strain ε_{sy} or an intermediate strain $\mu\varepsilon_{sy}$ in tensile reinforcement (considering steel ductility μ as representative of sectional ductility).

The neutral axis position is given by Eq. (1), and according to the constitutive nonlinear relationship of steel and concrete it is possible to determine the resultant forces N_S and N_C.

$$x' = \frac{d'\,\varepsilon_{cu}}{\varepsilon_{su} + \varepsilon_{cu}} = k_u d' \tag{1}$$

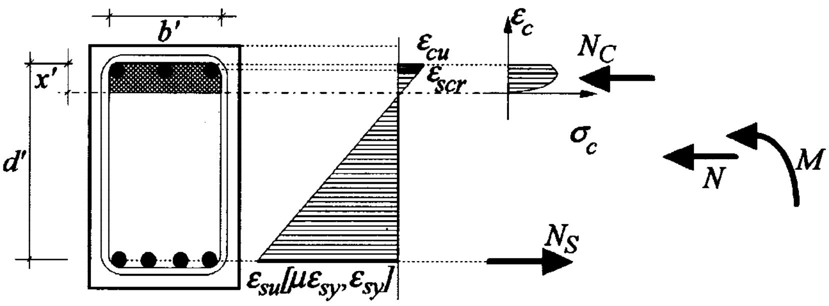

Fig. 1. Cross section, strain and stress distribution in a typical r.c. member at ultimate condition.

If the tensile force N_S is equal to N_C-N the section is a "balanced" section and ultimate strains in concrete and tensile reinforcement are attained; if N_S is greater than N_C-N the actual tensile steel strain is lower than the ultimate value and the thickness of the compressive block is greater than required; the section is a steel-strong one and the balanced condition is obtained by increasing N_S and decreasing N_C; on the contrary if N_S is lower than N_C-N the actual compressive strain is lower than the ultimate value and the section is a steel-weak one.

In order to evaluate the concrete compressive strain influence upon tensile reinforcement failure three critical conditions can be defined considering these phenomena:

a. concrete cover spalling;

b. first buckling of longitudinal reinforcement;

c. global buckling of longitudinal reinforcement with sudden confinement decay and concrete core failure.

The ratio of tensile to compressive reinforcement for each critical condition can be evaluated and design curves can be obtained which characterize the member failure.

Balanced section at concrete cover spalling (critical curve for condition a.). This critical condition can be obtained considering the following forces:

$$N_C = N_{s'} + N_{cc} + N_{cs} + N_{ci}; \quad N_S = \sigma_s A_s \quad [\sigma_s = f_t,\ f_y,\ \sigma_s(\mu\varepsilon_{sy})] \tag{2}$$

where $N_{s'} = \sigma_{s'}A_{s'}$ is the compressive reinforcement force, N_{cc} the concrete core force, N_{cs} and N_{cl} the top and lateral concrete cover forces while the tensile reinforcement force N_s depends on the actual stress considered.

Balanced section at first reinforcement buckling (critical curve for condition b.). In this case, if $N_{s'cr}$ is the compressive reinforcement force and the top concrete cover is crushed, the resultant forces are:

$$N_C = N_{s'cr} + N_{cc} + N_{cl};\quad N_S = \sigma_s A_s \quad [\sigma_s = f_t,\ f_y,\ \sigma_s(\mu\varepsilon_{sy})] \tag{3}$$

Balanced section at global buckling and concrete failure (critical curve for condition c.). In this case:

$$N_C = N_{s'u} + N_{cc} + N_{cl};\quad N_S = \sigma_s A_s \quad [\sigma_s = f_t,\ f_y,\ \sigma_s(\mu\varepsilon_{sy})] \tag{4}$$

where the compressive reinforcement force $N_{s'u} = \Sigma_n \sigma_{s'} A_{s'} + \alpha\Sigma_m \sigma_{s'} A_{s'}$ takes into account the reduced stress in the *m* reinforcement bars involved in the first buckling (condition *b.*); this reduced stress is defined by means of a factor α depending on bar slenderness (Monti *et al.*, 1992) and stirrups involved in the longitudinal buckling (Albanesi *et al.*, 1995.a); the global buckling causes the sudden decay in lateral confinement and the section failure (Albanesi *et al.*, 1995.a, Papia *et al.*, 1989).

