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SEISMIC DRIFT ESTIMATES FOR RC STRUCTURES

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ABSTRACT

The maximum nonlinear displacement of RC structures subjected to strong ground motion may be estimated from a response-spectrum analysis using a linear model with appropriate values of effective stiffness, damping and strength. It is shown that seismic drift for moderate-rise structures calculated using an effective stiffness based on half of the initial stiffness and the displacement response spectrum determined for a linear model having a damping factor of 0.02, would provide a safe bound to nonlinear drift. For short-period structures, strength plays a major role and the linear model estimates may be a fraction of the result from nonlinear analysis. A method for estimating seismic drift is proposed based on the use of idealized linear response spectrum modified by a factor that accounts for nonlinear action. The method is evaluated using results from nonlinear dynamic analysis for various hysteresis models. Additionally, comparisons are made with seismic drift observed in earthquake simulation tests. Results of the evaluation indicate that the proposed method represents a safe bound to nonlinear drift of RC structures.

KEYWORDS

Structural engineering; earthquake resistant design; displacement design; drift control; response spectrum; characteristic period; base shear; strength reduction factor.

GENERALIZED DISPLACEMENT SPECTRUM

Previous studies have shown that nonlinear displacement response can be approximated using linear estimates (Shibata and Sozen 1976; Shimazaki and Sozen 1984). The generalized displacement spectrum presented here is intended for drift estimates for reinforced concrete structures responding nonlinearly.

A linear displacement response spectrum can be defined based on a typical smoothed linear acceleration spectrum considering that the spectral values for displacements, velocities and accelerations, for damping less than 20% of critical damping, are related approximately by:

$$S_a \approx \omega^2 \cdot S_d = \left(\frac{2\pi}{T}\right)^2 S_d \tag{1}$$

$$S_v \approx \omega \cdot S_d = \left(\frac{2\pi}{T}\right) S_d \tag{2}$$

where,

S_a, S_v, S_d = spectral acceleration, velocity and displacement, respectively;
ω = circular frequency of vibration;
T = period of vibration of a single-degree-of-freedom system.

Linear acceleration spectra can be determined readily after estimating the maximum expected ground acceleration and using appropriate amplification factors. In this study, a system with 2% damping ratio is selected as the frame of reference for computing the linear response. Several studies have recommended values for spectrum amplification factors for linear response as a function of the damping ratio (Newmark *et al.* 1973; Riddell and Newmark 1979). For systems with a 2% damping factor, the acceleration amplification factor recommended in these studies vary from 3.1 to 4.3. The amplification factors have been found to be site dependent (Seed *et al.* 1974).

An idealized linear acceleration response spectrum can be defined using the following expression:

$$S_a = \begin{cases} F_a \cdot \alpha \cdot g & \text{for} \quad T < T_g \\ F_a \cdot \alpha \cdot g \cdot \dfrac{T_g}{T} & \text{for} \quad T \geq T_g \end{cases} \tag{3}$$

where,

F_a = acceleration amplification factor. A value of 3.75, for oscillators with 2% damping ratio, is representative of a wide range of earthquakes;
g = acceleration of gravity;
α = peak ground acceleration expressed as a coefficient of the acceleration of gravity;
T_g = characteristic period for ground motion. Period at which the assumed constant acceleration region ends.

The above definition is based on the assumption that for periods below T_g, the acceleration response is nearly constant and for periods above, the velocity response is nearly insensitive to change in period.

Using the relationships given by Eq. (1) and (2) and substituting into Eq. (3), gives an expression for an idealized linear displacement response spectrum:

$$S_d = \begin{cases} \dfrac{F_a \cdot \alpha \cdot g}{(2\pi)^2} \cdot T^2 & \text{for} \quad T < T_g \\ \dfrac{F_a \cdot \alpha \cdot g \cdot T_g}{(2\pi)^2} \cdot T & \text{for} \quad T \geq T_g \end{cases} \tag{4}$$

This expression can be rewritten as:

$$S_d = \begin{cases} D_g \cdot \left(\dfrac{T_{eff}}{T_g}\right)^2 & \text{for} \quad T_{eff} < T_g \\ D_g \cdot \dfrac{T_{eff}}{T_g} & \text{for} \quad T_{eff} \geq T_g \end{cases} \tag{5}$$

where,

D_g = characteristic displacement for ground motion;
T_{eff} = effective period of the oscillator.

