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STUDY ON NONLINEAR DYNAMIC ANALYSIS METHOD OF PILE SUBJECTED TO GROUND MOTION PART 1 : THEORY

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ABSTRACT

The purpose of this study is to propose a dynamic nonlinear analysis method of ground-pile-mass(structure) system subjected to ground motion. This analysis method is based on a static method which is applied the theory of beam on nonlinear solid to a pile and is able to calculate easily and in short time even if using micro computer.

KEYWORDS

Pile; ground; nonlinear analysis method of response; nonlinear soil reaction; p-y curve; practical analysis.

INTRODUCTION

When piles are designed, it is needed to grasp behavior of ground-pile system in great earthquake. Therefore, It is important to regard nonlinearity of soil properties for analysis method of pile subjected earthquake motion. In the past, a variety of analysis methods are proposed, but most are related to linear analysis method, and are not convenient to use practically. Consequently, it is required the analysis method that can not only deal with nonlinearity of soil properties but also calculate easily. In this paper, the authors propose a dynamic analysis method of ground-pile-mass(structure) system regarding nonlinear soil reaction which is applied the theory of beam on nonlinear solid to a pile.

SOIL REACTION

1) p-y Curves

A curve which is made from relationships between the soil reaction of pile (p) and lateral displacement of pile (y) are called p-y curve(Fig.1). In this analysis method, a couple of p-y curves which are distributed along the pile depth are supposed at positive and negative sides of pile.
The p-y curves which have not been loaded yet are especially called "Initial p-y curves". Eq. 1 shows the initial p-y curve at positive side of a pile.

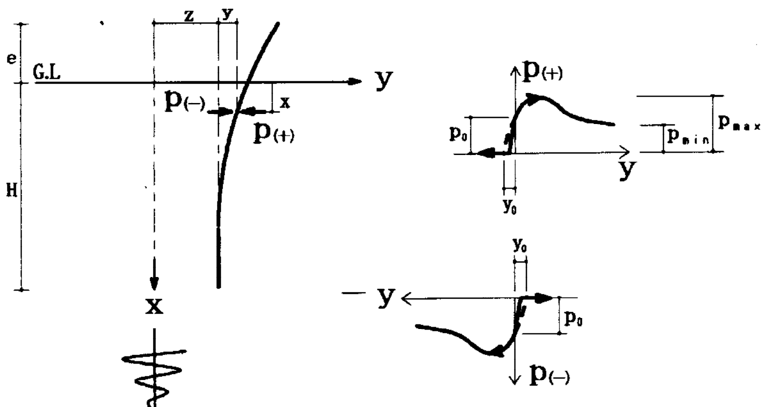

Fig.1 p-y Curve

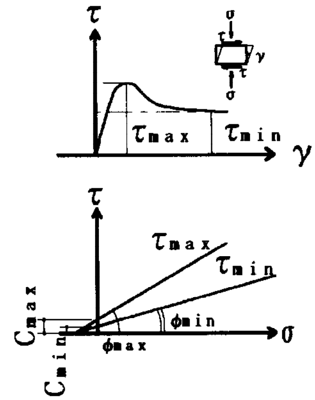

Fig.2 ϕ_{max} and ϕ_{min}

$$p_{(+)}=a_0+a_1e^{-\kappa(y+yo)}+a_2e^{-2\kappa(y+yo)}+a_3e^{-3\kappa(y+yo)}+a_4e^{-4\kappa(y+yo)} \quad (1)$$

$$a_0 = p_{min}$$
$$a_1 = -7.44\,p_{min} + 7.05\,p_{max}$$
$$a_2 = 19.26\,p_{min} - 17.06\,p_{max}$$
$$a_3 = 20.22\,p_{min} + 16.98\,p_{max}$$
$$a_4 = 7.39\,p_{min} - 6.97\,p_{max}$$
$$\kappa = k_{ho}/(4\,p_{max})$$

where, p_{max}, p_{min} : maximum and minimum values of soil reaction of a pile.
y_o , p_o : displacement and soil reaction at initial condition(Fig.1)