The balanced condition depends on the ultimate strains; of these, the reinforcement critical tensile strain (ε_{su}, ε_{sy}, $\mu\varepsilon_{sy}$) is directly obtained experimentally while the concrete critical strain is influenced by geometrical and mechanical characteristics (particularly the lateral confinement and the strain gradient across the section).

The longitudinal buckling and nonlinear reinforcement behavior depend on the arrangement and characteristics of lateral reinforcement; conditions *b.* and *c.* are significantly distinct and represent the anelastic stress level for a structural member; according to buckling analysis it is possible to quantify the lateral reinforcement involved and to control the actual postyield softening branch in compression for compressive bars (Monti *et al.*, 1992).

Disregarding concrete tensile strength in order to simulate cyclic strength decay and to obtain a result which is independent of the load history, the section equilibrium is controlled considering the stress distribution in the cross section and a nonlinear stress-strain model for both concrete [Kent-Park modified, for confined and unconfined concrete (Scott *et al.*, 1982)] and steel [Mahin-Bertero and Monti-Nuti for the hardening and softening branches respectively].

Assigning the area of tensile longitudinal rebars A_s, the reinforcement ratio $\rho_s = A_s/A_c$ and the ultimate curvature $\chi_u = \varepsilon_{cu}/(k_u d')$, the area of the compressive longitudinal rebars $A_{s'}$ can be determined through equilibrium and the critical curve ($\rho_s, A_{s'}/A_s$) is consequently obtained. By means of these critical curves the proper detailing and the sectional ductility can be evaluated in terms of critical strains and code requirements.

This criterion is an approximate one since a monotonic approach was assumed and bond-slip was disregarded; it does however seem that through this approach a correct evaluation of structural capacity and member failure is possible, in which the geometrical and mechanical characteristics are taken into account.

CONCRETE COVER FAILURE AND REINFORCEMENT BUCKLING

Each critical condition can be defined in terms of compressive reinforcement strain and hence sectional ductility can be quantified as a function of the stress level in tensile reinforcement.

Critical condition a.: concrete cover spalling. This condition is very severe because cover spalling can enhance reinforcement buckling; the critical strain can be assumed as the minimum of the ultimate compressive strain for unconfined concrete, ε_{cou}=0.008 (Mander *et al.*, 1992, Scott *et al.*, 1982), and a specific compressive strain in longitudinal reinforcement, ε_{sco}=0.004, (Krauthammer *et al.*, 1982);

$$\varepsilon_{cra.} = min\{\varepsilon_{sc}:\ \varepsilon_c = \varepsilon_{cou};\ \varepsilon_{sc} = \varepsilon_{sco}\} \tag{5}$$

of which the latter is generally appropriate.

Critical condition b.: first reinforcement buckling. The transversal reinforcement can offer different contributions in resisting the outward motion of a longitudinal bar: referring to Fig. 1. the longitudinal bars located at the corner of the tie are constrained by an extensional stiffness, while those located at the center of the tie's leg are constrained by the lateral stiffness of a flexural element having a point load at its center. This second arrangement is less efficient and therefore the first buckling occurs in this longitudinal bar.

Considering the reduced modulus E_{rt} and moment of inertia I_t of the tie cross section, the lateral stiffness is determined assuming a fixed-fixed condition:

$$\mu_t = \frac{192 E_{rt} I_t}{b'^3} \tag{6}$$

This lateral tie stiffness must compared to the critical minimum stiffness for the longitudinal bar modeled as a fixed-fixed column supported by n lateral springs; in order to obtain buckling the column length has to be $(n+1)l_t > 5d_s$ (Monti *et al.*, 1992) where l_t is the tie spacing and d_s the diameter of longitudinal bars.

$$\mu_{cr} = \eta_{cr} \frac{\pi^4 E_r I}{\left[(n+1)l_t\right]^3} \tag{7}$$

In (7) E_r is the reduced modulus and I the moment of inertia of longitudinal bars; the adimensionalized stiffness η_{cr} depends on which tie is involved in the buckling [n=1, η_{cr}=64/π^2; n=2, η_{cr}=2,54; etc. (Albanesi *et al.*, 1995.a)]. Longitudinal buckling occurs if the tie lateral stiffness is lower than the critical stiffness or when the actual compressive stress is greater than the critical stress of a fixed-fixed column of length $(n+1)l_t > 5d_s$:

$$\sigma_{cr} = \frac{\pi^2}{4} \frac{E_r d_s^2}{\left[(n+1)l_t\right]^2} \tag{8}$$

