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SOIL-STRUCTURE INTERACTION SYSTEMS ON THE BASE OF THE GROUND IMPEDANCE FUNCTIONS FORMED INTO A CHAIN OF IMPULSES ALONG THE TIME AXIS

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ABSTRACT

It is well known that structural constructions are interacted with their surrounding soil ground under dynamic excitations. The interaction phenomena are, however, so complicated as to obtain the analytical manner, especially in the time domain. The present study is concerned to introduce the practical method for describing the dynamic properties on the base of the causal conditions along the time and frequency axes.

KEYWORDS

soil-structure interaction; causality; ground impedance functions.

INTRODUCTION

An infinite number of researches have been vigorously carried out for the dynamic interaction problems between structural constructions and their underlaying soil ground. The interacted systems are usually divided into the tentative subset composed only of upper-structures with base foundations and the one of a soil ground concaved with foundation-shaped cavities. According to the numerical analysis of the frequency compliance functions or the impedance ones defined on the interface between foundations and a soil ground, the first stage of the analysis is set on the spectral characteristics and the response processes are left below the second. Moreover, it is an essential condition for the response analysis to be consistent with the physical causality in the time and frequency domain.

In the present investigation, the ground impedance functions are simulated on the series expansions with causal representations and transformed into a chain of impulses along the time axis. The total equations of motion are formulated with the displacement vector and its delayed ones of the upper structures and the base foundations. The modal components are described in practice through the approximate method with little time behinds or the reduced procedure selecting the major modes on the complex plane. The numerical analysis is carried out in the interacted responses along the frequency and time directions for some system properties.

FORMULATION OF THE PROBLEM

The soil-structure systems are modeled on the composition of an upper-structure with concentrated masses and base foundations in tight contact with the surface of the elastic soil ground spread over a half space, as shown in

Fig.1. The equations of motion are written by the displacement vector $\mathbf{x}(t)$ of the upper-structure and their sustained base foundations in the following sway-rocking type,

$$\sum_{m=0}^{2} \mathbf{A}_m \mathbf{x}^{(m)}(t) = -\mathbf{A}_2 \mathbf{e} x_G^{(2)}(t) - \mathbf{f}(t) \qquad (1)$$

in which $\mathbf{A}_2$, $\mathbf{A}_1$ and $\mathbf{A}_0$ are the matrices of mass, damping and stiffness of upper and base structures, $\mathbf{e}$ gives the modified unit vector with the exception of null values for the rotational components, $x_G(t)$ is the lateral displacement of the excited soil ground, $\mathbf{f}(t)$ shows the external force vector associated with the soil-structure interaction and the upper right script (i) corresponds to the derivative in respect of time t. According to the lateral excitations in the surrounding ground, the external forces are composed of a shearing force $P_G(t)$ and a bending moment $M_G(t)$ around the horizontal axis passing through the base foundation, which are written in the convolution integrals along the time direction, constituent of the relative displacement vector $\mathbf{x}_0(t)$ of the foundation and the impedance functions $\mathbf{D}_0(t)$ specified on the interface between the foundation and its underlaid soil ground,

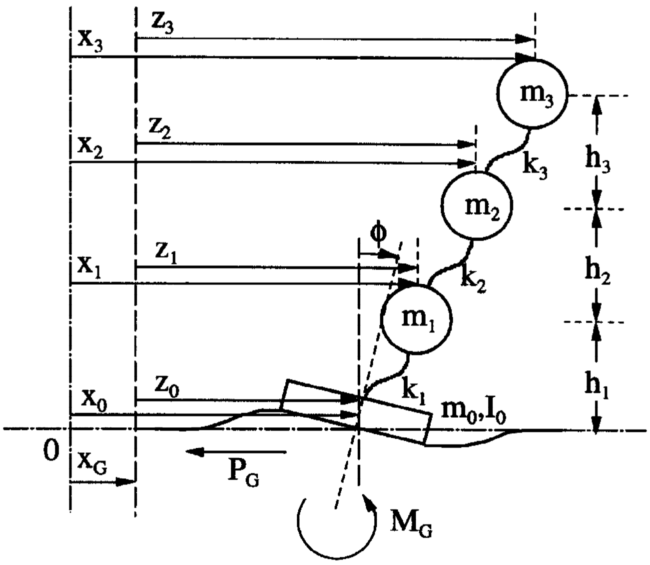


Fig.1 Configuration of soil-structure interaction systems.