The characteristic displacement for ground motion, in this study, represents the linear displacement of an oscillator with period T_g and a damping factor of 2%, it can be defined in terms of the peak ground acceleration using:

$$D_g = F_a \cdot \alpha \cdot g \cdot \left(\frac{T_g}{2\pi}\right)^2 \tag{6}$$

The effective period, T_{eff}, for reinforced concrete structures, may be assumed to be approximately $\sqrt{2}$ times the period that corresponds to uncracked member section properties. It represents an intermediate value between the periods corresponding to uncracked and fully cracked section.

The idealized response spectra defined by Eq. (3) and (5) are compared to the actual response spectra for linear systems subjected to El Centro 1940 NS (Imperial Valley Earthquake). The results shown in Fig. 1 indicate an adequate smooth representation of the response spectra with results generally on the higher side.

The characteristic period for ground motion, T_g, is determined using the input energy spectrum. The maximum input energy, expressed as equivalent velocity, is given by:

$$V_{eq} = \sqrt{\frac{2 \cdot E_i}{m}} \tag{7}$$

and,

$$E_i = -m \cdot \int_0^{tf} \ddot{x}_g \cdot \dot{x} \cdot dt \tag{8}$$

where,

V_{eq} = equivalent velocity;
E_i = input energy;
m = mass of the oscillator;
$\ddot{x}_g$ = ground acceleration;
$\dot{x}$ = velocity of the oscillator relative to the ground;
tf = time duration used to calculate the response.

The input energy, E_i, equals the maximum of the sum, over time t_f, of kinetic energy, dissipated energy and strain energy. T_g is defined as the lowest period where the input energy spectrum, for a damping factor of 10%, tends to level off. This definition of T_g, based on the energy response spectrum, coincides closely with the period at which the nearly constant acceleration range ends. The rationale for using the energy spectrum for defining T_g is that nonlinear displacement response lengthens the period of the oscillator and if this results in an increase of input energy, the nonlinear system has to displace further in order to dissipate the increase in input energy. This interpretation suggests that linear and nonlinear systems with initial periods more than T_g, where input energy remains nearly constant, are likely to experience nearly equal maximum displacements. For practical applications, T_g can be taken equal to 0.6 sec. for stiff and 1.2 for soft soil.

Several studies recommend that the response spectrum to use in design be constructed through the use of controlling ground motion parameters: peak acceleration (A), peak velocity (V) and peak displacement (D) . Relationships between the ground motion parameters, such as V/A and AD/V^2 have been defined (Newmark *et al.* 1973; Newmark and Rosenblueth 1971). The region of nearly constant displacement response occurs approximately at a period of AD/V^2 times T_g. Values for AD/V^2 are dependent, among other factors, on the focal distance of the earthquake. A value of 6 has been recommended to accommodate a wide range of earthquakes, regardless of soil conditions (Newmark *et al.* 1973). Theoretically, an infinite focal distance would yield a value of 1.

For practical applications, in the domain of low- to moderate-rise buildings, the nearly constant displacement region may be ignored. In this study, the constant velocity region is assumed to extend to a period of not less than 3 seconds.

To account for nonlinear response, Eq. (5) is modified and rewritten in nondimensional form:

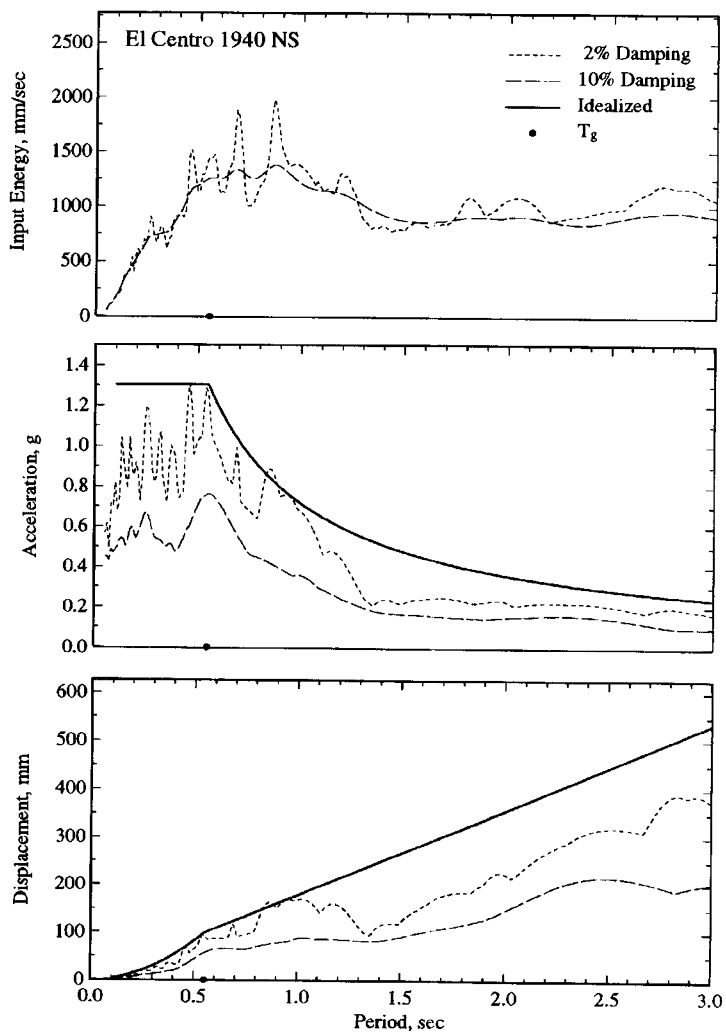

Fig. 1. Linear Response Spectra for El Centro 1940 NS

$$\Delta_s = \frac{S_d \cdot DR}{D_g} = \begin{cases} TR^2 \cdot DR & \text{for} \quad TR < 1 \\ TR & \text{for} \quad TR \geq 1 \end{cases} \tag{9}$$

where,

- Δ_s = nonlinear displacement spectrum ordinate normalized by the characteristic displacement for ground motion, D_g, given by Eq. (6);
- DR = displacement ratio, normalizes the maximum nonlinear displacement in relation to the smoothed spectral displacement, S_d, for a linear system with 2% damping ratio;
- TR = period ratio, normalizes the effective period of the system, T_{eff}, in relation to the characteristic period for ground motion, T_g.

Equation (9) implies that for systems having period ratios more than one, linear and nonlinear displacements are nearly the same, so the displacement ratio, *DR*, is taken as equal to unity. For systems in the short-period range ($TR < 1$), values of *DR* are more than one. These trends have been discussed previously (Shimazaki and Sozen 1984).

The expression for the nonlinear displacement, Eq. (9), is not intended for prediction of maximum displacements. It is a bound. The following expression is proposed as a bound to the displacement ratio, *DR*:

$$DR = \begin{cases} \dfrac{1 - SR}{TR} + SR & \text{for} \quad TR < 1 \\ 1 & \text{for} \quad TR \geq 1 \end{cases} \tag{10}$$

where,

- SR = strength ratio, normalizes the strength of the system in relation to the maximum force that the system would develop if it remained linear (at a damping factor of 2%). A value of *SR* equal to or greater than unity corresponds to systems responding linearly.

In Eq. (9), the displacement ratio, *DR*, represents a modification factor for the linear displacements to account for nonlinear response. Figure 2 contains a plot of Eq. (9), using Eq. (10) to define *DR*. For low values of *SR*, *DR* approaches to $1/TR$ and Eq. (9) results in $\Delta_s = TR$, a straight line through the origin.