The stiffness of transversal reinforcement depends on actual deformation; the tie tensile strain, ε_{st}, and the longitudinal compressive strain, ε_{sc}, are connected by means of an apparent Poisson ratio, ν_{app}, which depends on the global stress state of the cross section. A minimum condition due to tie yielding, ε_{syt}, should also be imposed. The critical strain in this case can therefore be determined as:

$$\varepsilon_{crb.} = min\{\varepsilon_s\text{: } \sigma_s(\varepsilon_s) = \sigma_{scr}\text{; } \mu_t(\varepsilon_{st} = \nu_{app}\varepsilon_s) = \mu_{cr}\} \geq \varepsilon^*_{sc} = \varepsilon_{syt}/\nu_{app} \tag{9}$$

Critical condition c.: global reinforcement buckling. The critical strain for the longitudinal bar located along the leg of the tie is determined, (9), considering the softening compressive branch in force equilibrium, (4); the least extensional stiffness for longitudinal bars located at the corner can be expressed in terms of tangent modulus E_r and cross-section diameter d_r of the transversal tie:

$$\mu_t = \frac{\pi d_t^4 E_t}{4b'} \tag{10}$$

The critical compressive strain is determined as section failure (Papia *et al.*, 1989, Albanesi *et al.*, 1995.a).:

$$\varepsilon_{crc.} = min\{\varepsilon_s\text{: } \sigma_s(\varepsilon_s) = \sigma_{scr}\text{; } \mu_t(\varepsilon_{st} = \nu_{app}\varepsilon_s) = \mu_{cr}\} \geq max\{\varepsilon_{cuo}\text{; } \varepsilon_{sh}\} \tag{11}$$

This critical condition depends on the apparent Poisson ratio value due to stress and strain gradients in the cross section; in particular it is necessary to use a Poisson ratio greater than the elastic value in order to consider the softening behavior of confined concrete. In this paper a medium value ν_{app}=0.32 is considered which takes into account the strain distribution in the cross section and the transversal confinement (Albanesi *et al.*, 1995.a).

CRITICAL CONDITION ANALYSIS

In order to analyze various cross sections the actual reinforcement behavior is taken into account; considering more than five thousand bars of recent production in Italy and of various diameters (d_s=6-32 mm), the values of Tab. 1. are obtained. The FeB44k steel bars have a nominal strength of 430 MPa and the Italian Code prescribes that the characteristic value, f_k, be expressed in terms of the average deviation, δ, and the medium value, f_m,:

$$f_k = f_m \pm k\,\delta \tag{12}$$

Considering Tab. 1. four types of steel are defined for all bar diameters: type A: f_y=430 MPa, f_t=540 MPa, ε_{su}=0.120 (minimum Code values); type B: f_y=506 MPa, f_t=696 MPa, ε_{su}=0.226 (experimental medium values); type C: f_y=580 MPa, f_t=774 MPa, ε_{su}=0.226 (maximum stress, medium experimental strain values); type D: f_y=506 MPa, f_t=696 MPa, ε_{su}=0.267 (medium stress, maximum experimental strain values).

Italian Steel Production 1993 - 1994 (FeB44K type)													
		f_y				f_t				ε_{su}			
Spec. [No.]	k	Med. [MPa]	δ_y [MPa]	95% [MPa]	5% [MPa]	Med. [MPa]	δ_t [MPa]	95% [MPa]	5% [MPa]	Med. [-]	δ_u [-]	95% [-]	5% [-]
5015	1.64	506.1	45.0	579.8	432.3	695.8	47.6	773.9	617.7	0.226	0.025	0.267	0.185

Tab. 1. Italian Steel Production: medium and characteristics values for FeB44K steel bars.

Thirty six 30x50 cm² rectangular cross-sections were considered (No. 1-9 type A; No. 10-18 type B; No. 19-27 type C; No. 28-36 type D); for each steel type three tie spacings (l_t = 50, 75, 100 mm, d_t = 8 mm) and bar diameters (d_s = 12, 16, 20 mm) are taken into account; the compressive strength of concrete is assumed as f_c = 30 MPa. The medium values computed for the critical strains are: $\varepsilon_{cra.}$=0.004; $\varepsilon_{crc.}$=0.008; $\varepsilon_{crc.}$=0.0115. This last value is a conservative one and thirty percent lower than the medium value obtained through ultimate compressive strain as defined by Scott *et al.* (1982):

$$\varepsilon_{cu} = 0.004 + 0.90\rho_s(f_{yt} / 300) \quad (13)$$

In Figs. 2 to 6 some interesting results are shown; in all critical curves the dot branch refers to the theoretical solution while the thick one represents the actual reinforcement, which is consistent with the cross section arrangement and longitudinal bar spacing.