$$\mathbf{f}(t) = \mathbf{D}_0(t) * \mathbf{x}_0(t) \qquad (2)$$

The impedance functions or the compliance ones are usually obtained in the frequency domain and entertained numerically for the soil-foundation systems without upper-structures. To describe the impedance functions in the analytical manner, their digital data are simulated on the next series expansions through the least squares method within the frequency range spread enough for analyzing the interacted responses,

$$\tilde{\mathbf{D}}_0(\omega) = \sum_{m=0}^{M} \sum_{n=0}^{N} {}_0\mathbf{D}_{mn} (i\omega)^m \exp(-i\omega\tau_n) \quad : \ \omega_L \geq |\omega| \qquad (3)$$

in which any term is arranged to agree with the causality when the corresponding coefficient matrix ${}_0\mathbf{D}_{mn}$ is given in the real domain. The sampling time τ_n is limited positive and related with the cutoff frequency ω_L to guard the spectral functions from the aliasing confusion. When the series equation (3) is available in the entire frequency range, the simulated impedance functions are transformed into a chain of impulses and its derivatives behindhand with their responses along the forward time direction, namely the impulsive responses are divided into the segments concentrated at the discrete instants,

$$\mathbf{D}_0(t) = \sum_{m=0}^{M} \sum_{n=0}^{N} {}_0\mathbf{D}_{mn} \delta^{(m)}(t - \tau_n) \qquad (4)$$

The impulse chains are honed sharp in the case that the series expansion of the impedances are still effective in the higher frequency range exceeding the cutoff limit for the simulation procedure. The extended arrangement contributes the intensive distributions toward the feedback responses circulating around the convolution integrals in the time domain. The equations of the interacted motions are rewritten in the following form by the displacement vector and its delayed ones,

$$\sum_{m=0}^{M} \left\{ \mathbf{A}_m \mathbf{x}^{(m)}(t) + \sum_{n=0}^{N} \mathbf{D}_{mn} \mathbf{x}^{(m)}(t - \tau_n) \right\} = -\mathbf{A}_2 \mathbf{e} x_G^{(2)}(t) \quad : M=2 \qquad (5)$$

in which the impedance coefficient matrix $\mathbf{D}_{mn}$ is empty except for the subset ${}_0\mathbf{D}_{mn}$ associated with the responses $\mathbf{x}_0(t)$ of the base foundation. One of the upper bounds is settled with M=2 for the series expansions because the differentiation over two times is physically meaningless for the displacement components along the time axis. The other upper bound is restricted with N=1 due to the response components circulating not so behind the exciting motions, as far as the base foundation has the normal configuration.

MODAL ANALYSIS OF INTERACTION SYSTEMS

For the analysis of the response processes under earthquake excitations, wind turbulences or artificial controlling forces along the time direction, it is convenient to separate the dynamic systems into their modal elements. The strict separation is, however, difficult when the delayed components are included inside the motions as explained in the equation (5). In accordance, the following two types of practical method are recommended for the modal analysis of the soil-structure interaction systems.

