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EFFECT OF DIFFERENT LAYERED GROUND MODELS ON DYNAMIC AMPLIFICATION CONSIDERING THE SOIL PARAMETER VARIATIONS

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ABSTRACT

In this paper, we develop a technique to estimate a stochastic amplification function for a layered ground model taking into account soil parameter variation. By expanding the amplification function in the Taylor series about the mean values of soil parameters, we calculated the mean and the standard deviation of amplification. We introduced two different layered ground models from the site; i.e. one is a two layered ground model and the other a multi-layered one. The dynamic amplification characteristics obtained from both models are shown and compared with each other. We conclude that the difference of the models largely changes the amplification characteristics, and that the number of layers affects the amplification characteristics in the higher frequency range.

KEYWORDS

Horizontally Two-layered Soil Models, Soil Parameter Variations, Stochastic Dynamic Amplification, Amplification Characteristics of Two Models, Fluctuation of Higher Frequency Range

INTRODUCTION

For an earthquake-proof design, it is very important to know the dynamic characteristics of the soil ground underlying a site for building construction. Considerable attention has been paid to earthquake ground motions and dynamic amplification characteristics, because they relate very closely to earthquake structural damage. In order to obtain fundamental dynamic ground characteristics of the soil ground, a one dimensional layered model is frequently introduced in the analysis, while multi-dimensional layered and irregular ground models are introduced as models for *in situ* ground.

Shima (1974) found that resonance response amplification was determined by the shear wave velocities of the top soil layer and the bedrock, assuming a horizontal uniform layer model. However, it is well-known that soil parameters cannot be considered perfectly constant values. Hibino (*et al.*, 1988) and Ishida (*et al.*, 1988) estimated dynamic ground characteristics, taking into account the variation of shear wave velocity, stiffness- and damping-strain curves, and the Q value. Chavez-Garcia (*et al.*, 1989) took into account the randomness of the interface of the soil layer, and Tsujihara (1991) analyzed the dynamic ground characteristics with probabilistic variations of parameters using a finite element method. Sato (1994) and Harada examined the influence of the irregularity of surface ground. Recently, three dimensional and irregular ground models have been introduced in the analyses. However, a slight change of soil properties, such as different conditions pertaining at two neighboring sites, will give rise to considerable differences in the dynamic ground characteristics.

It is necessary to examine dynamic ground characteristics by taking into account the soil parameter variation due to the fact that: 1) it is almost impossible to make soil parameters perfectly clear for *in situ* ground; and 2) there is a problem as to how to introduce the soil parameters into the analysis, even if the associated parameters were possible to obtain. It is therefore reasonable for the analysis to take into account the variations of uncertain soil parameters in order to discuss the dynamic ground characteristics.

In this paper, we develop a technique to estimate the dynamic stochastic amplification function taking into account the soil parameter variations (Suzuki *et al.*, 1992). This technique is similar to that of Sawada, but has the advantage that it is applicable to other amplification functions. Numerical calculations are carried out, and the different effects of the two analytical ground models on the dynamic ground characteristics are compared. We present two interesting results for the mean and standard deviations of amplification based on the Haskell matrix assuming a stationary vertical SH incident wave. We treat the thickness and the shear wave velocity in our work as stochastic parameters. These parameters are assumed to be Guassian and independent of each other because of the lack of sufficient evidence for their distribution probability law, heretofore.

ANALYTICAL METHOD

General Expression

By expanding a function of $H(\mathbf{X})$ with random variables $\mathbf{X}$ in the Taylor series about the mean values, $H(\mathbf{X})$ is expressed as follows (Amin *et al.*, 1975) :

$$\begin{aligned} H(\mathbf{X}) = {} & H(\overline{\mathbf{X}}) + \sum_{\kappa=1}^{m} \left.\frac{\partial H}{\partial X_\kappa}\right|_{\overline{\mathbf{X}}} (X_\kappa - \overline{X_\kappa}) \\ & + \frac{1}{2}\sum_{j=1}^{m}\sum_{\kappa=1}^{m} \left.\frac{\partial^2 H}{\partial X_j \partial X_\kappa}\right|_{\overline{\mathbf{X}}} (X_j - \overline{X_j})(X_\kappa - \overline{X_\kappa}) \\ & + \frac{1}{3!}\sum_{i=1}^{m}\sum_{j=1}^{m}\sum_{\kappa=1}^{m} \left.\frac{\partial^3 H}{\partial X_i \partial X_j \partial X_\kappa}\right|_{\overline{\mathbf{X}}} \\ & \times (X_i - \overline{X_i})(X_j - \overline{X_j})(X_\kappa - \overline{X_\kappa}) \\ & + \cdots \end{aligned} \tag{1}$$

in which $\mathbf{X}$ is the random variable vector, $\overline{X}$ expresses the mean of X, and m is the number of random variable. The mean of $H(\mathbf{X})$ is obtained from Eq.(1).