In Fig. 2 the results for all the cross sections are presented, considering maximum ductility and tensile failure. The comparison to Eurocode 8 shows that when the steel type A is considered the critical curves are less severe than those obtained for the actual steel; this occurs both for the proposed model and for the Seismic Code. When the ratio $A_{s'}/A_s$ assumes low values (less than 0.7), the reinforcement ratio ρ for actual steel is lower than the minimum value ρ_{min} prescribed in EC8 while, for steel type A, this ratio is close to the value

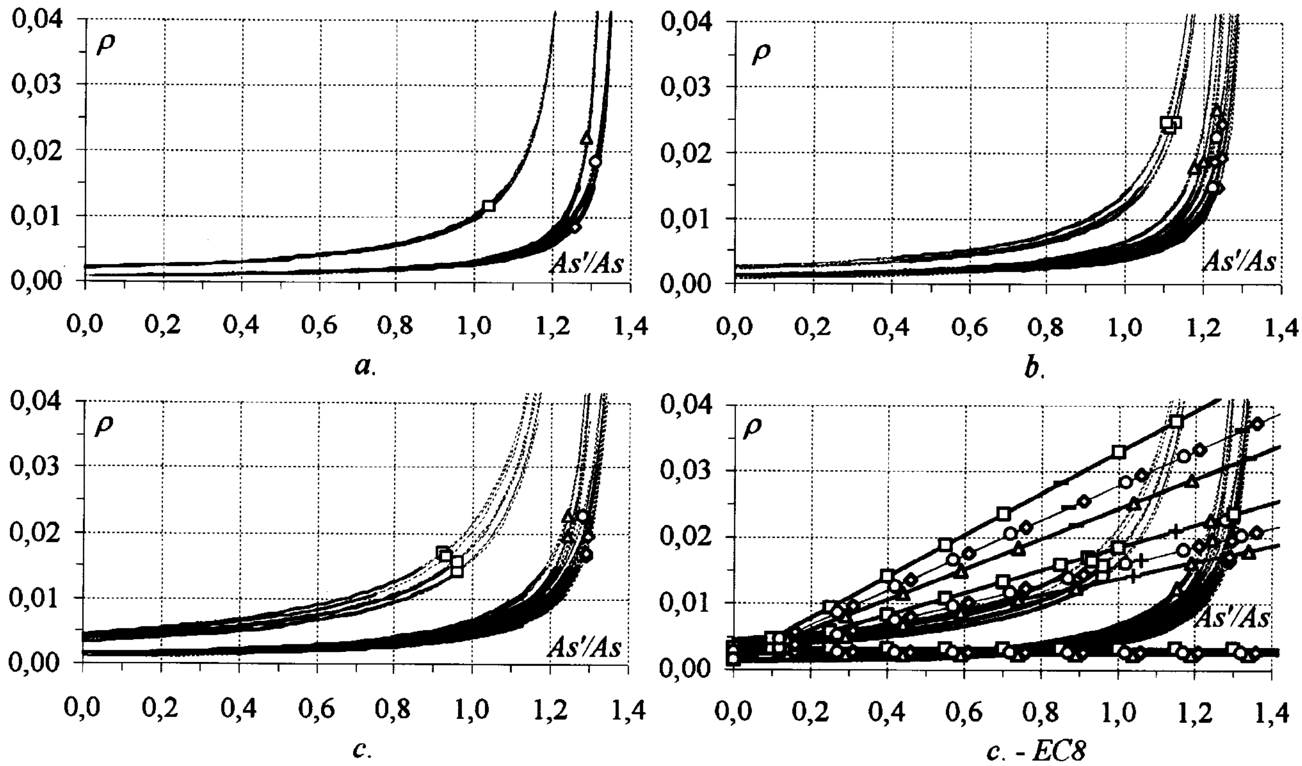

Fig. 2. Critical curves *a.*, *b.* and *c.* for steel types A (□), B (○), C (△) and D (◇) at ultimate tensile strain, considering d_s=12, 16, 20 mm and l_t=50, 75, 100 mm; the *c.* curves are compared to EC8: the higher straight lines (–) represent ρ_{max} for ductility class M and the lower ones (+) for ductility class H; the lowest constant curves represent ρ_{min}.

