# FLEXURE AND SHEAR BEHAVIOR OF HIGH STRENGTH REINFORCED CONCRETE COLUMN SUBJECTED TO HIGH AND FLUCTUATING AXIAL LOAD

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## ABSTRACT

This paper presents a macroscopic analysis model which is able to simulate not only flexural behavior but also shear behavior of reinforced concrete columns subjected to high or fluctuating axial load as well as horizontal load. Then, high strength reinforced concrete column specimens tested in the past were analyzed using the proposed model and the caluculated results were compared with the test results to investigate validity of the proposed model. Furthermore, parametric study was conducted using the proposed model and effect of such variable parameters as magnitude of axial force on strength and deformation capabilities of reinforced concrete columns was investigated.

## KEYWORDS

High strength reinforced concrete column; high axial force; fluctuating axial force; flexural deformation; shear deformation; macroscopic model; coupled model; cyclic load.

## INTRODUCTION

In recent years, technology for designing and constructing high-rise reinforced concrete(hereafter, refers to as "R/C") buildings has been actively developed. When buildings are under eathquake ground motion, high or fluctuating axial force may occur in columns at the lower stories or at the corner. For this reason, shear deformation, which is normally negligible, grows significantly in addition to flexural deformation and this leads to complex behavior. Therefore, it is an urgent business to develop a reliable macroscopic model for R/C columns, which can take shear defromation and mechanical properties of high strength materials into consideration, to investigate earthquake responses of such buildings. The authors(Lejano *et al.*, 1995) conducted the horizontal cyclic loading test on R/C column specimens under high or fluctuating axial load. Horizontal deformation of column spcimen was successfully decomposed into the flexural, shear and other components on the basis of measured deformations and strains. In the present study, models for flexural and shear behaviors of R/C columns shall be formulated independently and macroscopic model, which couples both models in series, is presented. In the next place, monotonic and cyclic horizontal loading analyses on the R/C column specimens are conducted using the proposed model and validity of the model is studied by comparing the calculated results with the test results. Furthermore, parametric analyses with variable parameters such as magnitude of axial force, amount of shear reinforcement and concrete strength are conducted and effect of these parameters on strength and deformation capabilities of R/C columns is investigated.


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## ANALYSIS MODEL FOR REINFORCED CONCRETE COLUMN

The R/C column, which is subjected to the horizontal and axial forces simultaneously as shown in Fig. 1, can be modeled as the system which couples the flexural spring expressing flexural behavior under bending moment(M) and axial force(N) with the shear spring expressing shear behavior under shearing force(Q) and axial force(N) in series. In this study, the former shall be referred to as "the flexure model", the latter "the shear model" and the combined system "the coupled model".

### Flexure Model

It is known that the flexural behaviors of R/C members can be simulated fairely well by the fiber method even under high or fluctuating axial force (Kanda *et al.*, 1988, Ono *et al.*, 1989). The authors *et al.*(Kanda *et al.*, 1988) developed the fiber analysis method. It was formulated on the basis of the modified Endochronic theory (Tanaka *et al.*, 1994) as the constitutive law for concrete, in which the original Endochronic theory(Bazant *et al.*, 1980) was extended so that it could be applied to high strength concrete, and the Ciampi's model(Ciampi *et al.*, 1982) for expressing cyclic stress–strain hysteresis relation of reinforcement. Furthermore, the method was applied to the analysis of felxure behaviors on the R/C column specimens under high or fluctuating axial force and validity of the method was investigated(Ono *et al.*, 1989). In this study, the fiber method stated in the above shall be adopted as the flexure model.

### Shear Model

With reference to the past test results(Lejano *et al.*, 1995), it is assumed that shear deformation of R/C columns is almost identical independent of positions along the member axis. Therefore, shear behavior of the R/C column can be simulated by solving the constitutive equation for R/C element, which is representative of the R/C column, under the combined state of axial and shear stresses. That is, axial and shear deformations of R/C column can be evaluated by multiplying axial and shear strain responses for the R/C element by an effective length of R/C column.