$$\begin{aligned} \mathrm{E}[H(\mathbf{X})] = {} & H(\overline{\mathbf{X}}) \\ & + \frac{1}{2}\sum_{j=1}^{m}\sum_{\kappa=1}^{m} \left.\frac{\partial^2 H}{\partial X_j \partial X_\kappa}\right|_{\overline{\mathbf{X}}} \mathrm{E}[(X_j - \overline{X_j})(X_\kappa - \overline{X_\kappa})] \\ & + \cdots \\ = {} & H(\overline{\mathbf{X}}) + \frac{1}{2}\sum_{j=1}^{m}\sum_{\kappa=1}^{m} \left.\frac{\partial^2 H}{\partial X_j \partial X_\kappa}\right|_{\overline{\mathbf{X}}} \mathrm{Cov}[X_j, X_\kappa] \\ & + \cdots \end{aligned} \tag{2}$$

$\mathrm{Cov}[X_j, X_\kappa]$ in Eq.(2) is the co-variance.

$$\mathrm{Cov}[X_j, X_\kappa] = \mathrm{E}[(X_j - \overline{X_j})(X_\kappa - \overline{X_\kappa})] \tag{3}$$

E[·] in Eq.(2) is the ensemble averaging operator. The mean square of $H(\mathbf{X})$ is also obtained from Eq.(1).

$$
\begin{aligned}
&\mathrm{E}[(H(\mathbf{X})-\mathrm{E}[H(\mathbf{X})])^2] \\
&\quad=\sum_{j=1}^{m}\left(\left.\frac{\partial H}{\partial X_j}\right|_{\overline{\mathbf{X}}}\right)^2 \mathrm{E}[(X_j-\overline{X_j})^2] \\
&\quad+\frac{1}{4}\sum_{j=1}^{m}\sum_{\kappa=1}^{m}\left.\frac{\partial^2 H}{\partial X_j \partial X_\kappa}\right|_{\overline{\mathbf{X}}} \mathrm{Cov}^2[X_j, X_\kappa] \\
&\quad-\frac{1}{4}\sum_{j=1}^{m}\sum_{\kappa=1}^{m}\left(\left.\frac{\partial^2 H}{\partial X_j \partial X_\kappa}\right|_{\overline{\mathbf{X}}}\right)^2 \mathrm{Cov}^2[X_j, X_\kappa] \\
&\quad+\sum_{j=1}^{m}\sum_{\kappa=1}^{m}\left.\frac{\partial H}{\partial Xj}\right|_{\overline{\mathbf{X}}}\left.\frac{\partial^2 H}{\partial X_j \partial X_\kappa}\right|_{\overline{\mathbf{X}}} \\
&\quad\times \mathrm{E}[(X_j-\overline{X_j})^2(X_\kappa-\overline{X_\kappa})] \\
&\quad+\cdots
\end{aligned}
\tag{4}
$$

<u>Application of an Amplification Function</u>

In order to directly obtain stochastic amplification, we introduced a horizontal layered soil ground model and a stationary vertical incident wave. $H(\mathbf{X})$ in Eqs.(2) and (4) is substituted by the amplification function of the Haskell transmitted matrix method at the free surface. Vector $\mathbf{X}$ presents the shear wave velocity, the thickness and the density of the soil layer ($\mathbf{X}=\{\mathrm{v_s}, \mathrm{z}, \rho\}, \mathrm{m}=3$). By neglecting the higher order terms of Taylor expansion expressions for Eqs.(2) and (4) and limiting those of the second order, we can obtain the approximate mean and mean square of $H(\omega;\mathbf{X})$.

