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DUCTILITY AND THE P - DELTA EFFECT

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ABSTRACT

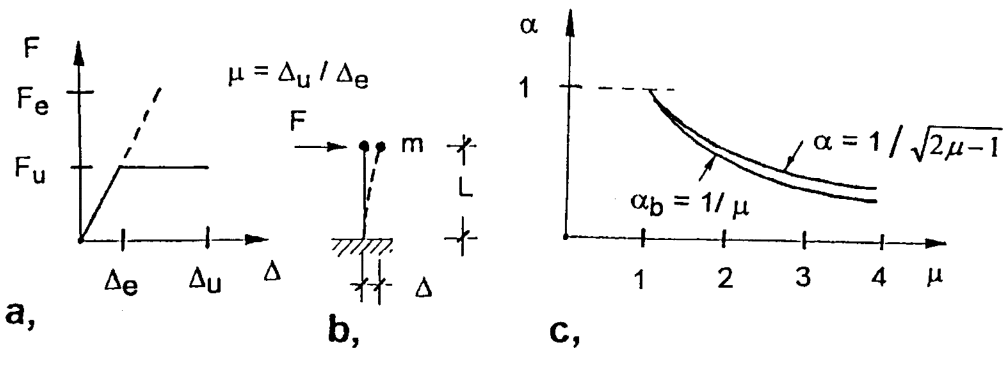
Investigating the earthquake stability of buildings, the effect of ductility, supposed to be favourable, is considered throughout the world: the force F substituting the horizontal earthquake impulse acting on the elastic system is devided by the $\mu = \Delta_u/\Delta_e$ ductility factor. This ductility factor varies between 2 and 12, e.g. in in the US for reinforced concrete structures $\mu = 8$. As a result, buildings are analysed on a low load level in elastic state, where displacements are small. Thus the unfavourable effect of the vertical load is either neglected or considered only approximately (as a linear effect). In fact, plasticity and buckling are both nonlinear and they must not be superposed. According to correct analysis, this is a serious mistake at the expense of safety. Due to this usual method, throughout the world many earthquake-engineered buildings fall or collapse, burying thousands of people under themselves. For more detailed analysis, we conducted different investigations considering the moment-increasing effect of the vertical load, applying a factual elasto-plastic model with a ductility factor $\mu = 12$. Calculations were made by the energy method and also by solving the differ-ential equation of motion with step-by-step method. Results were checked by experi-ments. It can be seen from the results, that the impulsive force has a critical value, thereafter the displace-ment is going to infinity, whereas the velocity is not decreasing to zero, thus vibration will not stop. The plastic spectrum curve (dynamic factor) is bending to infinity, whereas the elastic spectrum is decreasing with the vibration period. Our investigations indicate that the earthquake design applied throughout the world is false. Certain formulaes were derived for correct design. Design is proper if the favourable effect of ductility that sometimes occurs is considered as a reserve and not taken into account.

KEYWORDS

critical force; ductility; deflection spectra; elastic-plastic; energie methode; large deflection; P-delta effect; stability