(1) *Approximate procedure on little time differences* :

When the delayed duration τ_n is infinitesimal, the function x(t) and its derivatives at t are generally related to those at t-τ_n through the central difference method,

$$\left(\frac{\tau_n}{2}\right)^2\left\{x(t)-x(t-\tau_n)\right\}^{(2)}-\tau_n\left\{x(t)+x(t-\tau_n)\right\}^{(1)}+2\left\{x(t)-x(t-\tau_n)\right\}=0\left(\tau_n{}^3\right) \tag{6}$$

By combining the difference equation (6) with the equation (5) of the interacted motions, the next form is approximately obtained in respect to the unknown vector $\boldsymbol{\zeta}(t)$ enlarged with the delayed components and without the positive implication of the time difference τ_n, similar to that of the structural systems fixed at the base foundation,

$$\sum_{m=0}^{M}\mathbf{U}_m\boldsymbol{\zeta}^{(m)}(t)=\boldsymbol{\gamma}x_G{}^{(2)}(t)+0\left(\tau_n{}^3\right) \tag{7}$$

where the coefficient matrices $\mathbf{U}_m$ are unsymmetric and unsuccessful in making the orthogonal conditions for the complex mode vectors. In spite of the disadvantage,the differential equation (7) has the capacity for changing into the state form at a single instant as far as the delayed period is short enough. The characteristic values and the mode vectors are approximately obtained on the complex plane through the ordinary procedure.

(2) *Sorting procedure with major modes*:

The vector equation (5) of motion is directly exchanged for the following state formula under single derivatives,

$$\sum_{n=0}^{N}\left\{\mathbf{B}_n\mathbf{y}(t-\tau_n)+\mathbf{C}_n\mathbf{y}^{(1)}(t-\tau_n)\right\}=\mathbf{g}x^{(2)}{}_G(t) \tag{8}$$

Because of the interacted properties with the continuous ground, the whole number of the characteristic values are counted more than the total of the response components in the upper structure and the base foundation. When selecting the characteristic values and the mode vectors coincident with the dimension size of the state equation, it is acceptable that the state vector and its modified one traveling τ_n behind are composed in the following forms with the matrix $\boldsymbol{\Phi}$ having the mode vectors in the row,

$$\mathbf{y}(t)=\boldsymbol{\Phi}\mathbf{q}(t),\quad \mathbf{y}(t-\tau_n)=\boldsymbol{\Phi}\mathbf{E}\mathbf{q}(t) \tag{9}$$

in which $\mathbf{E}$ corresponds to the diagonal matrix containing the exponential functions exp($-\lambda_i\tau_n$) with the characteristic values λ_i, and $\mathbf{q}(t)$ is the unknown vector with the time parameter t. Under the preparation, the state equation (8) is modally divided into the next formation,

$$\mathbf{q}^{(1)}(t)-\Lambda\mathbf{q}(t)=\boldsymbol{\alpha}x^{(2)}{}_G(t) \tag{10}$$

in which Λ is the spectral matrix diagonalized with the characteristic values. The vector $\boldsymbol{\alpha}$ contributes to participating the excitations among the modal equations.

NUMERICAL ANALYSIS

For regulating the numerical results, the next dimensionless parameters are adopted with the over script ($\bar{\ }$),