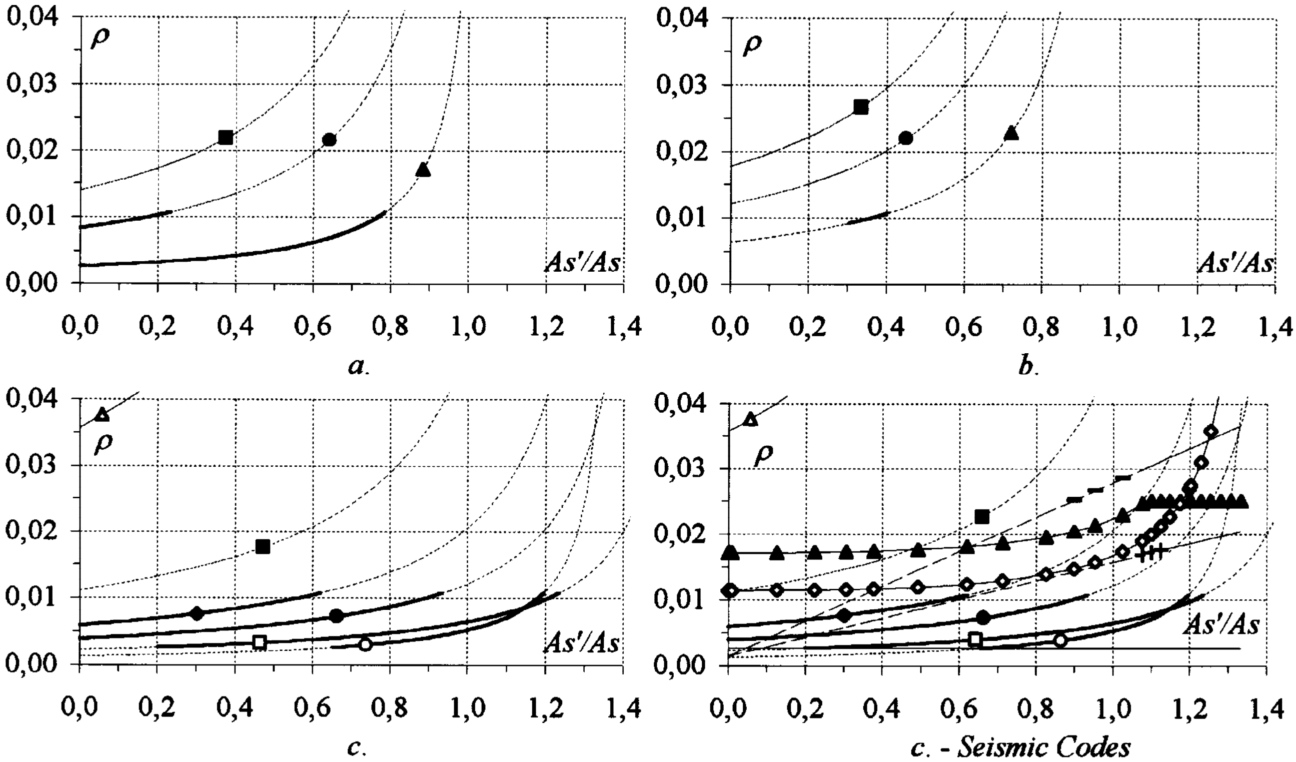

Fig. 3. Critical curves *a.*, *b.* and *c.* for cross-section No. 14: μ=1 (△), μ=10 (■), μ=20 (◆), μ=30 (●), μ=50 (□), μ=89.3 (max) (○); the *c.* curves are compared to Seismic Codes: the straight lines (–, +) represent ρ_{max} for EC8-M, H; the lowest constant curve represents ρ_{min} for EC8 and ACI; the curves (◇)and (▲) represent ρ_{max} in ACI318-77 and in ACI318-83 respectively.