1. INTRODUCTION

A basic question of investigating earthquake stability of buildings is the determination of stresses caused by the horizontal earthquake impulses. As long as the structure remains in elastic state, the customary methods of dynamics can be used, and the calculations, although through many steps, can be done. Some decades ago it was revealed that in special cases plastic deformation is capable of absorbing a considerable amount of energy, and this effect was introduced into earthquake analysis [1,2,3]. The plastic behaviour of structural materials was denominated to ductility, and a ductility factor μ was introduced, what can be calculated as the ratio of the limiting plastic deformation Δ_u, and the limiting elastic deformation Δ_e: $\mu = \Delta_u/\Delta_e$. This phenomenon is shown on Fig. 1. Here F_e is the substitute static force, that causes a static deformation equal to the deformation caused by the dynamic effect, when the impulse excitation is constant and lasts for time t_0. $F_e = \beta m a_g$, where m is the concentrated mass at the tip of the bar, a_g is the exciting acceleration, that, when multiplied by the mass, gives the force of the momentum, β is the dynamic factor. Investigating an elastic structure $\beta = 2$, when $t_0/T > 1/2$, and $\beta = 2\sin\pi t_0/T$ when $t_0/T < 1/2$. Here T is the period of time of the structure. F_u is the plastic limiting force, Δ_e is the elastic limiting deformation of the ideal elasto-plastic bar, Δ_u is the plastic limiting deformation. We must note that a substitute mass can also be determined at the end of a bar with continuous mass distribution, that makes dynamic calculations simpler. The structure shown on Fig. 1. is the simplest single-mass structure of one degree of freedom, but the dynamic behaviour of a building can be well represented with it. The P-Δ effect is the characteristic of an elastic bar, that the bar loaded by the force P is also stressed by an increment of bending moment $M=P\Delta$, what causes further deformation. As the force P approaches EULER's critical force P_{cr}, Δ to infinity. If the stress of the bar reaches its stress at fracture, during the increase of the deformation, the bar failures. Suppose that $P = 0$ in the direction of axis, and the horizontal static force F causes the static displacement Δ_0 of the tip of the bar. In seismology the force F, in general, is the product of the exciting acceleration a_g that substitutes dynamic effects, and the mass m. It is to be noted here that if the acceleration excites the capture cross-section of the bar instead of the mass, than the acceleration a_g excites the mass m with $F = m a_g$ for $\Delta < \Delta_e$ values of deformation, but after this with F_u with opposite sign as it would be accelerating mass m directly. In case a vertical force P in the direction of the axis also acts on the bar, the deformation Δ_0 increases to the value $\Delta_0\Psi = \Delta_0(1 - P/P_{cr})^{-1}$. Here $P_{cr} = \pi EI/(2L)^2$ is the critical force, where E is the elastic coefficient and I is the moment of inertia of the cross-section. This increase is called the P-Δ effect. For further simplification suppose that the bar is substituted with a bar fixed with a spring at its lower end, than the spring constant must be selected to describe the behaviour of an elasto-plastic bar. In this simplified case the displacement caused by the force F is $\Delta_0 = F/K$, and the critical force $P_{cr} = KL$. Up to this point the teoretical examinations show that in some cases the behaviour of an elasto-plastic structure differs from the behaviour described by the references. The point of this behaviour is that not only the vertical force, but the horizontal force also has a critical value. A force greater than this critical value causes the destruction of the structure. This phenomenon is the consequence of the force P, what has not been considered properly in the researches. Whereas this phenomenon has not been described in the preferences known to us, we found it proper to check the phenomenon by proper calculations and experiments considering the effect of the force P and the plastic behaviour. This will be introduced in the followings.



$$F_u = \alpha \cdot F_e = \alpha \cdot \beta \cdot F$$

Fig. 1. a, Ductile modell for earthquake analysis.
b, Deflection. c, Effect of the ductilility factor

2. THE CONVENTIONAL DUCTILE ANALYSIS

Investigating earthquake stability of buildings, the effect of ductility, supposed to be favour-able, is considered throughout the world. That means the multiplication of the force F_e acting on the elastic system by a reducing factor α, which determining for the different materials. The reducing factor α was gained by equalising the energy E(elastic) done by the force F_e with the work E(plastic) done by the force F_u. Equalising the deformations, the relation for α_b is the result. Fig.1. shows the comparison of these two factors. American earthquake stability investigations use the factor α_b on condition that $\mu = 8$. The P-Δ effect is either neglected when the deformation is limited, or considered at the determination of the multiplicator Ψ of the increment of elastic buckling. Thus the seismic force is calculated by the formula $F = F_e \alpha \Psi$. The fault in the customary method described above is the fact that it superposes the results of non-linear systems, but superposition can be used only on linear systems. An additional fault of the customary method is the decreasing of the load instead of increasing the opposition of the structure by the effect of ductility. In this case the calculated deformations are so small, that the non-linear effects seem to be insignificant, and the problem remains hidden.