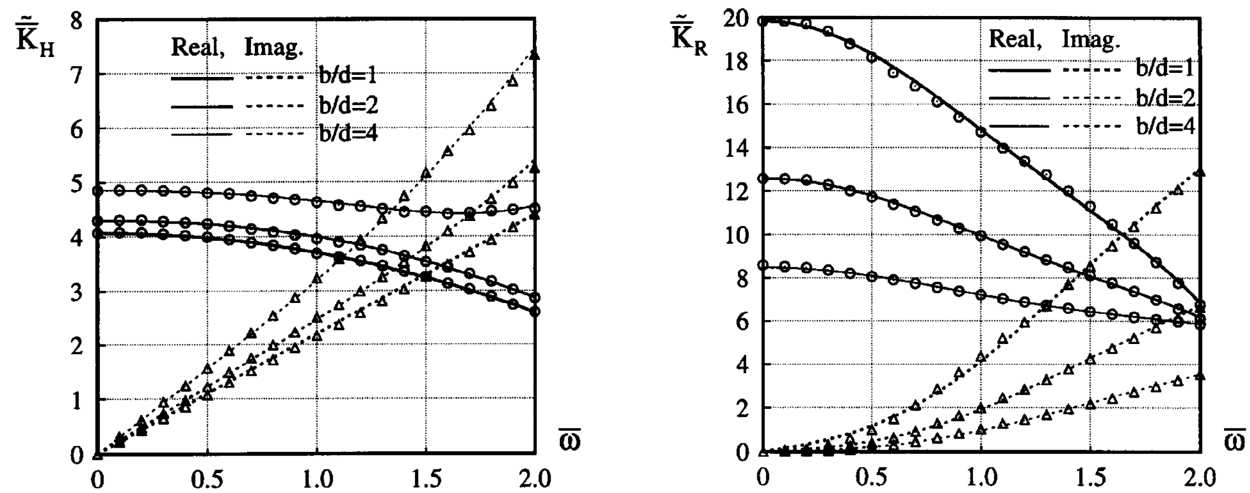
$$\bar{m}_i = \frac{m_i}{\rho b d^2},\ \bar{k}_i = \frac{k_i}{\mu d},\ \bar{c}_i = \frac{c_i}{\mu d \omega_G},\ \bar{r}_o = \frac{r_0}{d},\ \bar{h}_i = \frac{h_i}{d},\ \bar{h} = \frac{h}{d},\ \bar{b} = \frac{b}{d},$$

$$\bar{k}^H_{mn} = k^H_{mn}\frac{\omega_G{}^m}{\mu b},\ \bar{k}^R_{mn} = 3k^R_{mn}\frac{\omega_G{}^m}{\mu b d^2},\ \bar{x}_i = \frac{x_i}{u_G},\ \bar{\phi} = \frac{\phi h}{u_G},\ \bar{\omega} = \frac{\beta\omega}{\omega_G},\ \bar{t} = \frac{t}{\beta\omega_G},\ \omega_G = \frac{1}{d}\sqrt{\frac{\mu}{\rho}},$$

in which m_i, c_i, k_i and h_i are respectively the concentrated mass, the damping coefficient, the stiffness coefficient and the height at the i-th floor level of the upper-structure, r_0 is the radius of moment inertia about the lateral axis passing the thin and rectangular foundation ($2b \times 2d$), and ρ, μ, ν are the mass density, the shearing rigidity and the Poisson's ratio of the soil ground. The displacement components x_i, ϕ correspond to the lateral translation at the i-th floor and the rotation at the foundation, relative to the ground motions. The standard measures ω_G, u_G, h are found in the frequency, the ground displacement and the story height of upper-structures. The scales of time and frequency parameters are adjusted with the factor β to the infinitesimal delay period.

Ground impedances: The simulated solutions are shown in Fig.2 for the lateral and rotational impedance functions on the interface between a rectangular foundation and an elastic half ground, which quite agree with the analytical data within the utilizable frequency range. When the shape of the foundation is confined in square (b=d), the delayed components almost disappear in the lateral translation, as exhibited on Table 1.

Characteristic properties: The characteristic values and the mode vectors are given on Table 2 for the soil-structure systems with one-story upper constructions and square foundations, plotted on the second quadrant of



(a) Horizontal translation (b) Rotation

Fig.2 Dimensionless impedance functions : b/d=1,2,4, ν=0.25.

Table 1 Dimensionless factors expanded in the impedance functions

(a) Horizontal translation

	b/d=1	b/d=2	b/d=4
$\bar{k}^H_{00}$	4.0700	3.4411	5.3709
$\bar{k}^H_{01}$	0.	0.8558	-0.5212
$\bar{k}^H_{10}$	2.2007	3.2500	3.0086
$\bar{k}^H_{11}$	0.	0.5519	-0.7208
$\bar{k}^H_{20}$	0.3688	-0.0003	-0.0005
$\bar{k}^H_{21}$	0.	0.0721	-0.3335