Constitutive law for the R/C element can be formulated on the basis of the following assumptions, equiliburium condition and compatibility condition;

(1) Reinforcement such as main bars and shear reinforcing bars is uniformly distributed over the concrete element, and thus the compatibility between steel strain $\{\varepsilon_s\}$ and concrete strain $\{\varepsilon_c\}$ is satisfied; $\{\varepsilon\} = \{\varepsilon_c\} = \{\varepsilon_s\}$. $\{\varepsilon\}$ stands for the strain vector of the R/C element.
(2) Cracks are allowed to occur in the multiple directions. Thus, the so-called "multi-directional smeared crack concept" (De Borst *et al.*, 1985, Rots, 1986) is adopted.
(3) The axis of principal stresses in concrete including cracks coincides with that of principal strains. This conditon of coaxiality, which allows the principal axis to rotate, leads to the so-called "rotating crack concept"(Rots, 1986).
(4) Average strain $\{\varepsilon_c\}$ in concrete including cracks can be defined as the sum of strain $\{\varepsilon_{co}\}$ in the solid part and strain $\{\varepsilon_{cr}\}$ in the cracking part. Hereafter, $\{\varepsilon_{co}\}$ shall be referred as the solid strain and $\{\varepsilon_{cr}\}$ as the crack strain.
(5) The tension stiffening and the shear transfer shall be considered to model behaviors in the cracking part.
(6) The modified Endochronic theory and the Ciampi's model shall be adopted as the constitutive laws for solid concrete and reinforcement as in the flexure model, respectively.

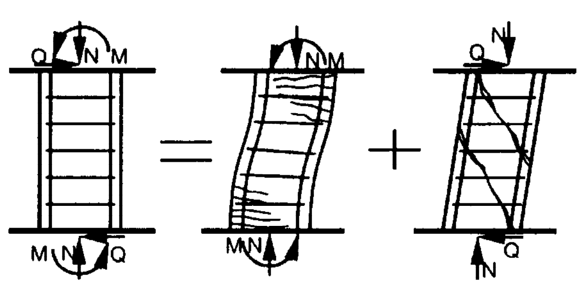



Fig. 1 Modeling concept of R/C column

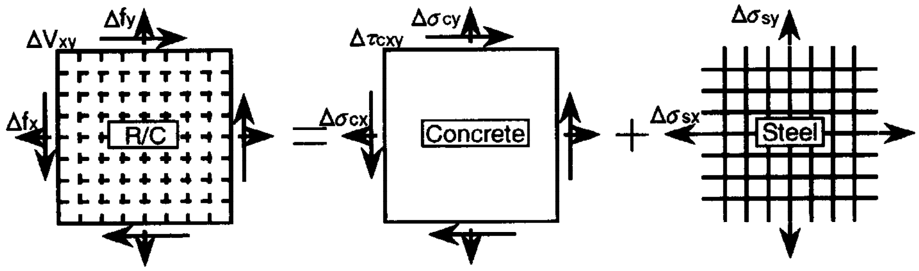



Fig. 2 Modeling of R/C column for shear analysis

The model proposed for simulating shear behavior of the R/C column is schematically shown in Fig. 2. The equilibrium condition of R/C element leads to the following relation;

$$\Delta f_x = \Delta\sigma_{cx} + p_s\Delta\sigma_{sx},\ \Delta f_y = \Delta\sigma_{cy} + p_w\Delta\sigma_{sy},\ \Delta v_{xy} = \Delta\tau_{cxy} \tag{1}$$

where, $\Delta f_x$, $\Delta f_y$ and $\Delta v_{xy}$ indicate the applied normal stress increments in the X and Y directions and the applied shear stress increment, $\Delta\sigma_{cx}$, $\Delta\sigma_{cy}$ and $\Delta\tau_{cxy}$ the normal stress increments in the X and Y directions and the shear stress increment in concrete, $\Delta\sigma_{sx}$ and $\Delta\sigma_{sy}$ the stress increments in reinforcement in the X and Y directions, and $p_s$ and $p_w$ the steel ratios in the X and Y directions. In the prsent specific case of R/C column, $\Delta f_x = N/A$, $\Delta f_y = 0$ and $\Delta v_{xy} = 1.5Q/A$ provided that the shear stress distribution across the depth of R/C column is parabolic. Where $N$ and $Q$ indicate the applied axial and shearing forces and $A$ the cross sectional area of R/C column.