3.THE PROPER METHOD OF DUCTILE DESIGN

It is well known, that the critical state of an elastic structure can even be valued by finding the critical force F_{cr}, at a constant P force, that takes the structure, through an indifferent state, to an unstable state by causing an increasing deformation. The point is that, if an earthquake impulse causes plastic deformation (yields) in one direction, than the next plastical yielding happens in the direction of the original plastic deformation, even in case of a small-er impulse. In this essay, a finitely short term impact on a unimass system excited by the impulse Ft_0 is examined, considering the work Ps done by the force P during the vertical displacement s caused by the displacement Δ. Here t_0 is the finitely short period of time, s is the displacement of the tip of the rod in the direction of force P.

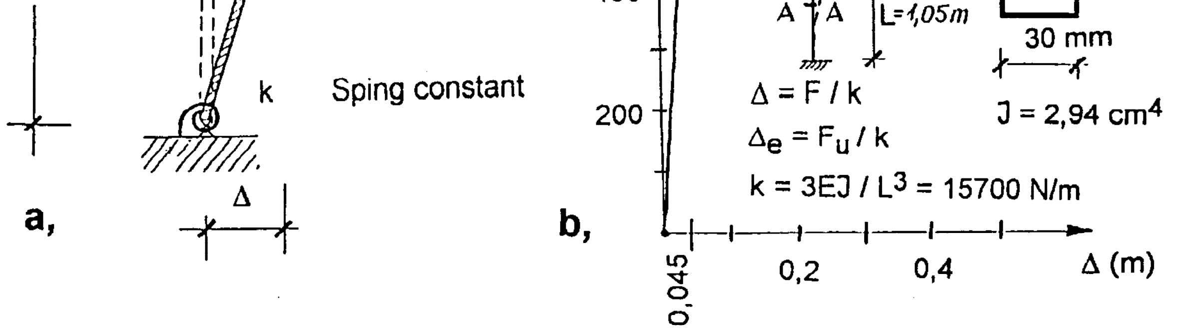


Fig2. a, Simple modell for the analysis with vertical force.
b, Statical force-deflection diagram of the experimental modell.