(b) Rotation

	b/d=1	b/d=2	b/d=4
$\bar{k}^R_{00}$	8.1175	6.9963	6.2494
$\bar{k}^R_{01}$	11.7199	5.5786	2.2387
$\bar{k}^R_{10}$	13.2422	6.3156	2.7362
$\bar{k}^R_{11}$	6.7113	2.9617	0.9710
$\bar{k}^R_{20}$	0.0003	-0.0004	-0.0007
$\bar{k}^R_{21}$	2.6187	1.1881	0.4627

the complex plane (their conjugate values are also on the third quadrant) and missed on the first and the fourth quadrants which establishes the dynamic stability of the interaction systems. The plots of the characteristics are counted to four for the fundamental systems, in correspondence with two sets of the lateral translation at each floor level, and two sets of the rotation and its delayed one at the foundation.

Frequency responses: The spectral properties are shown in Fig. 3, which are the displacement amplitudes of three-story structures under the ground excitations. They are dominant only at the natural frequencies of the lateral translation on the upper floor levels, while another dominance is also distinguished in the higher frequency range on the base foundation due to the rotating motions. When the inner damping is perfectly left out of the upper structures, the amplified dominance is violent on the natural frequencies especially within the lower range regardless of the radiation damping for the far field of the soil ground, and has the tendency to be put down by adding a bit of inner viscosity.

Transient responses: The response processes are given in Figs. 4 and 5 for the interaction systems mentioned above when the unit displacement is initially forced at the top floor. The foundation motions are found diminishing more rapidly than those on the upper floors, especially in respect of the velocity components. It is comparable with the spectral characteristics that the high frequency notches are added to the harmonic responses which decrease soon when the inner damping is included in the upper structures.

CONCLUDING REMARKS

For the dynamic interaction systems between structural constructions and their surrounding soil ground, the equations of motion are formulated with the displacement vector and its delayed ones through the ground impedance functions simulated in the present proposal on the base of the physical causality. It is no simple task to obtain the characteristic values and their associate mode vectors in the differential equations with the behindhand components. The major sets of the characteristics are, however, countable in the finite numbers so that some experiments are put into practice in the modal separation under the permissible restriction. By referring to the numerical results, the complicated subject is well researched without the serious injury to the dynamic interaction characteristics.

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Table 2 Complex eigen values and eigen vectors

(a) Approximate method : β=5.0

<eigen values>

Real	Imag.	Real	Imag.	Real	Imag.	Real	Imag.
-0.05304	0.61001	-0.26947	1.18364	-0.49754	2.09519	-1.33004	1.77509

<eigen vectors>

Amplitude	Phase angle	Amplitude	Phase angle	Amplitude	Phase angle	Amplitude	Phase angle
1.0	0.	1.0	0.	1.0	0.	1.0	0.
0.420	-0.072π	3.612	-0.777π	1.222	-0.926π	1.167	1.000π
0.224	0.042π	2.962	0.194π	2.659	-0.763π	4.787	-0.448π
0.245	-0.274π	4.964	-0.457π	5.057	0.066π	33.041	0.175π