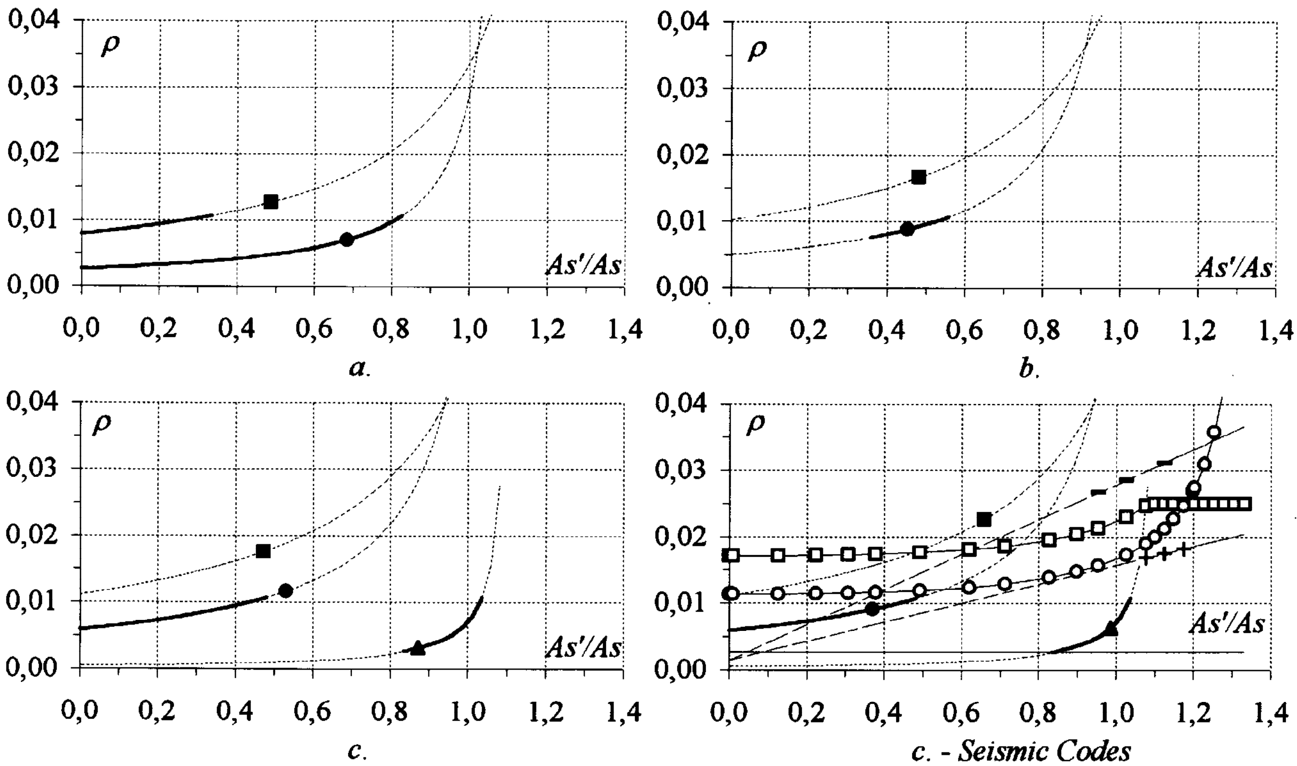

Fig. 4. Critical curves *a.*, *b.* and *c.* for cross-section No. 14 : μ=10 - ν=0 (■), ν=0.1 (●) and ν=0.2 (▲); the *c.* curves are compared to Seismic Codes: the straight lines (–, +) represent ρ_{max} for EC8-M, H; the lowest constant curve represents ρ_{min} for EC8 and ACI; the curves (○) and (□) represent ρ_{max} in ACI318-77 and in ACI318-83 respectively.