3.1 Investigation by Energy Method

In the followings a rigid, weightless console, shown on Fig.2a, will be investigated by energy method. Attached to the console there is a unimass m weight on the upper end, and an elasto-plastic spring, that has a spring constant k, at the lower end. According to the principle of the conservation of energy, the energy equation $\Pi = \Pi_{ex} - \Pi_{in} = E_{kin}$ can be written, where Π is the total potential energy, Π_{in} is the potential energy of the internal, Π_{ext} is potential energy of the external forces and E_{kin} is the kinetic energy. It is to be noted that $\Pi_{ex} = L_{ex1}$, where L_{ex1} is the work done by the external forces, and $\Pi_{in} = L_{in}$, where L_{in} is the work done by the internal forces. The external potential energy : $\Pi_{ex} = L_{ex1} + L_{ex2}$, where L_{ex1} is the work done by the horizontal force $F = ma_g$, that is : $L_{ex} = \int_0^{\Delta} a_g \cdot m \cdot d\Delta$. Here a_g is the exciting acceleration, m is the mass and Δ is the displacement. In case a_g and m are constants, than $L_{ex1} = a_g m\Delta$. If t_o is the period of the effect of the constant force $F = ma_g$, than the impulse is : $I_t = a_g m t_o$. In case of a short duration of impulse t_o ($t_o \leq T/5$), neglecting the little work done by the spring force during this short time, the velocity is $v = I_t/m$, and using this, $L_{ex1} \approx a_g\, mt_o(0.5a_g t_o^2) = 0.5I_t^2/m$, the work of the force giving the momentum can be calculated by the equation. Here T is the period of time. The second component of external work can be calculated by the work done by force P along s falling displacement, that is $L_{ex2} = Ps$. Here $P = mg$ is the force of gravity, where g is the gravitational acceleration. The mass m approximative moves along a circular arc, and the value of s comes from the equation $\Delta^2 + (L-s)^2 = L^2$. In case the equation $\Delta \leq L/2$ is true, and $s \approx \Delta^2/2L$ is acceptable. Considering this, the external work in case of a short, constant momentum $I_t = a_g m t_o$ is $L_{ex} = L_{ex1} + L_{ex2} = 0.5I_t^2/m + 0.5P\Delta^2/L$, while in case of a constant, long term impulse it is $L_{ex} = a_g m\Delta + 0.5P\Delta^2/L$. The internal potential energy of the structure acting against the motion is given by the equ. $\Pi_{in} = L_{in,e} + L_{in,pl} + E_d$. Here, in the elastic section of $\Delta < \Delta_e$, $L_{in,e} = 0.5F_b\Delta$ is the internal elastic work. Force F_b can be expressed by the spring constant k, $F_b = k\Delta$. The internal plastic work in the $\Delta > \Delta_e$ section is: $L_{in,pl} = F_u(\Delta - \Delta_e/2)$, where $F_u = k\Delta_e$ is the plastic critical force. The internal plastic work $L_{in,pl}$ is converted into heat during the motion of the structure, thus there is no elastic energy accumulated from this and it does not form returning force. E_d is the energy used up during the internal damping of the structure, that is neglected in this investigation because of its low value. The elasto-plastic internal potential energy Π_{in} in the state is : $\Pi_{in} = L_{in} + L_{in,1} + L_{in,2} = 0.5F\Delta_e + F_u(\Delta - 0.5\Delta_e)$. As for the kinetic energy, it is calculated from the equation $E_{kin} = 0.5mv^2$, where v is the first differential quotient of Δ according to time. Examining the motional state of a structure, the followings can be stated : Considering the conditions of equlibrium the structure remains in motion until $\Pi > 0$, because kinetic energy is present in the system. When $\Pi = 0$, motion stops, and equilibrium ensues. After this, the equilibrium must be examined, whether it is stable or indifferent. In case $d\Pi_{ex} = d\Pi_{in}$, namely the external work in the course of a short displace-ment $d\Delta$ is equal to the internal work, the system is in an indifferent state. This state is called the critical state, and the displacement Δ_{cr} that belongs to this state is the critical displacement. In case $\Pi = 0$ exists, and $d\Pi = d\Pi_{ex} - d\Pi_{in} \leq 0$, than the investigated system is in equilibrium and is stable, since after this short displacement it returns to its original state. On the other hand, when $\Pi > 0$ or $\Pi = 0$ and $d\Pi = d\Pi_{ex} - d\Pi_{in} > 0$, the system is unstable and keeps on moving. Thus, to find the critical state, the requirements $\Pi = 0$ and $d\Pi = 0$ must be satisfied. Values of $d\Pi$ are usually examined by the differential quotient. These are shown on Fig.3. plotted against the displacement Δ *The* L_{ex} external work can be seen on Fig.3/a. When $P = 0$, than after the displacement value $\Delta(t_o)$, L_{ex} is constant, on the other hand, when a vertical force P exists, the increasing Δ results in an increasing L_{ex}. Fig.3/b shows the work L_{in}. The larger the spring constant k is, the larger the internal work L_{in} is, and the smaller the k is, the smaller the internal work L_{in} is, if Δ remains the same. On Fig.3/c, the differences between L_{ex} and L_{in}, and the kinetic energy E_{kin} is shown. As long as it is not equal to zero, motion keeps on. Take a look at Fig.3/d. Here a smaller force P is combined with a larger spring constant, thus the two curves are intersecting and motion stops. This happens in all cases when $P = 0$ and t_o is small. In case of a larger P force or a smaller spring constant, the case shown on 3/e is reached, when the two lines are touching each other at the point Δ_{cr} (*critical displacement*). The result of a further increase of P or a further decrease of spring constant is that, the two curves are not intersecting, always there is a remaining kinet-ic energy E_{kin}, namely $\Pi > 0$, so the motion of the structure never stops, it fails.It is to be noted that if $P \geq P_{cr}$, the structure suffers deformation even without horizontal force. Here $P_{cr} = kL$, the EULER's critical force is.