The strength ratio may be defined as the inverse of a strength reduction factor *R*. It is reasonable to adopt a policy for determining the tolerable level of *R*, the ratio of linear response at a damping factor of 2% to available strength. It is well established that for structures with effective periods exceeding the characteristic period of the ground motion, *DR* is insensitive to strength. Ideally, for $TR > 1$, *R* may be as large as desired. If *TR* approaches zero, *R* has to approach unity. For a rigid structure there is no amplification or deamplification of the ground acceleration. It is reasonable to set the value of *R* at 1 for $TR = 0$ (hypothetical value).

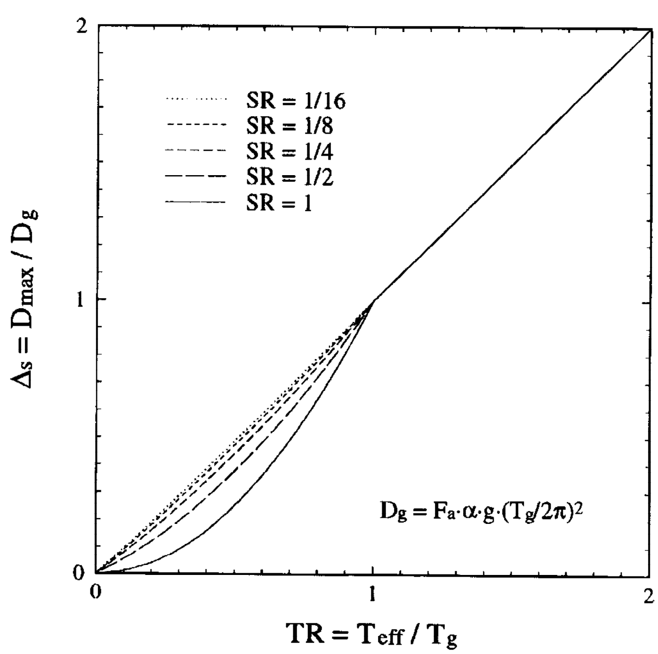

Fig. 2. Generalized Displacement Spectra

It has been shown that the value of *R* increases as *TR* goes from 0 to 1 (Miranda and Bertero, 1994). Conceding that *R* must vary between $TR = 0$ and $TR = 1$, the simplest policy is to make the variation linear. The value for *R* beyond $TR = 1$ must be set at a level that produces tolerable values at *TR* approaching 1. Pilot studies showed that Eq. (10) would be satisfactory for values of R_t not exceeding 16. The strength ratio expressed as a function of a tolerable strength-reduction factor is given by Eq. (11):

$$SR = \begin{cases} \dfrac{1}{(R_t - 1)\cdot TR + 1} \le \dfrac{1}{F_a} & \text{for} \quad TR < 1 \\ \dfrac{1}{R_t} & \text{for} \quad TR \ge 1 \end{cases} \tag{11}$$

where,

R_t = tolerable strength-reduction factor.

The acceleration response of systems with very low period is nearly equal to the ground acceleration, and therefore the strength ratio need not exceed $1/F_a$.

The strength ratio is related to the base shear strength coefficient, C_y, by the following expression:

$$C_y = \frac{Shear\ Strength}{Weight} = \frac{V_y}{W} = \frac{S_a \cdot SR}{g} \tag{12}$$

For values of F_a = 3.75 and R_t = 16, substituted in Eq. (3) and (11) and then into Eq. (12), a system with period T_g would require a minimum C_y of nearly 12% for a 0.5g earthquake.

Evaluation of the goodness of the generalized displacement spectrum, defined by Eq. (9) combined with Eq. (10), is performed by analytical and experimental tests. These are presented next.

ANALYTICAL TEST OF DRIFT ESTIMATES

Description of Analyses

In this section, the applicability of the generalized spectrum is evaluated using calculated data. The analytical test consists of the comparison between the maximum nonlinear displacement computed for a series of single-degree-of-freedom systems and the estimated displacement given by Eq. (9). Computed maximum response is obtained using the Newmark Beta Method with beta set to 1/6 (Newmark 1959), a numerical time-step procedure to evaluate dynamic response.

Results will be presented in the form of displacement response spectra for the earthquakes listed in Table 1.