2) The Values of p_{max} and p_{min}

The values of pmax and pmin are expressed as Eqs. 2 and 3, regarding the influence of progressive failure of soil.

$$p_{max} = p_{max0} - (p_{max0} - p_{min0})/5 \quad (2)$$
$$p_{min} = p_{min0} \quad (3)$$

where, p_{max0}, p_{min0}: ultimate soil reaction (passive earth pressure) applied from maximum and minimum internal friction angle of soil ϕ_{max} and ϕ_{min}(Fig.2)

3) Hysteresis Rule

In quite cohesionless soil, when a pile moves, any gaps do not appear between the pile and the soil. On the other hand, in a shallow area of cohesion soil, when pile moves, the gaps between the pile and the soil appear because the soil is able to stand alone. However, in a deep area, gaps do not appear because of ground failure like coheisionless soil. In this paper, the ground in which the gaps appear is called "C-type soil", and the ground in which the gaps do not appear is called "S-type soil". Boundary depth of these (H_1) is represented as follows:

$$H_1 = \alpha_k c_{max}/\gamma \cdot \tan(\pi/4 + \phi_{max}/2) \quad (4)$$

where, c_{max} : cohesion of soil at ϕ_{max} , γ : unit weight of soil, $\alpha_k = 4$

(a) Hysteresis rule of C-type soil

Fig.3-a～3-c show the hysteresis rule of C-type soil. Present step of p-y curve is drawn by thin line and will be replaced p-y curve at next step is drawn by bold line. Fig.3-a shows in the case that the present point (y,p) is located on initial p-y curve ($p \neq 0$), and Fig.3-b shows in the case that the present point (y,p) is located on linear line inclined k_{h0}, crossing to the y-axis , and Fig.3-c shows in the case that the point (y,p) is located on the y-axis.

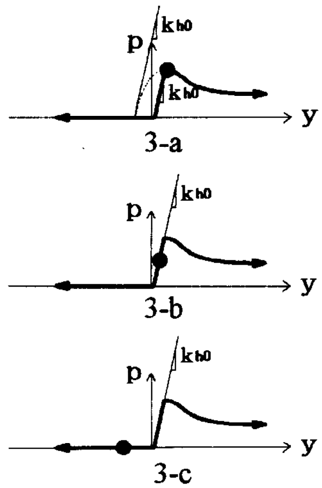

Fig.3 Hysteresis Rule of C-Type Soil

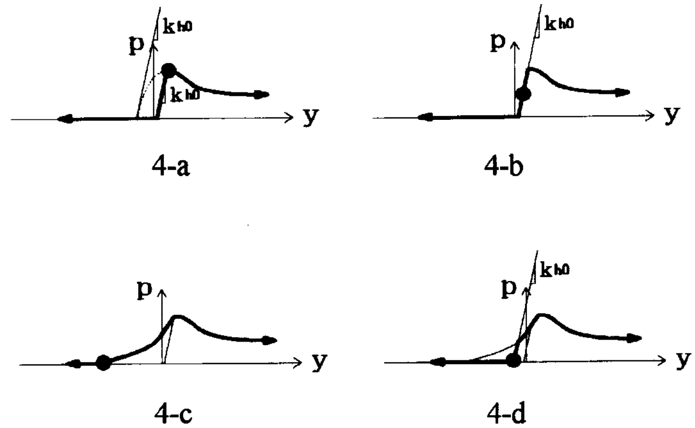

Fig.4 Hysteresis Rule of S-Type Soil

<u>(b) Histeresis rule of S-type soil</u>

Fig.4-a~4-c show the hysteresis rule of S-type soil. As the same as mentioned above, present step of p-y curve is drawn by thin line and will be replaced p-y curve at next step is drawn by bold line. In the case that the present point (y,p) locate on initial p-y curve ($p\neq 0$) is shown in Fig.4-a, in the case that (y,p) is located on linear line inclined k_{h0}, crossing to the y-axis is shown in Fig.4-b. And in the case that the point (y,p) is located over experienced area in negative direction on the y-axis($p=0$) is shown in Fig3-c, and in the case the point (y,p) is located inner experienced area in negative direction on the y-axis(Fig.4-d).