The stiffnesses of concrete and steel elements can be coupled in parallel on the basis of the assumption of (1). In the present paper, only formulation of constitutive law for the concrete element shall be described. According to the Endochronic theory, the constitutive law for concrete before cracking can be written as follows;

$$\{\Delta\sigma_{co}\} + \{\Delta\sigma''_{co}\} = [D_{co\,xy}]\{\Delta\varepsilon_{co}\} \tag{2}$$

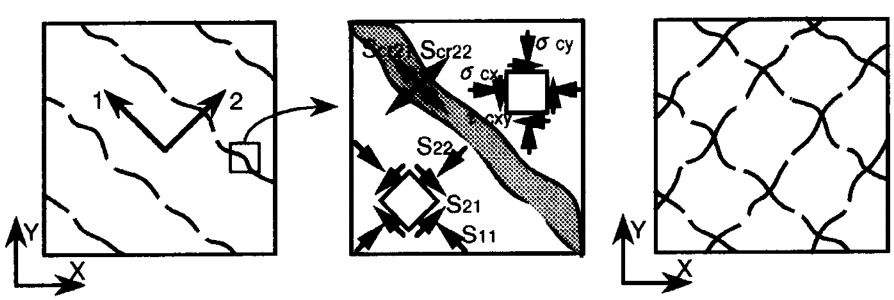
where $\{\Delta\sigma_{co}\}$, $\{\Delta\sigma''_{co}\}$ and $\{\Delta\varepsilon_{co}\}$ indicate the vectors of stress increment, inelastic stress increment and strain increment, and $[D_{co\,xy}]$ the material stiffness matrix in the X-Y coordinate system.

In the next place, if crack occurs, the average strain increment $\{\Delta\varepsilon_c\}$ in concrete including crack can be decomposed into the solid strain increment $\{\Delta\varepsilon_{co}\}$ and the crack strain increment $\{\Delta\varepsilon_{cr}\}$ on the basis of the assumption of (4) as;

$$\{\varepsilon_c\} = \{\varepsilon_{co}\} + \{\varepsilon_{cr}\} \tag{3}$$

The cracking and stress states in the concrete element can be schematically shown in Fig. 3. The directions of 1 and 2 mean the principal axes. At initial cracking, one of these principal axes coincides with the cracking direction. However, following the subsequent loading, the principal axis rotates and thus it deviates from the cracking direction. The rotation of principal axes result in creation of new crack whose direction is different from the initial one. The total crack strain increment $\{\Delta\varepsilon_{cr}\}$ in the concete element including multi-directional cracks can be defined as the sum of strain increments in the cracking parts of each direction as follows;

$$\{\Delta\varepsilon_{cr}\} = \Sigma[N_k]\{\Delta e_{cr,k}\} \tag{4}$$




a) Smeared crack in single direction b) Strain representation c) Smeared crack in two directions

Fig. 3 Multi-directional smeared crack concept where $\{\Delta\varepsilon_{cr}\} = \{\Delta\varepsilon_{cr\,x}, \Delta\varepsilon_{cr\,y}, \Delta\gamma_{cr\,xy}\}^T$ indicates the total crack increment, $\{\Delta e_{cr,k}\} = \{\Delta e_{cr22,k}, \Delta\gamma_{cr21,k}\}^T$ the crack strain increment in the $k$-th crack, $[N]$ the transformation matrix, $k(= 1, 2, \cdots, ncr)$ the crack number, $ncr$ the number of cracking directions. Similarly, the local stress increment corresponding to the crack strain increment in the $k$-th cracking part can be written as follows;

$$\{\Delta S_{cr,k}\} = [N]^T\{\Delta\sigma_{cr}\} \tag{5}$$

where $\{\Delta S_{cr,k}\} = \{\Delta S_{cr22,k},\ \Delta S_{cr21,k}\}^T$ indicates the local stress increment in the $k$-th cracking part, $\{\Delta\sigma_{cr}\} = \{\Delta\sigma_{cr\,x},\ \Delta\sigma_{cr\,y},\ \Delta\tau_{cr\,xy}\}^T$ the total stress increment in the cracking part with respect to the X-Y coordinate system.
The constitutive equation for the $k$-th cracking part can be written as follows;

$$\{\Delta S_{cr,k}\} + \{\Delta S''_{cr,k}\} = [D_{cr,k}]\{\Delta e_{cr,k}\}, \qquad [D_{cr,k}] = \begin{bmatrix} D_I & 0 \\ 0 & D_{II} \end{bmatrix} \tag{6}$$

where $\{\Delta S''_{cr,k}\} = \{\Delta S''_{cr22,k},\ \Delta S''_{cr21,k}\}^T$ indicates the inelastic crack stress increment, and $D_I$ and $D_{II}$ the tension stiffness and the shear stiffness in the cracking part.