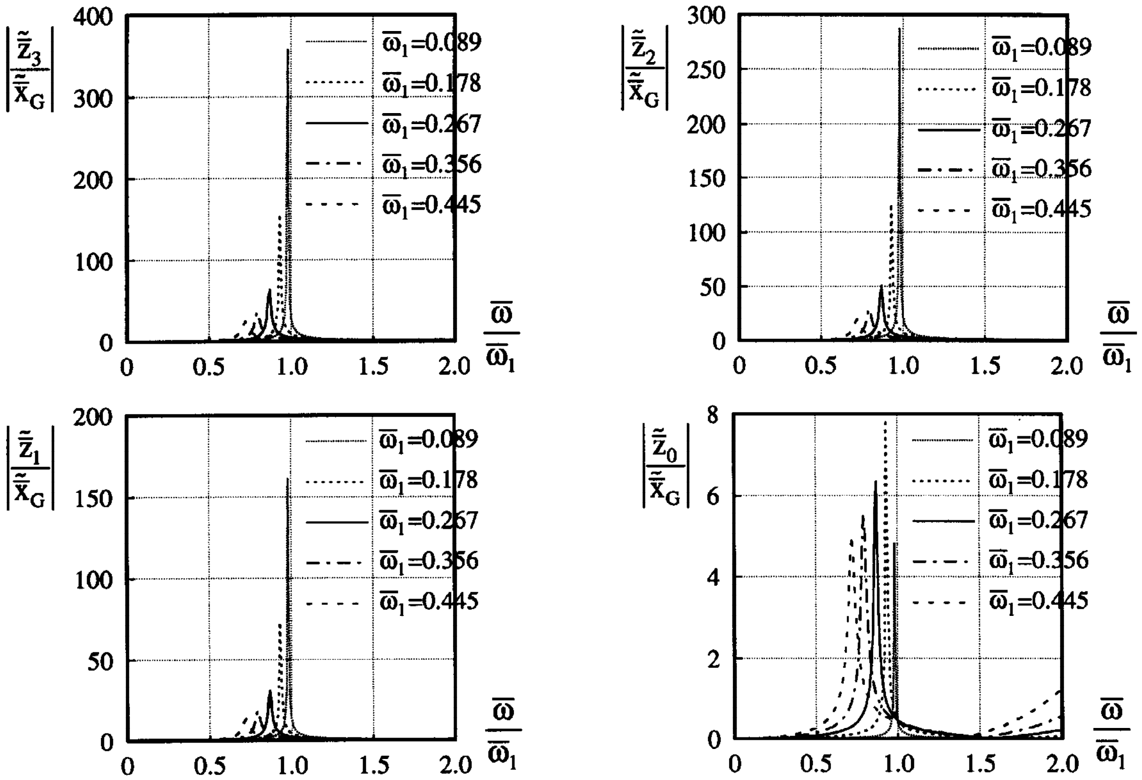
(b) Reduced method

<eigen values>

Real	Imag.	Real	Imag.	Real	Imag.	Real	Imag.
-0.05452	0.61011	-0.30932	1.21846	-0.42624	2.29953	-1.18078	1.70872

<eigen vectors>

Amplitude	Phase angle	Amplitude	Phase angle	Amplitude	Phase angle	Amplitude	Phase angle
1.0	0.	1.0	0.	1.0	0.	1.0	0.
0.421	-0.070π	3.226	-0.843π	1.150	0.924π	1.215	-0.999π
0.222	0.036π	2.572	0.097π	3.432	-0.836π	4.098	-0.446π



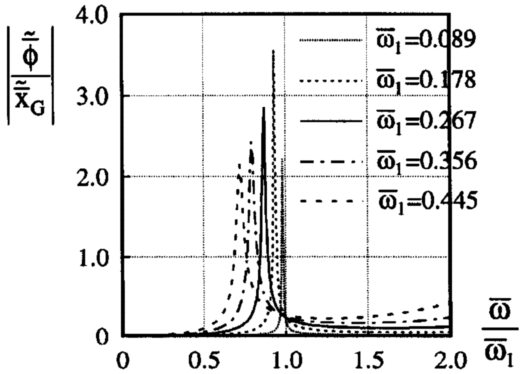


Fig.3 Relative response amplitudes of the upper structures having the uniform distributions of mass and stiffness. The first natural frequency $\bar{\omega}_1$ and the damping constant ζ are defined in the constructions fixed at the base :