BASIC EQUATIONS OF ANALYSIS MODEL

<u>1) Analysis Model</u>

An analysis model of this calculation is shown in Fig.5.

<u>2) Equations of Oscillation of Pile</u>

<u>・Pile Head</u>

$$\left(EI\frac{\partial^3 y}{\partial x^3} + \eta_p I\frac{\partial^4 y}{\partial x^3 \partial t} + m\frac{\partial^2 (y+z_s)}{\partial t^2}\right)_{x=-e} = 0 \quad (5)$$

$$\left(EI\frac{\partial^2 y}{\partial x^2} + \eta_p I\frac{\partial^3 y}{\partial x^2 \partial t}\right)_{x=-e} = 0 \quad (6)$$

<u>・Pile Body</u>

$$EI\frac{\partial^4 (y+z)}{\partial x^4} + \eta_p I\frac{\partial^5 (y+z)}{\partial x^4 \partial t} + D\cdot(sy+t) + A_p\cdot\rho_p\frac{\partial^2 (y+z)}{\partial t^2} = 0 \quad (7)$$

where, $x<0$: $s=0$, $t=0$ (upper ground surface)

<u>・Pile Tip</u>

$$\left(EI\frac{\partial^3 (y+z)}{\partial x^3} + \eta_p I\frac{\partial^4 (y+z)}{\partial x^3 \partial t}\right)_{x=H} = 0 \quad (8)$$

$$\left(EI\frac{\partial^2 (y+z)}{\partial x^2} + \eta_p I\frac{\partial^3 (y+z)}{\partial x^2 \partial t}\right)_{x=H} = 0 \quad (9)$$

where,

- E : Young's modulus of pile,
- η_p : damping constant for the pile,
- D : diameter of pile,
- ρ_p : mass density of pile,
- z : lateral deflection of ground motion,
- e : height of eccentricity,
- zs : ground motion at free ground surface
- s : inclination of tangent line of a point on p-y curve(Fig.3)
- t : intercept value of tangent line of a point on p-y curve(Fig.3)
- I : second moment of area of pile section
- m : mass on pile head
- A_p : sectional area of pile
- y : lateral displacement of pile
- x : depth
- H : pile depth,

3) Equations of Oscillation of Ground

・Free Ground Surface

$$\left(G\frac{\partial z}{\partial x} + \eta\frac{\partial^4 z}{\partial x \partial t}\right)_{x=0} = 0 \tag{10}$$

・Inner Ground

$$G\frac{\partial^2 z}{\partial x^2} + \eta\frac{\partial^3 (y+z)}{\partial x^2 \partial t} - \rho\frac{\partial^2 z}{\partial t^2} = 0 \tag{11}$$

・Plane Earthquake Motion Input

$$\left(G_N\frac{\partial z}{\partial x} + \eta_N\frac{\partial^2 z}{\partial x \partial t} - C_B\left(\frac{\partial z_B}{\partial t} + \frac{\partial z}{\partial t}\right)\right)_{x=H_B} = 0 \tag{12}$$

where, G : shear modulus , η : damping constant for the ground

ρ : mass density of pile, C_B: damping constant at the plane earthquake motion input

NUMERICAL ANALYSIS METHOD

1) Basic Equations

・Eqs. of finite difference method(Fig.5)

$$\left(\frac{dy}{dx}\right)_{x=xi} = \frac{y_{i+1} - y_{i-1}}{2\lambda}, \quad \left(\frac{d^3 y}{dx^3}\right)_{x=xi} = \frac{y_{i+2} - 2y_{i+1} + 2y_{i-1} - y_{i-2}}{2\lambda^3}$$

$$\left(\frac{d^2 y}{dx^2}\right)_{x=xi} = \frac{y_{i+1} - 2y_i + y_{i-1}}{\lambda^2}, \quad \left(\frac{d^4 y}{dx^4}\right)_{x=xi} = \frac{y_{i+2} - 4y_{i+1} + 6y_i - 4y_{i-1} + y_{i-2}}{\lambda^4}$$