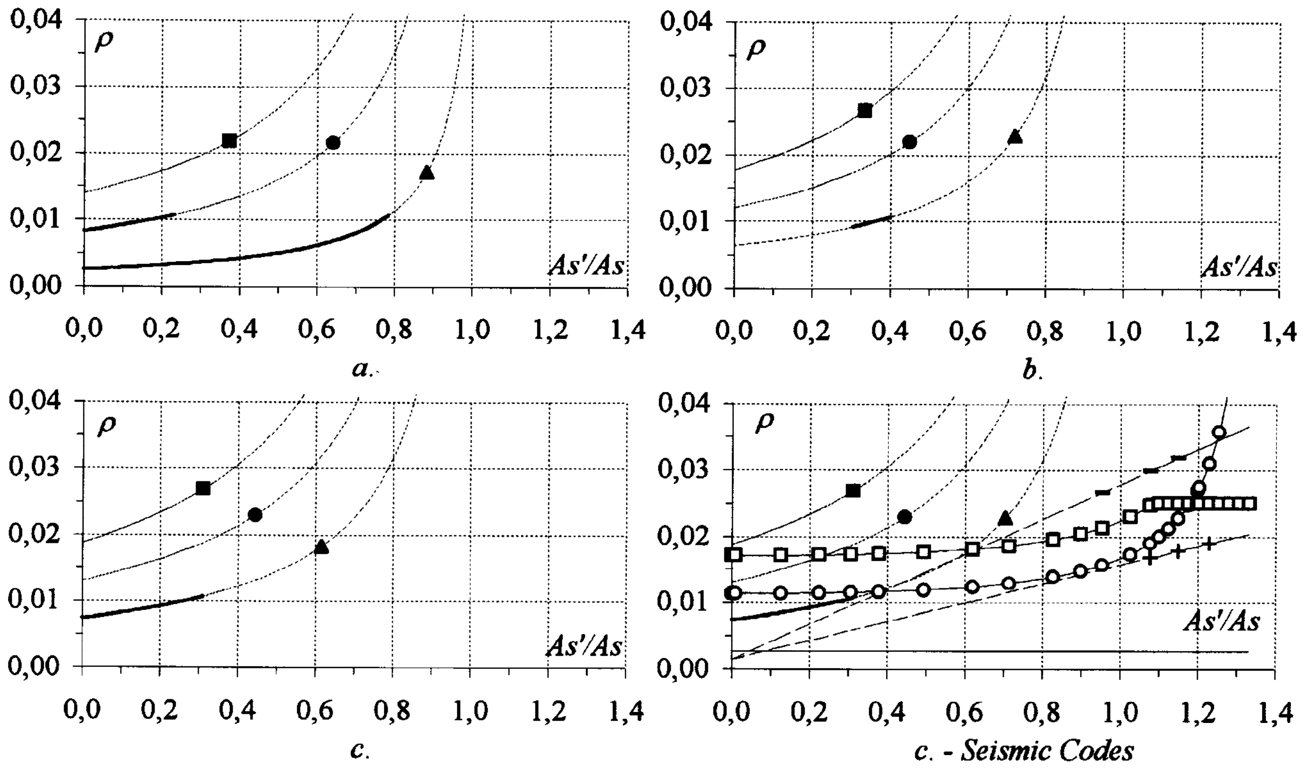

Fig. 5. Critical curves *a.*, *b.* and *c.* for cross-section No. 14: μ=5 - ν=0 (■), ν=0.1 (●) and ν=0.2 (▲); the *c.* curves are compared to Seismic Codes: the straight lines (–, +) represent ρ_{max} for EC8-M, H; the lowest constant curve represents ρ_{min} for EC8 and ACI; the curves (○) and (□) represent ρ_{max} in ACI318-77 and in ACI318-83 respectively.