Properties of Oscillators Analyzed

The force-displacement relations that characterize the single-degree-of-freedom systems considered in this study, are based on two hysteresis models: the bilinear and the reducing-stiffness (loading and unloading stiffness reduce with increase in displacement beyond yield) models. Both models are described in detail by Otani (1981).

The reducing-stiffness model used in this study is based on a bilinear primary curve. For maximum response amplitude much larger than the yield displacement, this model can produce displacement waveforms very similar to that of more elaborate models (Otani 1981).

To represent a broad range of structures, each model was considered with post-yield stiffness K_u, of 0% and 10% of the initial stiffness K_y. For each earthquake, an idealized linear displacement spectrum was determined following Eq. (5), where D_g was obtained using Eq. (6) with $F_a = 3.75$, $\alpha = 0.5$ and T_g from Table 1. The effective period, T_{eff}, for each oscillator was based on the initial stiffness, K_y, of the assumed bilinear force-displacement primary curve.

The expression for the displacement ratio, DR, given by Eq. (10) was evaluated. The strength ratio, SR, for each oscillator, was computed from Eq. (11), with R_t values of 16, 8 and 4. The SR values selected, correspond to base shear strength coefficient C_y, of nearly 12, 25 and 50 percent for systems with period T_g responding to earthquakes with peak acceleration of 0.5g.

Viscous damping included was small, as is generally accepted for undamaged structures. A damping factor equal to 2% of the critical was based on the initial stiffness K_y. Damping was assumed to remain unchanged during the entire analysis.

Table 1. Ground Motions Considered

Earthquake	Station	Component	Peak Ground Acceleration (g)	Record Duration t_f	Characteristic Period T_g
San Fernando 02-09-1971	Castaic Old Ridge Route, California	N21E	0.32	30	0.35
Imperial Valley 05-18-1940	El Centro Irrigation District, California	NS	0.35	45	0.55
Kern County 07-21-1952	Santa Barbara Courthouse, California	S48E	0.13	60	1.03
Tokachi-Oki 05-16-1968	Hachinohe Harbor, Japan	EW	0.19	36	1.14

Sources: California records: Earthquake Engineering Research Laboratory, California Institute of Technology.
Japan record: Association for Science Documents Information, Tokyo, Japan.

Computed and Estimated Drift

Linear and nonlinear displacement response spectra were computed for each earthquake record listed in Table 1. Results are presented in Fig. 3, where the estimated bound is given by Eq. (9) and (10), and the strength ratio is based on Eq. (11) for R_t values of 16.

All figures are plotted in nondimensional form with period ratio in the horizontal axis. The vertical axis represents the maximum displacement response normalized in relation to D_g, the idealized linear displacement for a system with period T_g and a damping factor of 2%.

For all earthquakes considered, in the period range where $TR \geq 1$, the estimated bound represented a reasonable upper bound for the nonlinear response. In the short-period range ($TR < 1$), the average response of the models considered was kept below the estimated bound. Systems with zero post-yield stiffness ($K_u = 0$) tended to experience the largest displacements and occasionally exceeded the estimated bound.