・Eqs. of numerical difference

$$\dot{y}_j = v \cdot y_j + V_{i,j}$$

$$\ddot{y}_j = w \cdot y_j + W_{i,j}$$

where,

j : adding character for series of time history

$j=1$: $v = \frac{1}{\Delta t}$, $V_{y,j} = \frac{-y_{j-1}}{\Delta t}$, $w = 0$, $W_{y,j} = 0$

$j=2$: $v = \frac{3}{2\Delta t}$, $V_{y,j} = \frac{-4y_{j-1} + y_{j-2}}{2\Delta t}$, $w = \frac{1}{\Delta t^2}$, $W_{y,j} = \frac{-2y_{j-1} + y_{j-2}}{\Delta t^2}$

$j \geqq 3$: $v = \frac{11}{6\Delta t}$, $V_{y,j} = \frac{-18y_{j-1} + 9y_{j-2} - 2y_{j-3}}{6\Delta t}$, $w = \frac{2}{\Delta t^2}$, $W_{y,j} = \frac{-5y_{j-1} + 4y_{j-2} - y_{j-3}}{\Delta t^2}$

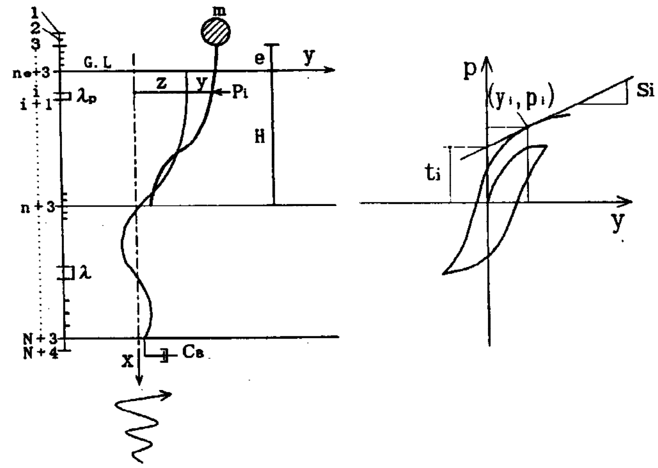

Fig.5 Analysis Model

2) Equations of Finite Difference Method

・Pile

$$
\begin{Bmatrix} T_1 \\ T_2 \\ T_3 \\ \vdots \\ T_{i-1} \\ T_i \\ T_{i+1} \\ \vdots \\ T_{n+3} \\ T_{n+4} \\ T_{n+5} \end{Bmatrix}
+
\left|\begin{array}{ccccccccccccc}
0 & 1 & -2 & 1 & 0 & & & & & & & & \\
-1 & 2 & S_2 & -2 & 1 & & & & & & & & \\
1 & -4 & S_3 & -4 & 1 & & & & & & & & \\
 & & & & \ddots & & & & & & & & \\
 & & & & 1 & -4 & S_{i-1} & -4 & 1 & & & & \\
 & & & & & 1 & -4 & S_i & -4 & 1 & & & \\
 & & & & & & 1 & -4 & S_{i+1} & -4 & 1 & & \\
 & & & & & & & & & \ddots & & & \\
 & & & & & & & & 1 & -4 & S_{n+3} & -4 & 1 \\
 & & & & & & & & 0 & 1 & -2 & 1 & 0 \\
 & & & & & & & & -1 & 2 & 0 & -2 & 1
\end{array}\right|
+
\begin{Bmatrix} y_1 \\ y_2 \\ y_3 \\ \vdots \\ y_i \\ \vdots \\ y_{n+3} \\ y_{n+4} \\ y_{n+5} \end{Bmatrix}
= 0 \qquad (13)
$$

where,

・i = 1, 2

$$T_1 = \frac{\eta_P}{EI + v\eta_p I}\left(V_{y,4} - 2V_{y,3} + V_{y,2}\right)$$

$$S_2 = -w\ m\frac{2\lambda^3}{EI} W_{z,ne+3}$$

$$T_2 = \frac{\eta_P}{EI + v\eta_p I}\left(V_{y,5} - 2V_{y,4} + 2V_{y,2} - V_{y,1}\right) - m\frac{2\lambda^3}{EI} W_{z,ne+3}$$