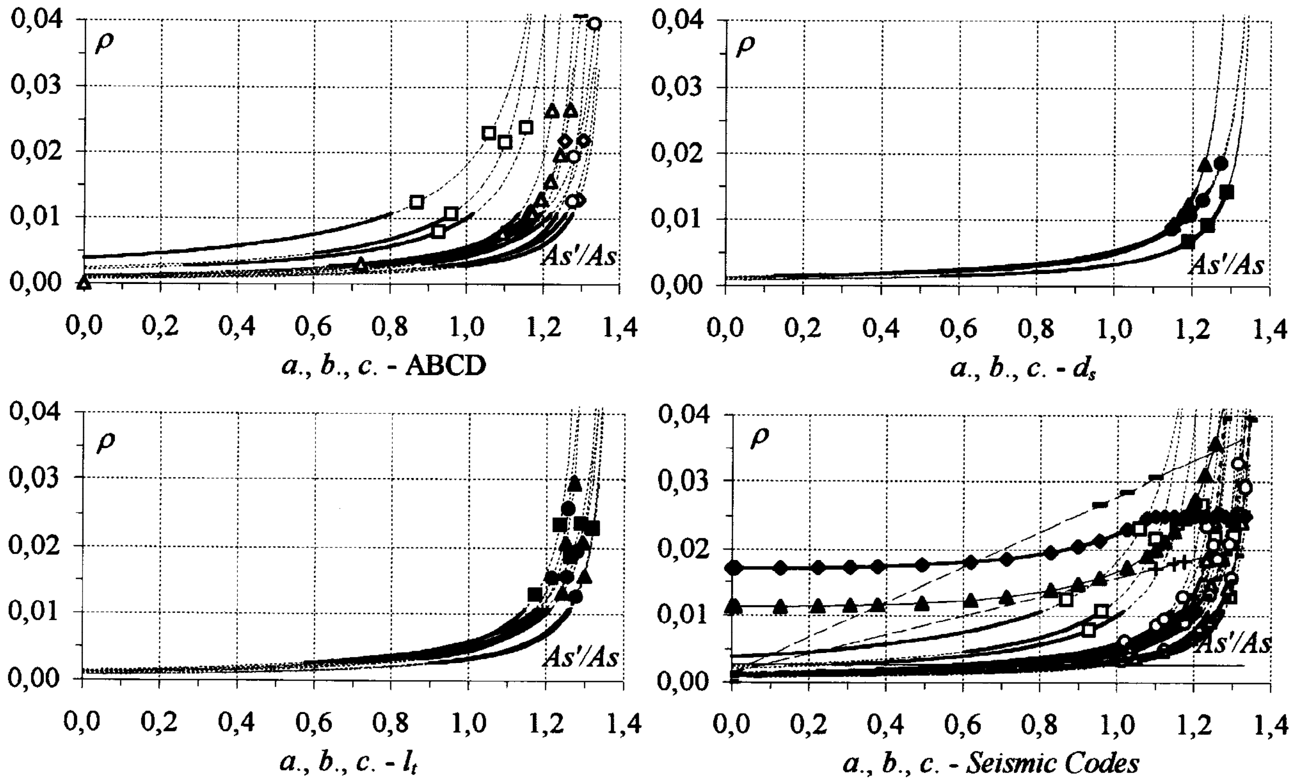

Fig. 6. Critical curves *a.*, *b.*, *c.*: The first diagram refers to specimens No. 5, 14, 23 and 32, with steel type A (□), B (○), C (△) and D (◈) (d_s=16 mm, l_t=75 mm); the second refers to specimens No. 11, 14,17, with d_s=12 (■), 16 (●) and 20 (▲) mm; the third refers to specimens No. 13, 14, 15, with l_t=50 (■), 75 (●), 100 (▲) mm. The comparison with Seismic Codes is detailed in the last figure.

ρ_{max} prescribed for H ductility class. For high values of $A_{s'}/A_s$ (greater than 1.2) however, the critical curves obtained are less severe than any Seismic Code requirements; the variation of bar diameter d_s and tie spacing l_t are non significant for condition *a.*, while they are increasingly meaningful for conditions *b.* and *c.*

In order to determine the reinforcement ratio ρ_{max} that can activate a prefixed ductility the actual strain $\mu\varepsilon_y$ is considered, as shown in Fig. 3 for specimen 14; the critical curves for μ=1 represent the yielding in tensile bars while the curves $\mu=\mu_{max}$ refers to tensile failure. Considering steel type B, for μ=10 the *c.* curve is near to ACI318-83 for $A_{s'}/A_s$=0.5, while for μ=20 the same curve is close to ACI318-77 and to EC8 for ductility class H; for the same steel type and in order to obtain μ_{max}, the requirements on ρ_{min} in EC8 and in ACI codes are close to the reinforcement ratio determined for $A_{s'}/A_s$<0.8.

The role of the axial load $\nu=N/(bhf_c)$, is shown in Fig. 4 (μ=10) and in Fig. 5 (μ=5). An increase in ν determines a severe condition in terms of ρ_{max}, so that in many cases not even the minimum reinforcement requirements can be met. It should be observed that a comparison to Seismic Codes for high values of ν is not representative.

In conclusion, in this paper post-elastic behavior is checked and the failure mechanism is detected, considering in particular the reinforcement buckling. It has been shown that the actual tensile behavior in reinforcement bars is very important for correct seismic design and in predicting critical conditions. The comparison to Seismic Codes shows that the prescribed reinforcement ratio is in fact relative to characteristic steel values, while in order to obtain the actual ultimate strains or to develop an acceptable ductility a lower reinforcement ratio is required.

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