$$
\begin{aligned}
&\mathrm{E}[H(\omega;\mathbf{X})] \\
&\quad= H(\omega;\overline{\mathbf{X}}) \\
&\quad+\frac{1}{2}\sum_{j=1}^{m}\sum_{\kappa=1}^{m}\left.\frac{\partial^2 H}{\partial X_j \partial X_\kappa}\right|_{\overline{\mathbf{X}}} \mathrm{Cov}[X_j, X_\kappa]
\end{aligned}
\tag{5}
$$

$$
\begin{aligned}
&\mathrm{E}[(H(\omega;\mathbf{X})-\mathrm{E}[H(\omega;\mathbf{X})])^2] \\
&\quad=\sum_{j=1}^{m}\left(\left.\frac{\partial H}{\partial X_j}\right|_{\overline{\mathbf{X}}}\right)^2 \mathrm{E}[(X_j-\overline{X_j})^2] \\
&\quad+\frac{1}{4}\sum_{j=1}^{m}\sum_{\kappa=1}^{m}\left.\frac{\partial^2 H}{\partial X_j \partial X_\kappa}\right|_{\overline{\mathbf{X}}} \mathrm{Cov}^2[X_j, X_\kappa] \\
&\quad-\frac{1}{4}\sum_{j=1}^{m}\sum_{\kappa=1}^{m}\left(\left.\frac{\partial^2 H}{\partial X_j \partial X_\kappa}\right|_{\overline{\mathbf{X}}}\right)^2 \mathrm{Cov}^2[X_j, X_\kappa] \\
&\quad+\sum_{j=1}^{m}\sum_{\kappa=1}^{m}\left.\frac{\partial H}{\partial Xj}\right|_{\overline{\mathbf{X}}}\left.\frac{\partial^2 H}{\partial X_j \partial X_\kappa}\right|_{\overline{\mathbf{X}}} \\
&\quad\times \mathrm{E}[(X_j-\overline{X_j})^2(X_\kappa-\overline{X_\kappa})]
\end{aligned}
\tag{6}
$$

As mentioned above, neglecting the correlation between the parameters and assuming the variation of the soil parameters to be Guassian, we can approximate the expression of Eqs.(5) and (6).

$$\mathrm{E}[H(\omega;\mathbf{X})]$$

$$\cong H(\omega;\overline{\mathbf{X}}) + \frac{1}{2}\sum_{j=1}^{m} \left.\frac{\partial^2 H}{\partial X_j^2}\right|_{\overline{\mathbf{X}}} \mathrm{Var}[X_j] \tag{7}$$

$$\begin{aligned}
&\mathrm{E}[(H(\omega;\mathbf{X}) - \mathrm{E}[H(\omega;\mathbf{X})])^2] \\
&\cong \sum_{j=1}^{m} \left(\left.\frac{\partial H}{\partial X_j}\right|_{\overline{\mathbf{X}}} \right)^2 \mathrm{Var}[X_j] \\
&+ \frac{1}{4}\sum_{j=1}^{m} \left.\frac{\partial^2 H}{\partial X_j^2}\right|_{\overline{\mathbf{X}}} \mathrm{Var}^2[X_j] \\
&- \frac{1}{4}\sum_{j=1}^{m} \left(\left.\frac{\partial^2 H}{\partial X_j^2}\right|_{\overline{\mathbf{X}}} \right)^2 \mathrm{Var}^2[X_j] \\
&+ \sum_{j=1}^{m} \left.\frac{\partial H}{\partial X_j}\right|_{\overline{\mathbf{X}}} \left.\frac{\partial^2 H}{\partial {X_j}^2}\right|_{\overline{\mathbf{X}}} \mathrm{E}[(X_j - \overline{X_j})^3]
\end{aligned} \tag{8}$$

in which,

$$\mathrm{Var}[X_j] = \mathrm{E}[(X_j - \overline{X_j})^2] \tag{9}$$

$\mathrm{Var}[X_j]$ in Eq.(9) is the variance matrix.

One of our objectives in this paper is to examine the dynamic amplification characteristics. Then we can easily obtain the standard deviation of amplification (σ_H) from Eq.(8). Lastly, we can approximate the mean and the standard deviation of amplification as follows:

$$\overline{H} = H(\omega;\overline{\mathbf{X}}) \tag{10}$$

$$\begin{aligned}
\sigma_H &= \sqrt{\mathrm{E}[(H-\overline{H})^2]} \\
&= \sqrt{D_1 + D_2}
\end{aligned} \tag{11}$$

in which,

$$D_1 = \sum_{j=1}^{m} \left(\left.\frac{\partial H}{\partial X_j}\right|_{\overline{\mathbf{X}}} \right)^2 \mathrm{Var}[X_j] \tag{12}$$

$$D_2 = \frac{1}{4}\sum_{j=1}^{m} \left.\frac{\partial^2 H}{\partial X_j^2}\right|_{\overline{\mathbf{X}}} \mathrm{Var}^2[X_j] \tag{13}$$

The second differential of Eq.(11) can be calculated by computer using the difference, in which $\Delta f = 0.01\ Hz$.