・i = 3 ～ n+3

$$S_i = 6 + \frac{Ds_i\lambda^4 + wA_P\rho_P\lambda^4}{EI + v\eta_p I}$$

$$T_i = z_{i+2} - 4z_{i+1} + \left(6 + \frac{Ds_i\lambda^4 + wA_P\rho_P\lambda^4}{EI + v\eta_p I}\right) z_i - 4z_{i-1} + z_{i-2}$$

$$+ \frac{\eta_p I}{(EI + v\eta_p I)}\left(V_{y,i+2} - 4V_{y,i+1} + 6V_{y,i} - 4V_{y,i-1} + V_{y,i-2}\right.$$

$$\left. + V_{z,i+2} - 4V_{z,i+1} + 6V_{z,i} - 4V_{z,i-1} + V_{z,i-2}\right)$$

$$+ \frac{1}{(EI + v\eta_p I)}\left(Dt_i\lambda^4 + A_P\rho_P\gamma^4 W_{y,i} + A_P\rho_P\gamma^4 W_{z,i}\right)$$

where, $x<0 : s_i=0 : t_i=0$

・i = n+4, n+5

$$T_{n+4} = z_{n+4} - 2z_{n+3} + z_{n+2}$$

$$+ \frac{\eta_p I}{(EI + v\eta_p I)}\left(V_{y,n+4} - 2V_{y,n+3} + V_{y,n+2} + V_{z,n+4} - 2V_{z,n+3} + V_{z,n+2}\right)$$

$$T_{n+5} = z_{n+5} - 2z_{n+4} + 2z_{n+2} - z_{n+1}$$
$$+ \frac{\eta_p I}{(EI + v\eta_p I)} (V_{y,n+5} - 2V_{y,n+4} + 2V_{y,n+2} - V_{y,n+1}$$
$$+ V_{z,n+5} - 2V_{z,n+4} + 2V_{z,n+2} - V_{z,n+1})$$

<u>• Ground</u>

$$\begin{Bmatrix} D_{ne+2} \\ D_{ne+3} \\ \\ D_i \\ \\ D_{N+3} \\ D_{N+4} \end{Bmatrix} + \begin{vmatrix} A_{ne+2} & B_{ne+2} & C_{ne+2} & & & & & & \\ A_{ne+3} & B_{ne+3} & C_{ne+3} & & & & & & \\ & & & A_i & B_i & C_i & & & \\ & & & & & & A_{N+3} & B_{N+3} & C_{N+3} \\ & & & & & & A_{N+4} & B_{N+4} & C_{N+4} \end{vmatrix} \begin{Bmatrix} z_{ne+2} \\ z_{ne+3} \\ \\ z_i \\ \\ z_{N+3} \\ z_{N+4} \end{Bmatrix} = 0 \qquad (14)$$

where,

<u>• $i = n_e+2$</u>

$A_{ne+2} = G_{ne+3} + v\,\eta_{ne+3}$, $B_{ne+2} = 0$

$C_{ne+3} = G_{ne+3} + v\,\eta_{ne+3}$, $D_{ne+2} = \eta_{ne+3}(V_{z,4} - V_{z,2})$

<u>• $i = n_e+3 \sim N+3$</u>

$$A_i = G_{i-1} + v\,\eta_{G(i-1)}$$
$$B_i = -\{G_i + v\,\eta_i + G_{i-1} + v\,\eta_{i-1} + \frac{1}{2}w(\rho_i\lambda_i^2 + \rho_{i-1}\lambda_{i-1}^2)\}$$
$$C_i = G_i + v\,\eta_i$$
$$D_i = \{\eta_i V_{z,i+1} - (\eta_i + \eta_{i-1})V_{z,i} + \eta_{i-1}V_{z,i-1}\}$$
$$- \frac{1}{2}(\rho_i\lambda_i^2 + \rho_{i-1}\lambda_{i-1}^2)W_{z,i}$$