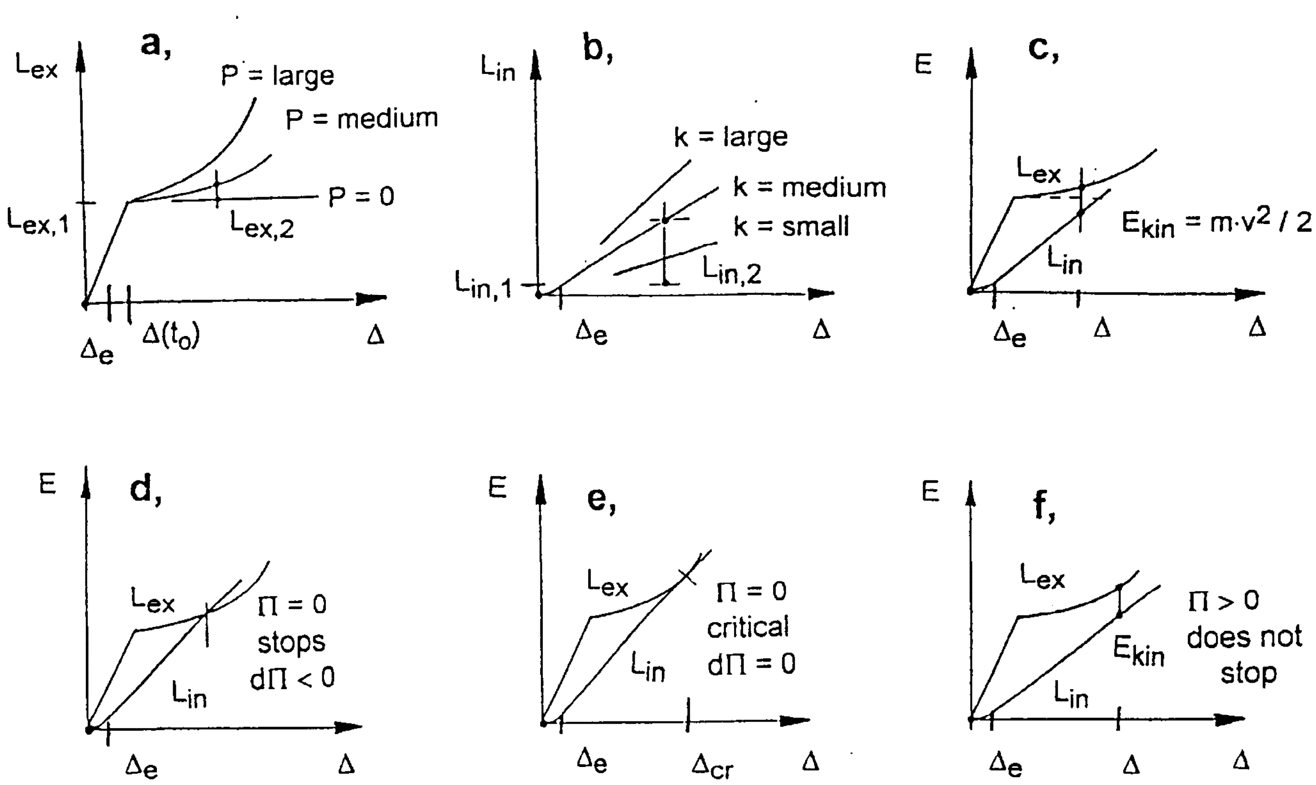


Fig3. Analysis by the energy methode. (Explaination is in the text.)

In the followings the short term momentum $t_0 \leq T/5$, and the the longer impact that keeps at least until Δ_{cr}, each acting on elastic or elasto-plastic structures, will be examined separately, for each $P=0$ and $P \neq 0$ cases. Since we want to find the condition when motion stops, the velocity $v = 0$ and the kinetic energy can be neglected. Thus the energy equation is the following : $L_{ex} = L_{in}$. With the help of the analysed works the equations of work can be stated and the limit displacements (Δ_{max}) can be determined. The results are summarised in Tabelle 1. and Tabelle 2.

3.2 Investigation Using the Differential Equation of Equilibrium

In the equilibrium-deformation differential equation the horizontal force is : $Pu/(L-s)$. In case of accepting a parabolic instead of a circular motion for the values of s when $\Delta < 0.5L$, the error is less then 1%. Beside this the internal damping occurring in the range of elasticity can also be neglected because of its low value.