$\bar{m}_i=3.2,\ \bar{h}_i=1,\ \zeta=0,\ \bar{b}=1.$

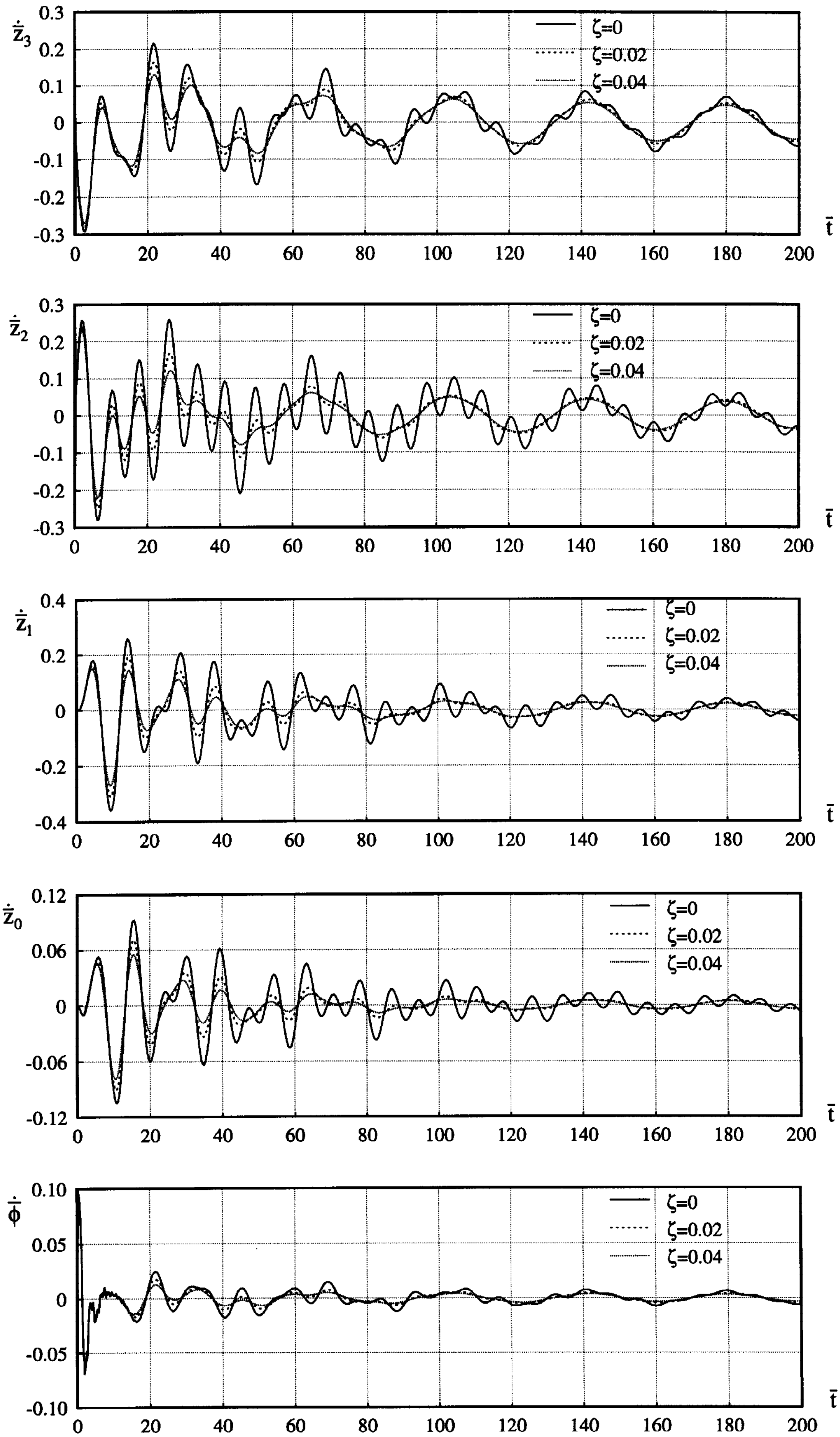


Fig.4 Transient responses of velocity under the initial conditions : $\bar{z}_3=1$.