NUMERICAL RESULTS

Analytical Model

We introduced a horizontally layered model of 1-dimension on infinite soil ground. $\overline{z_i}$, $\overline{v_{s_i}}$, and $\overline{\rho_i}$ are respectively the ith soil layer thickness, shear wave velocity, and density of the model, where the bar represents the average. Variance coefficients of soil parameters of 5% were chosen for this analysis. The variance coefficient ν is the fluctuation around the related mean value.

$$\nu = \frac{\sigma_X}{\overline{X}} \tag{14}$$

Comparison of Stochastic Amplifications

We introduced two different layered horizontal soil ground models, obtained the fundamental characteristics of stochastic amplification for the respective models, and discussed the difference between them. In particular, we examined the effects of the number of layer and the higher frequency on the stochastic amplification characteristics. In order to check this effect, we introduced, as an example, the following soil parameters for the basis of the properties of soil profile from Memphis, USA. As shown in Tables 1 and 2, Model-S (Sato, 1994) consisted of a two-layered model and Model-M (Sawada *et al.*), a multi-layered one (see Chang *et al.*, 1989 and Ahmad *et al.*, 1990). Both models were assumed to be horizontally layered. In our work, the constant Q value was chosen in all soil layers; i.e. $Q = 10$, 20, and 30. By taking into account the 5% variation in the soil parameters in Tables 1 and 2, and by carrying out the calculation of stochastic amplification with Eqs.(15) and (16), we presented the relationships between the amplification and the frequency ($f = 0.1 \sim 10\,Hz$).

Figures 1 and 2 show the relations between the amplification and frequency, where a 5% variation of thickness and shear wave velocity are taken into account for the two different soil layers, Model-S and Model-M. In the figure, (a), (b), and (c) respectively show the results of $Q = 30$, 20, and 10. The thick solid line represents the mean amplification ($\overline{H}$), and the thin solid lines above and below the thick one represent the summation and the difference of $\overline{H}$ and three times the standard deviation of the amplification ($\overline{H} \pm 3\sigma_H$). Fig. 1, the case of Model-S, shows that the fundamental frequency and the second modal frequency are respectively 1.2 and 3.6 Hz. The peaks for the higher modes are less evident when the damping is larger. The peak of fundamental modal amplification is the maximum value through all frequencies. Limitation of space prevents the showing of all of the results, but we found that the amplification fluctuated greatly in the higher frequency range, and that the maximum value for the corresponding variance coefficient was about 10%. Fig. 2 is the case of Model-M. The mean value of amplification for the fundamental mode is almost the same as that for Model-S. The peak value of the second mode, however, is larger than that of the fundamental mode. This becomes evident when the damping is small. Particularly, in the case of $Q = 30$, one can recognize that $\overline{H} + 3\,\sigma_H$ for the third mode is larger than that of the fundamental mode.

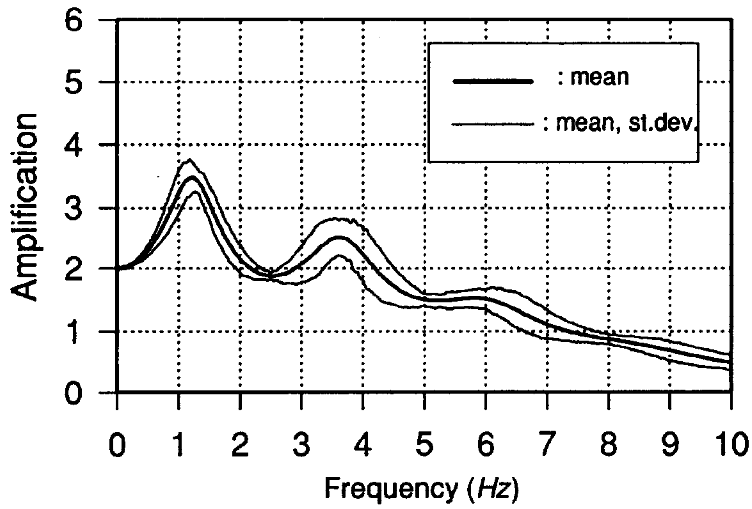
The variation of amplification in the higher frequency range is larger than at lower frequencies. The maximum value of the variation is almost 30%, and this value is large compared with the results shown in Fig.2. The number of soil layers and their properties strongly influence the stochastic amplification characteristics. The different results obtained are typical examples of different soil ground models, such as Model-S and Model-M. It is interesting that in Model-M the peak values of amplification for the higher mode ($\overline{H} \pm 3\,\sigma_H$ and $\overline{H}$) are larger than those for the fundamental mode.