Elastic structure **Tabelle 1.**

Pushing time	P = 0	P ≠ 0
short	$\Delta_{max} = \frac{I_t}{\sqrt{m \cdot k}} = a_g \cdot t_0 \sqrt{\frac{m}{k}}.$	$\Delta_{max} = \sqrt{\frac{L \cdot m \cdot a_g^2 \cdot t_0^2}{k \cdot L - P}}.$ $\frac{If \quad P = k \cdot L}{P = P_{cr}, \Delta \to \infty}$
long	$\Delta_{max} = \frac{2m \cdot a_g}{k}.$	$\Delta_{max} = \frac{2a_g \cdot m}{k - P/L} = \frac{2a_g \cdot m}{k \cdot (1 - P/P_{cr})}.$

Elastic - plastic structure **Tabelle 2.**

Pushing time	P = 0	$P \neq 0$
short	$\Delta_{max} = \frac{1}{2}\left(\Delta_e + \frac{m \cdot a_g^2 \cdot t_0^2}{F_u}\right).$	$\Delta_{max} = \frac{F_u \cdot L}{P}\left[1 - \sqrt{1 - \frac{P}{F_u \cdot L}\left(\frac{m \cdot a_g^2 \cdot t_0^2}{F_u} + \Delta_e\right)}\right].$ If $F_u \leq P \cdot \Delta_e / L$, or $P / P_{cr} \geq 1{,}0.$ $\Delta \to \infty.$
long	$\Delta_{max} = \frac{\Delta_e}{2}\left(\frac{1}{1 - \frac{a_g \cdot m}{F_u}}\right).$ If $a_g \cdot m \leq F_u / 2,$ elastic. If $a_g \cdot m \geq F_u / 2;$ plastic. If $a_g \cdot m \geq F_u,$ $\Delta \to \infty.$	$\Delta_{max} = B\left[1 - \sqrt{1 - \frac{F_u \cdot L \cdot \Delta_e}{P \cdot B^2}}\right],$ where $B = (F_u - a_g \cdot m)\frac{L}{P}.$ If $a_g \cdot m \geq F_u\left[1 - \sqrt{\frac{P \cdot \Delta_e}{L \cdot F_u}}\right],$ or $P \geq P_{cr} = k \cdot L,$ $\Delta \to \infty.$

Thus the differential equation of equilibrium in the range of elasticity is :

$$ma_g - m\Delta'' - k\Delta - P/(L-s)\Delta = 0. \qquad (1/a)$$

In the range of plasticity the difference is that the force increment of the rod after the elastic limit Δ_e stops, and the value $k\Delta = F_u$ takes over the place of the term $k\Delta$ in the equation. So the differential equation of the range of plasticity is :

$$ma_g - m\Delta'' - F_u + P/(L-s)\Delta = 0. \qquad (1/b).$$

Naturally the two equations are equal when $\Delta = \Delta_e$. If the value of s compared to L is neglected, the error originating from this increases quadratically to Δ, and it reaches 10% in the term expressing the effect of the force P when $\Delta < 0.5L$. Considering this, the value of s compared to L can be neglected. The analytic solutions of the (1) equations are not known yet. Therefore, to solve these equations, the different *step-by-step* solution methods have developed. One example of these is NEWMARK's method of average acceleration, that uses a constant acceleration on the investigated differentially small section, or the method of linear acceleration, that considers acceleration linear on the investigated differentially small section. These methods have developed for manual calculations, and use intervals of 0.1-0.5 seconds. Since computers are capable of handling the problem even with much smaller intervals, there is no point in specifying these methods. Therefore in our examples the results were calculated by transforming the differential equations into difference equations, and these were solved by *step-by-step* method. Thus the solution is similar to NEWMARK's method.The difference equations are the followings:The difference equation in the range of elasticity :

$$ma_{g(t)} - m\frac{\Delta_{i-1} - 2\Delta_i + \Delta_{i+1}}{\Delta t^2} - k\Delta_i + \frac{P}{L}\Delta_i = 0. \qquad (2/a)$$

The difference equation in the range of plasticity:

$$ma_{g(t)} - m\frac{\Delta_{i-1} - 2\Delta_i + \Delta_{i+1}}{\Delta t^2} - F_u + \frac{P}{L}\Delta_i = 0. \qquad (2/b)$$