<u>• $i = N+4$</u>

$$A_{N+4} = -(G_{N+3} + v\,\eta_{N+3})$$
$$B_{N+4} = 2vC_B\lambda_{N+3}$$
$$C_{N+4} = G_{N+3} + v\,\lambda_{N+3}$$
$$D_{N+4} = \eta_{N+3}\lambda_{N+3}(V_{z,N+4} - V_{z,N+2}) + 2C_B\lambda_{N+3}(V_{z,N+3} - v\,z_B - V_{z,B})$$

<u>3) Method of Numerical Analysis</u>

A flow chart of calculation is shown in Fig.6. First, solutions of ground response (zi) are applied from Eq. 14. Next, si,j , ti,j which satisfy pi-yi curves are obtained from Eq.13 by using iterative numerical calculation method. When these solutions are converge, response, yi,j can be acquired. The solutions of Eqs. 13 and 14 are obtained easily by Gleser's method.

EXAMPLES OF NUMERICAL ANALYSIS

Fig.7 ~10 show examples of the numerical analyses that are carried out with a micro computer. The results of calculations can take nonlinearity of ground properties into account. Through these analyses, it is found that this method is a practical one because the method is enabled to calculate easily and in short time.

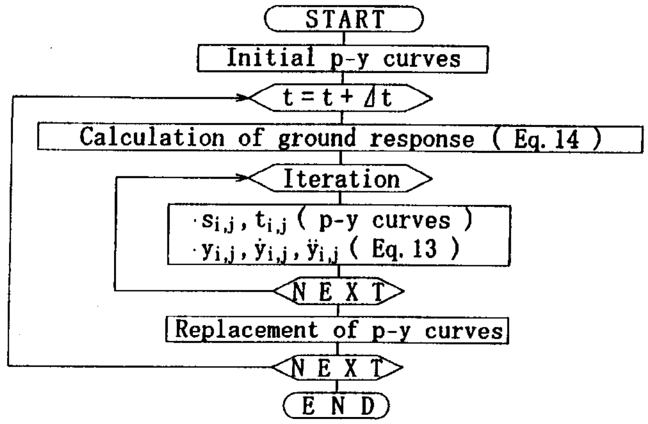

Fig.6 Flow Chart For Numerical Analysis

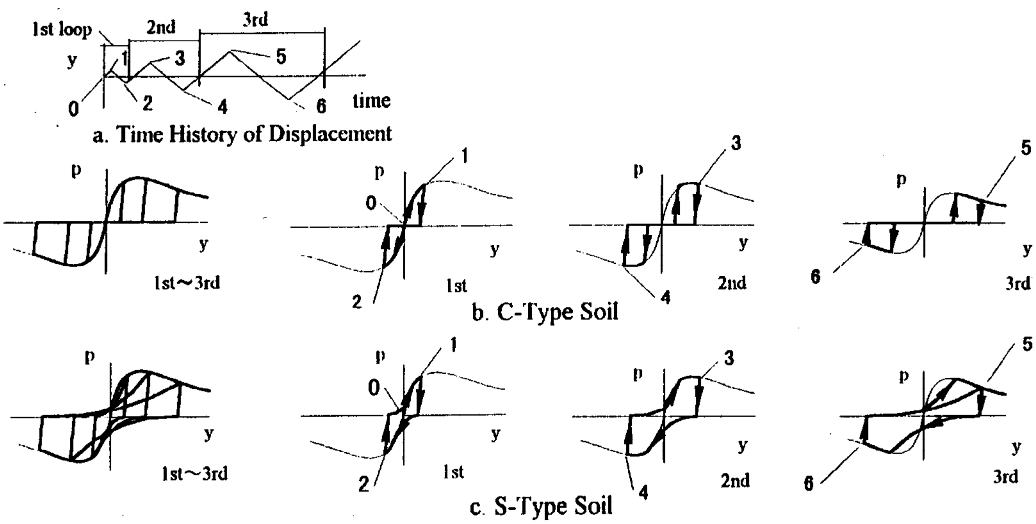

Fig.7 Examples of Hysteresis Loop

CONCLUSIONS

In this paper, although an analysis method of ground-pile-mass system regarding only nonlinearity of soil reaction of the pile is described, it is also possible to consider the nonlinerarity of ground properties in calculation of ground response by the same analysis method.

A characteristic of this analysis method are to be able to calculate easily and in short time even if micro computer is used.

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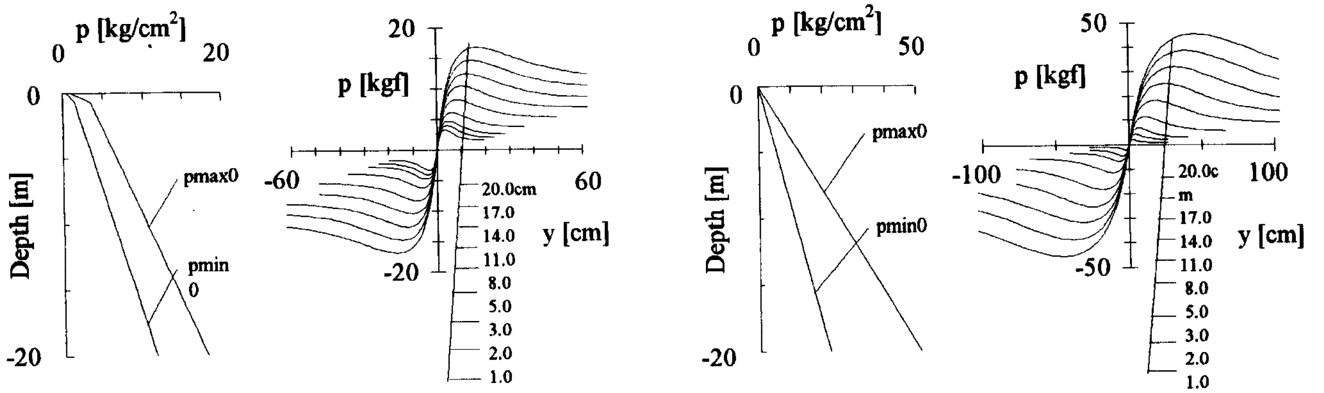
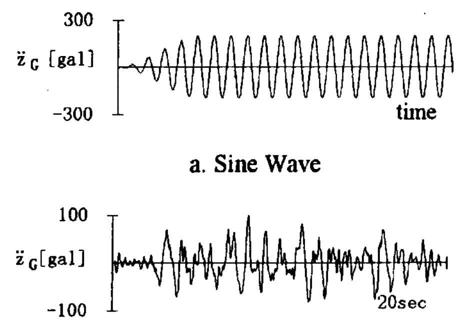


a. p_{max0} and p_{min0} b. Family of Initial p-y Curves
(1) Soil 1(C-type Soil)

a. p_{max0} and p_{min0} b. Family of Initial p-y Curves
(2) Soil 2(S-type Soil)