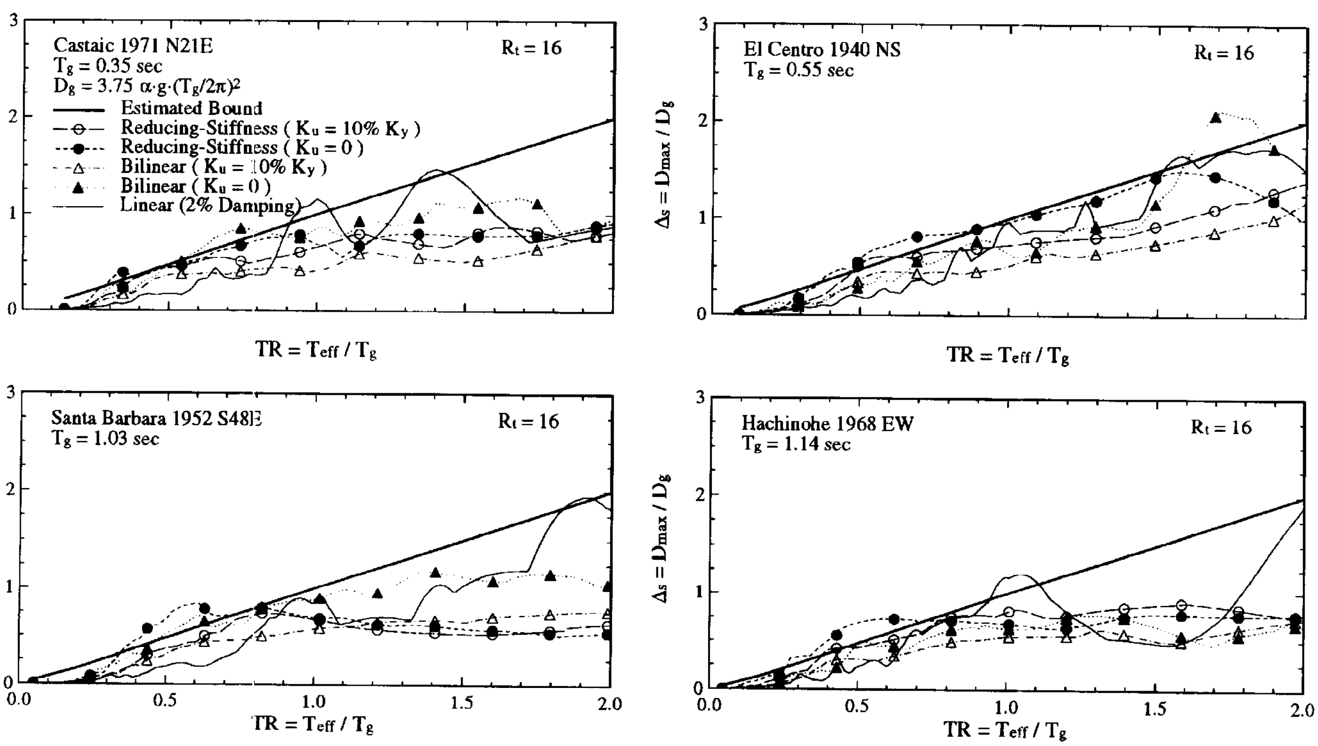

Fig. 3. Normalized Displacement Response Spectra for Selected Earthquakes

The estimated bound used was based on a smoothed linear displacement spectrum with an acceleration amplification factor of $F_a = 3.75$ with respect to the ground motion, a value typical of systems responding linearly with a damping ratio of 0.02. Had a larger factor been used, then all response maxima would have kept well within the estimated bound. On the other hand, use of a lower value, say $F_a = 2.7$, which is typical for a damping factor of 0.05, would have resulted in lower displacement estimates and lower shear strength. Consequently, nonlinear displacement maxima would have exceeded the estimated bound in a wider range of periods. It is important to emphasize that, in this study, the use of Eq. (9) and (10) are based arbitrarily on the 2-percent-damping response spectra. There are no implications in the choice about the physical state of the structure. Response spectra at another damping factor could have been used with appropriate but different statements about DR and/or T_{eff}.

EXPERIMENTAL TEST OF DRIFT ESTIMATES

Description of Experimental Data

Experimental testing of the calculated drifts was focused on short-period single-degree-of-freedom systems. A total of 22 small-scale reinforced concrete structures tested using the University of Illinois Earthquake Simulator are cited in the present study. The maximum displacement measured for each specimen is compared with the estimated bound provided by the proposed generalized response spectrum, Eq. (9) and (10). Results of the tests have been studied and reported in detail in a series of publications (Gulkan 1971; Morrison 1981; Bonacci 1989).

In general, test runs of a given specimen included repetitions of free-vibration test to determine low-amplitude natural frequencies followed by earthquake simulation. The sequence was repeated with the intensity of earthquake simulation being increased in successive runs. For each specimen, test runs with the maximum base accelerations are considered in this study.