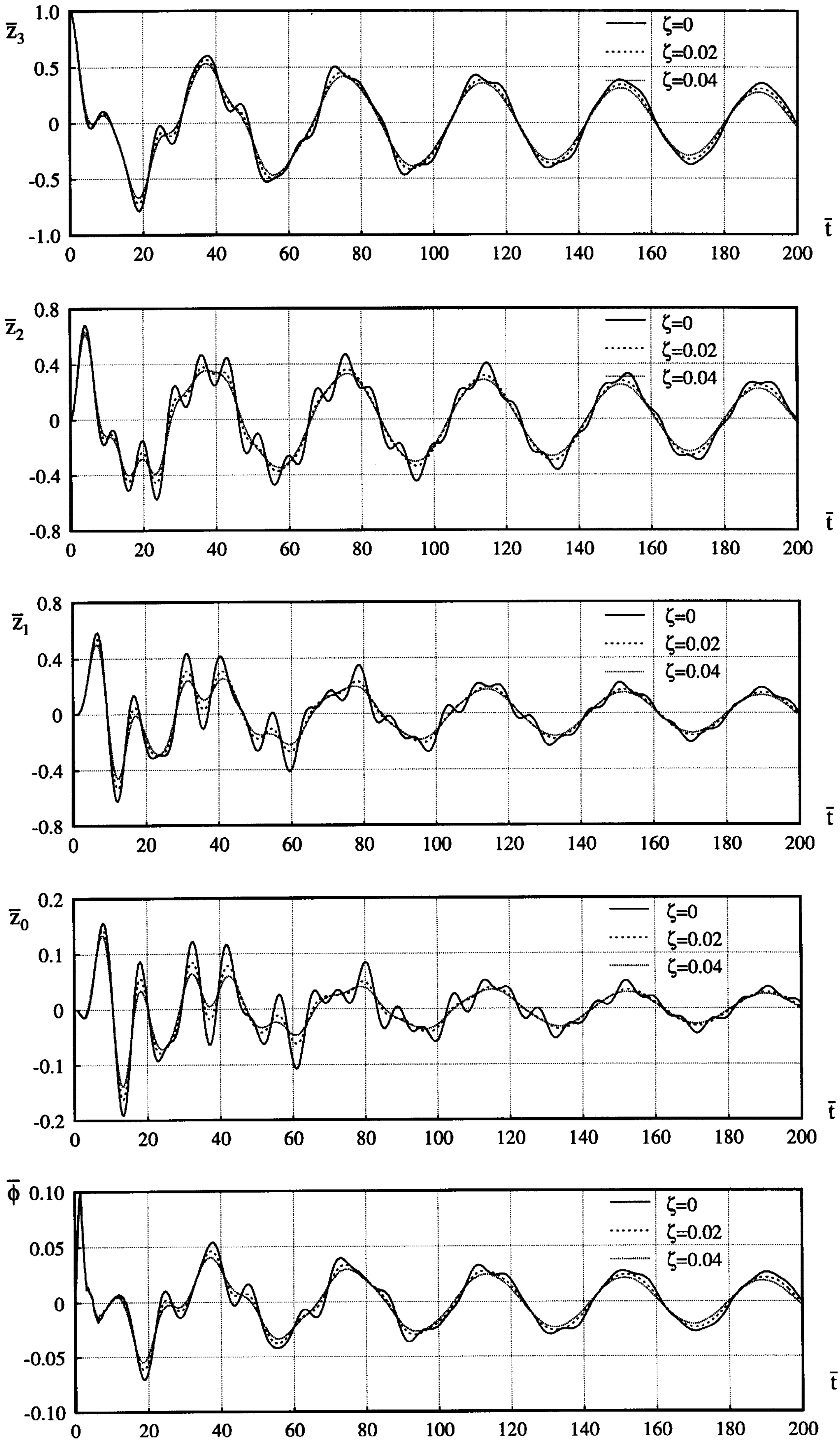


Fig.5 Transient responses of displacement under the initial conditions : $\bar{z}_3$=1.