A problem is in the solution, when all values at time t 0 are zero, because in that case the analysis fails in the beginning. In our case it did not occur, since acceleration a_g was a constant. In the case of the equations (2) the analysis begin at time t=0, when $\Delta_{i-1}=\Delta_i=0$, and Δ_{i-1} can be calculated. At the next interval of time Δt only $\Delta_{i-1}=0$, and from the third interval neither of them equals to zero. This means a small error at the beginning of the calculations, but this is neglected, since very small Δt can be used. In case of exciting the clamping instead of the mass, the calculation is modified. In the range of elasticity the acceleration of support has a similar effect to the case when the mass is accelerated with the same value but in the other direction. In the range of plasticity the level of excitability is limited by the value of F_u when $a_g\,m>F_u$.

4. EXPERIMENTS

The theoretical investigation described so far have revealed that in special cases the structure behaves differently from that the references describe. The essence of this difference is the fact that not only the vertical force, but even the horizontal force has a critical value. A force greater than this critical value causes the failure of the structure. This bilateral effect is the result of the force P, what has not been considered correctly in the references. Since this phenomenon has not been mentioned in the references known to us, it seemed proper to verify the analytical method by experiments. Extending the verification only to the model examined theoretically in the essay, seemed correct for the time. The model of the experiment used in the calculations, the cross section and the elasto-plastic model of the applied steel baris shown on Fig.2. In the course of the experiments the path and the acceleration were measured, while the velocity was determined by the differentiation of the path or by the integration of the acceleration (with the controlled coordination of these two methods). Originally 15 experiments were planned, so that, by setting the different parameters the critical phenomenon were encased. Unfortunately, owing to different faults (collision of parts, data loose of the controlling computer etc.) four of the experiments turned out to be unsuccessful. Therefore 11 of the results of the experiments were valid.The results of experiments were recorded with an interval of 0.02 sec. by the measuring instrument. The controlling calculations of the completed experiments were done with an interval of 0.005 sec. The calculated and the experimental values of displacement are shown on Fig.4.

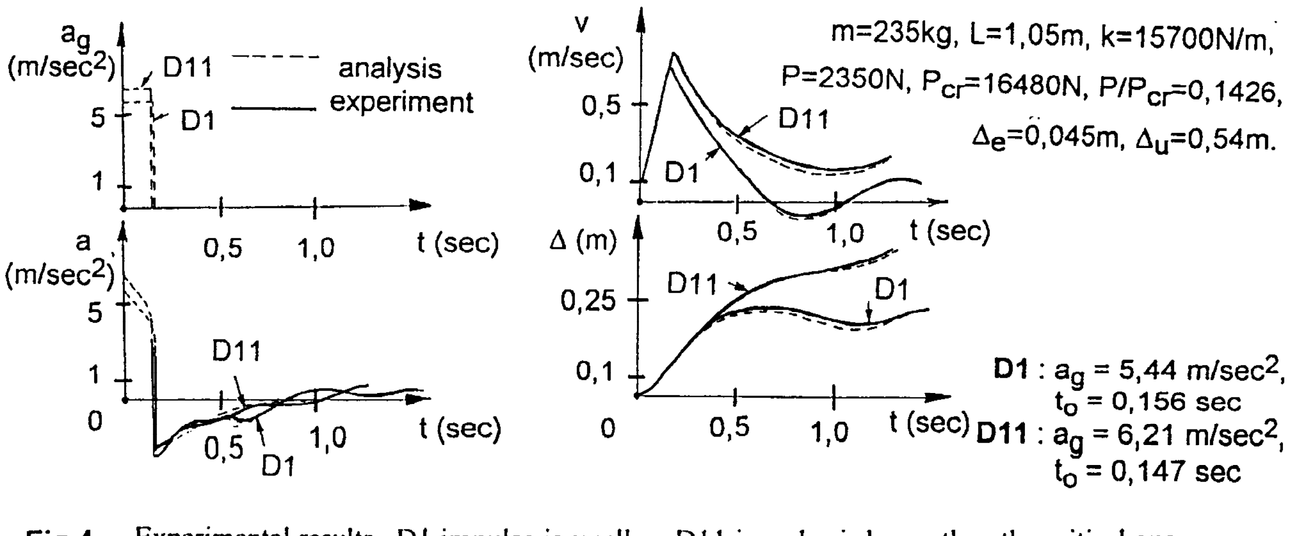


Fig.4. Experimental results. D1 impulse is smaller, D11 impulse is larger then the critical one.