Fig. 8 Distribution of Subgrade Reaction and Initial p-y Curves



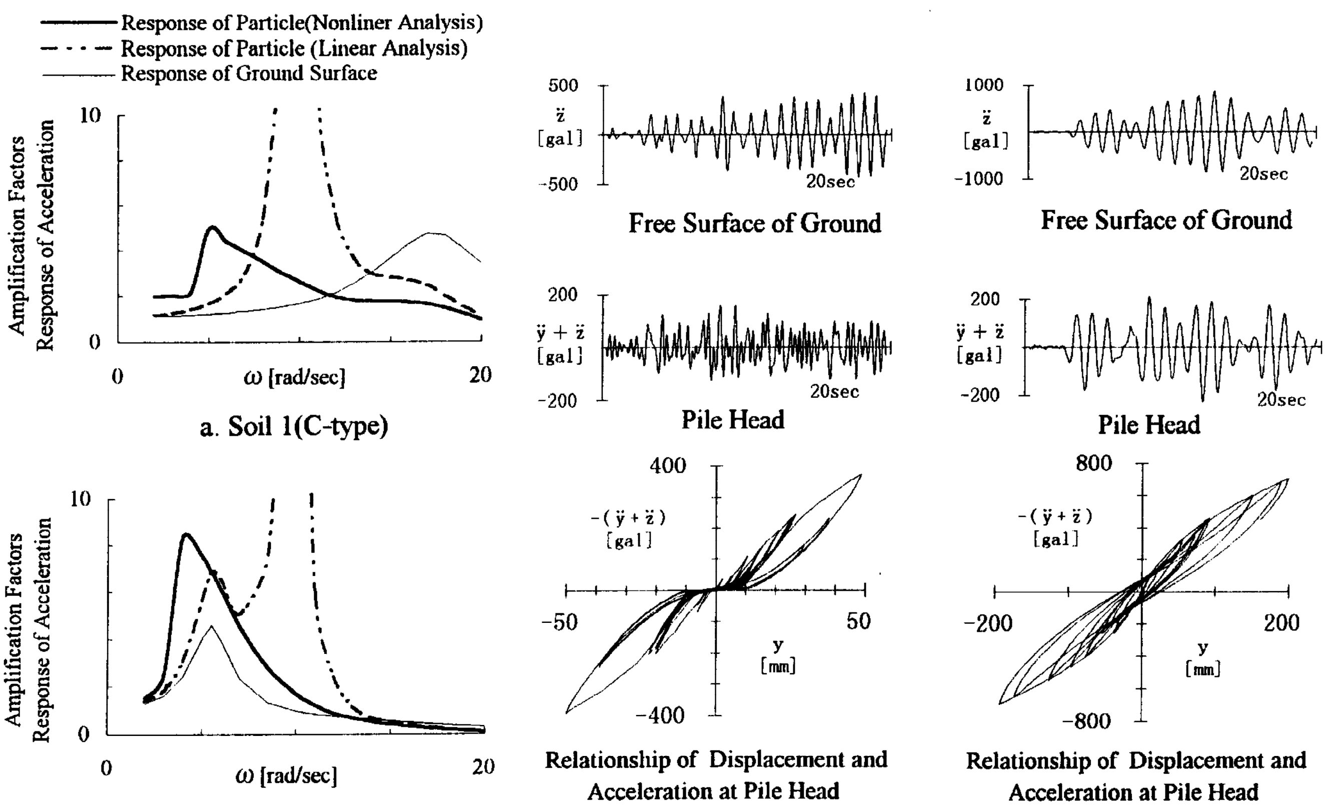
a. Sine Wave

b. Earthquake Motion

Fig.9 Input Motion

Table 1 Characteristics for Examples

Pile and Weight of particle		Ground Conditions	Soil 1	Soil 2
Pile diameter D [mm]	500	Internal friction angle ϕ max [deg]	20	35
Thickness t [mm]	10	ϕ min [deg]	15	25
Bending stiffness EI [kg/cm^2]	9.70×10^{10}	Cohesion Cmax [t/m^2]	2.0	0
Height of eccentricity e [mm]	1,000	Cmin [t/m^2]	1.0	0
Pile depth H [mm]	20,000	Unit weight γ [t/m^3]	1.8	1.8
Coefficient of soil reaction k_h [kg/cm^3]	4.0	Share modulus G [kg/cm^2]	1,000	100
Weight of mass W [ton]	50	Damping constant η_G [kg·sec/cm^3]	15	5



(1) Sine Wave Input

a. Soil 1(C-type) b. Soil 2(S-type)
(2) Earthquake Mosion Input Wave

Fig.10 Examples of Numerical Analysis