Table 1 Properties of soil ground of Model-S

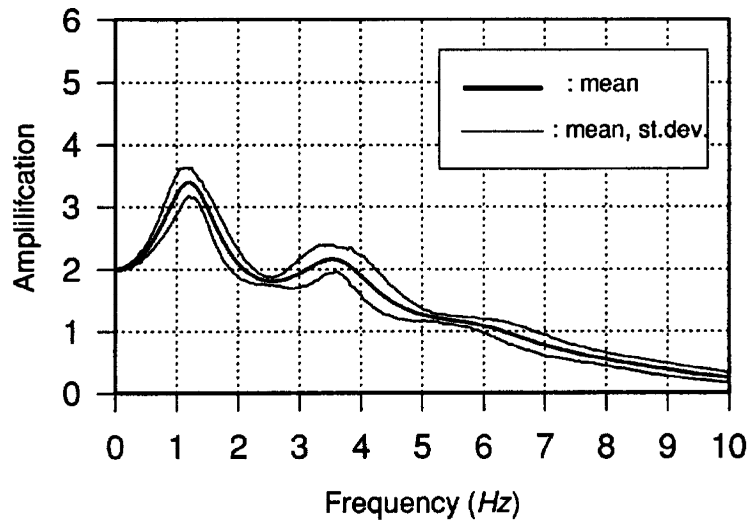
No. of stratum	Thickness ($\overline{z}$) (m)	Shear wave velocity ($\overline{v}$) (m/sec)	Density ($\overline{\rho}$) (t/m^3)
1	75.0	370.0	2.32
2	∞	610.0	2.56

Table 2 Properties of soil ground of Model-M

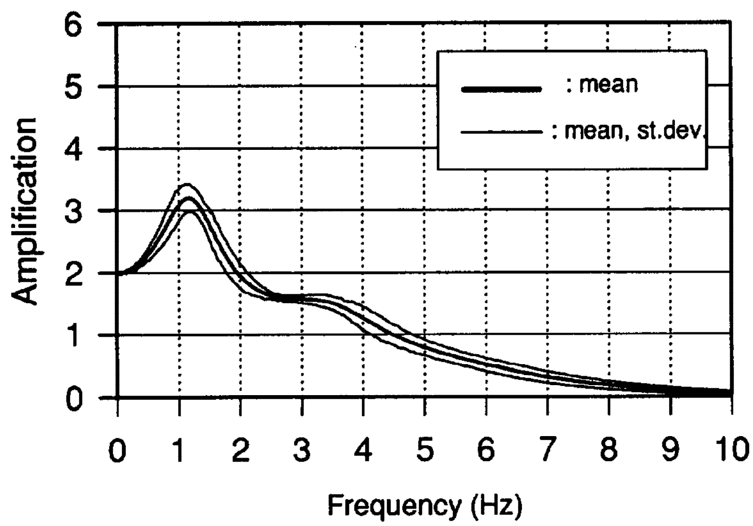
No. of stratum	Thickness ($\overline{z}$) (m)	Shear wave velocity ($\overline{v}$) (m/sec)	Density ($\overline{\rho}$) (t/m^3)
1	7.5	207.0	2.31
2	7.5	290.0	2.35
3	15.0	300.0	2.48
4	45.0	360.0	2.74
5	∞	600.0	2.74