Fig.4. shows the analytical and the experimental series of data gained from two experiments, one of which was before (D1), the other after (D11) the critical state. On the figures we stated the impulsive exciting acceleration as well as the time-history of the acceleration a, velocity v and displacement u of the mass m. The thick continuous line represents the experimental results, while the analytical values are indicated with the thin, dashed line. On the evidence of the figures the analytical and the expe-rimental results are nearly identical and the critical phenomenon exists. In the experiment D1, as an effect of the momentum

I_t 200 mkg/sec, the velocity decreases to zero at around 0.7 sec., displacement stops, so the phenomenon is stable. As a contrast, in the experiment D11 the effect of the momentum I_t 214 mkg/sec does not result in the decrease below zero, so the displacement Δ increases unlimited, the phenomenon is unstable. The critical state is between the two values. According to the requirement $ma_g^2 t_o^2 \; F_u \Delta_e(Pcr/p-1)$ the critical momentum is I_t 212 mkg/sec. These values prove that the analytical method is correct.

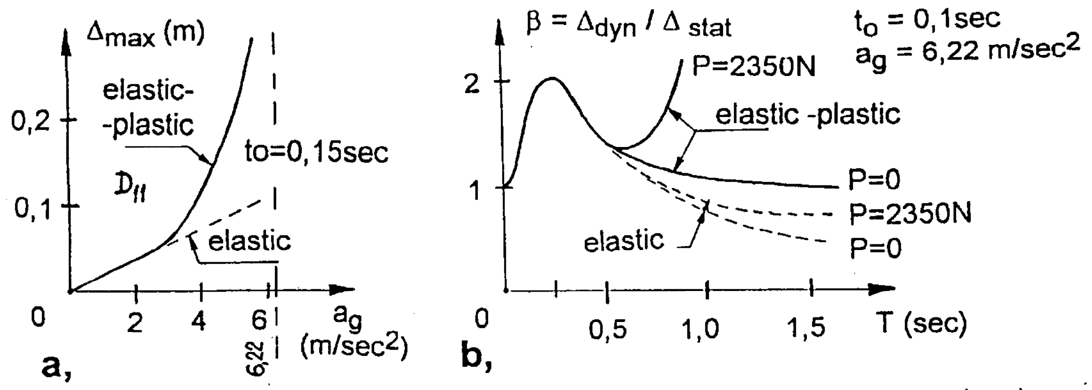


Fig.5. a, Maximum deflection of the modells in function of the exciting acceleration.
b, Elastic and elasto-plastic deflection spectra of the modells with and without vertical P force.

For the sake of interest the displacement values were calculated for different values of exciting acceleration a_g, when the time of exciting was t_o = 0.15sec. This diagram is shown on Fig.5a. Until the critical limit the displacement Δ_{max} is proportional to a_g, but after the critical limit it increases rapidly until it reaches the critical exciting acceleration $a_g = 6.22$ m/sec^2, from which point theoretically it increases to infinity. This infinity value comes from the approximations, since the tip of the bar moves along a circular curve instead of a parabolic one, thus Δ_{max} could be at most L. In case the spring constant k is changed, T period of time also changes, and, with the values of Δ_{max} spotted against T, the values of $\Delta_{dyn}/\Delta_{stat}$ give the deflection spectra. This calculation can be seen on Fig.5/b. It can be seen that while in case of $P = 0$, the spectra decreases when T increases, when $P \neq 0$ the decrease changes to an increase. This has a meaning that the softening of an elasto-plastic structure has damaging effect.

5. RESULTS

It is obvious from the research, that the customary method of investigating earthquake stability, that considers with the reduction of the loads, is faulty, and in some cases it causes significant undersdimensioning.

The issue of research indicate that the worldwide applied ductile analysis must be revised.

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