Measured and Estimated Drift

Drift estimate based on Eq. (9) is driven by three parameters: period ratio, TR; characteristic displacement, D_g; and strength ratio, SR. Evaluation of these parameters conformed to the definitions given below.

The structure effective period, T_{eff}, and the earthquake characteristic period, T_g, define the period ratio. The effective period was obtained as $\sqrt{2}$ times the initial period of the system, which was taken as the value measured in the low-amplitude free vibration test that preceded the first run. Lacking an energy response spectrum, the characteristic period for ground motion was obtained from the acceleration spectrum, for a 10% damping factor, due to recorded base acceleration. It was taken as the period where the last significant peak was attained.

The characteristic displacement for ground motion, D_g, was obtained from the linear response spectrum for a damping factor of 10%, scaled up by a factor of 2. The result is equivalent to a smoothed spectrum for 2% damping ratio.

The strength ratio was calculated from the relationship given in Eq. (12), where V_y was based on the calculated yield strength, and the linear response acceleration, S_a, was obtained from the idealized linear displacement, S_d, using Eq. (1) and (5).

Comparison of measured maximum displacements with estimated values, is shown in Fig. 4. The mean for the ratio of measured to estimated displacements resulted 0.72 with a sample standard deviation of 0.22. The maximum ratio of measured to estimated resulted 1.02 and the minimum 0.41.

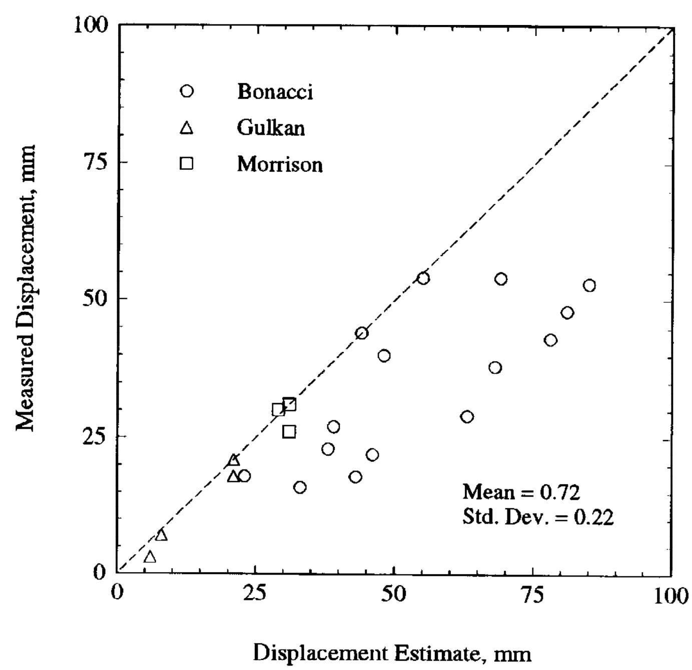

Fig. 4. Measured and Estimated Maximum Displacement

SUMMARY

The maximum nonlinear displacement of reinforced concrete structures subjected to strong ground motion may be estimated from a response-spectrum analysis using a linear model with appropriate values of effective stiffness, damping and strength. Seismic drift for moderate-rise structures ($TR > 1$) calculated using an effective stiffness based on half the initial stiffness and a displacement response determined for a linear model with a damping factor of 2%, would provide a safe bound to nonlinear drift.

For short-period structures ($TR \leq 1$), strength plays a major role and the linear model estimates may be a fraction of the result from nonlinear analysis. This is compensated by introducing a modification factor to the linear estimate. For structures complying with a minimum strength ratio based on Eq. (11), the modification factor may be obtained from Eq. (10). The factor is made dependent on the structure initial stiffness and strength and on the type of ground motion characterized by intensity and frequency content.

The proposed method was evaluated using results from nonlinear dynamic analysis performed on systems with different hysteretic properties. The evaluation was complemented by experimental data collected from 22 small-scale reinforced concrete structures tested on the University of Illinois Earthquake Simulator. Results of the analytical and experimental evaluation indicate that the proposed generalized displacement spectrum provides a safe bound to nonlinear drift estimates for reinforced concrete structures subjected to earthquake motions.

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