(a) Case of Q=30

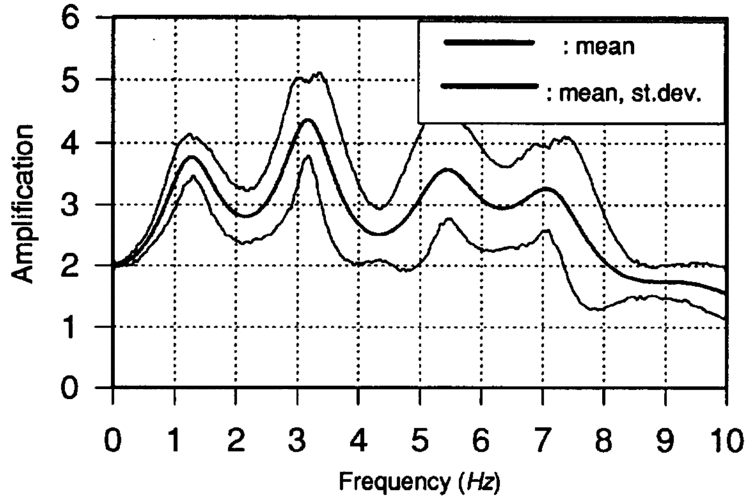


(b) Case of Q=20

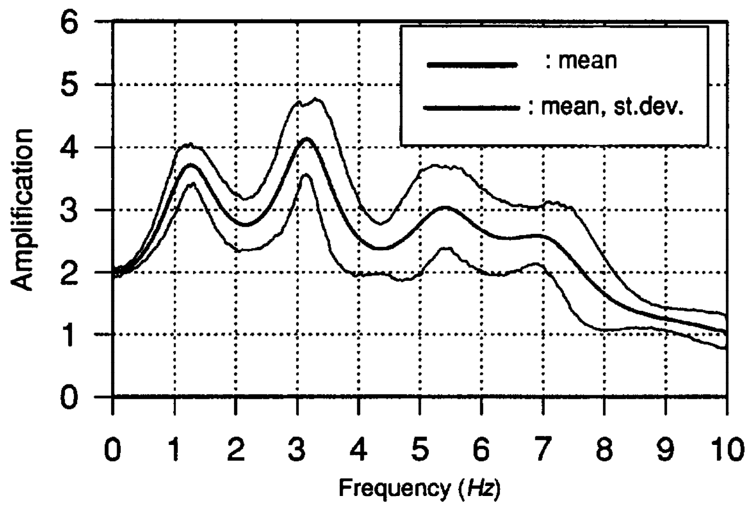


(c) Case of Q=10

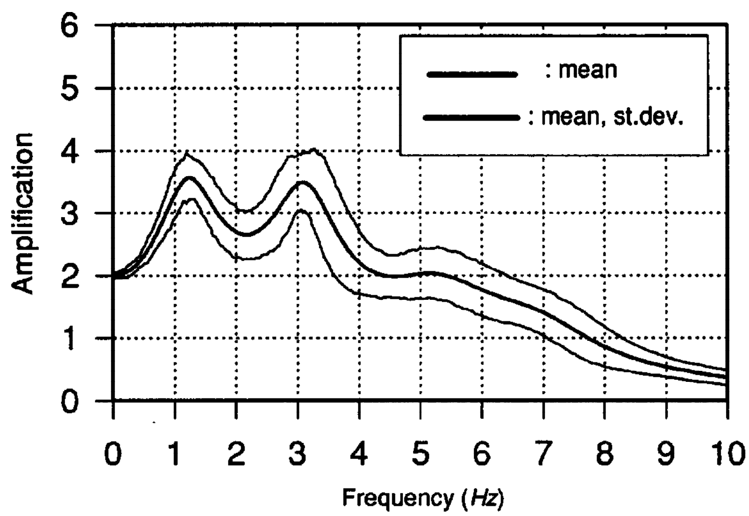
Fig. 1. Stochastic amplification in the case of Model–S with 5% variation of thickness and shear wave velocity



(a) Case of Q=30



(b) Case of Q=20



(c) Case of Q=10

Fig. 2. Stochastic amplification in the case of Model–M with 5% variation of thickness and shear wave velocity

The variation of stochastic amplification for Model-M was larger than for Model-S. This phenomenon is noticeable with small damping, such as $Q = 20$. The $\overline{H} \pm 3\ \sigma_H$ curves of Fig.3 envelope the amplification in the case of the ground surface layer, whose top has either a long period or a very small irregularity (see Sato 1994).

CONCLUSIONS

In this paper, we developed a technique for estimating the dynamic stochastic amplification function, by taking into account soil parameter variations. We discussed the dynamic amplification characteristics obtained from the two kinds of ground model based on the soil profile of Memphis, USA. One is a two-layered ground model (Model-S) and the other is a multi-layered one (Model-M).

Only the peak for the fundamental mode of amplification was recognized from the results in the case of Model-S. Its value was about 4.0. This phenomenon was not strongly influenced by change of the shear wave velocity or by the thickness of the layered model. Peaks for the 1-4 modes of amplification were apparent, however, from the Model-M results. It is interesting to note that the peak for the higher mode was larger than the peak for the fundamental one and that this differs from the deterministic amplification results.

The fluctuation of amplification in the case of Model-M was larger than that of Model-S. The amplification in the higher frequency range fluctuates much more than in the lower frequency range. This means that the number of soil ground layer affects the amplification characteristics in the higher frequency range.

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