Since the material stiffnesses of the solid and cracking parts in concrete are coupled in series from the assumption of (4), the total stress increment $\{\Delta\sigma_c\}$ in concrete including crack(s), the total stress increment $\{\Delta\sigma_{co}\}$ in the solid part and the total stress increment $\{\Delta\sigma_{cr}\}$ in the cracking part satisfy the following equiliburium condition;

$$\{\Delta\sigma_c\} = \{\Delta\sigma_{co}\} = \{\Delta\sigma_{cr}\} \tag{7}$$

Finally, substituting Eqs.(3) to (7) into Eq.(2), the constitutive equation for concrete element including cracks can be derived as follows;

$$\{\Delta\sigma_c\} = ([D_{co\,xy}]^{-1} + \Sigma[N_k][D_{cr,k}]^{-1}[N_k]^T)^{-1}(\{\Delta\varepsilon_c\} - \{\Delta\varepsilon_{cp}\}) \tag{8}$$

where $\{\Delta\varepsilon_{cp}\}$ is regarded as the inelastic strain increment corresponding to the inelastic stress increment $\{\Delta\sigma''_{co}\}$ in the solid part and the inelastic stress increment $\{\Delta S''_{cr,k}\}$ in the cracking part, and this can be defined as follows;

$$\{\Delta\varepsilon_{cp}\} = [D_{co\,xy}]^{-1}\{\Delta\sigma''_{co}\} + \Sigma[N_k][D_{cr,k}]^{-1}\{\Delta S''_{cr,k}\} \tag{9}$$

Furthermore, the total crack strain increment $\{\Delta\varepsilon_{cr}\}$ can be obtained by the following equation.

$$\{\Delta\varepsilon_{cr}\} = \Sigma[N_k][D_{cr,k}]^{-1}[N_k]^T\{\Delta\sigma_c\} \tag{10}$$

Now, let us discuss about modeling of the tension stiffness $D_I$ and the shear stiffness $D_{II}$ in the cracking part defined in Eq.(6). First, the tension stiffness is intended to express the tension softening of concrete itself and the tension stiffening due to the bond action between concrete and reinforcement. In the present study, the tension stiffness $D_I$ shall be evaluated on the basis of the tensile stress–strain hysteresis model, which was constructed by modifying the model proposed by Shirai(Shirai, 1978), as shown in Fig. 4. Consequently, $D_I$ can be defined in terms of the tangential modulus of solid concrete $E_c$ and the tangential stiffness $E$ on the hysteresis curve assumed in Fig. 4 as follows;

$$D_I = E_c \cdot E/(E_c - E) \tag{11}$$

In the next place, the shear stiffness $G_c$ of cracked concrete can be defined by applying the concept of rotating crack as follows;

$$G_c = (\sigma_{cx} - \sigma_{cy})/2(\varepsilon_x - \varepsilon_y) \tag{12}$$

Consequently, the shear stiffness in the cracking part $D_{II}$ can be defined in terms of the shear stiffness of solid concrete $G_{co}$ and the shear stiffness of cracked concrete $G_c$ as follows;

$$D_{II} = G_{co} \cdot G_c/(G_{co} - G_c) \tag{13}$$

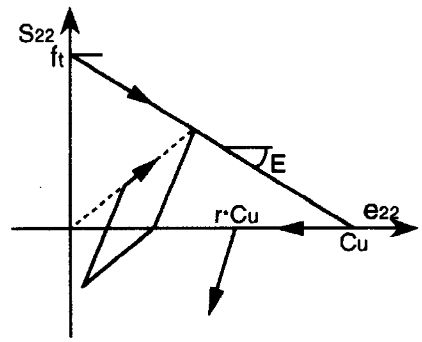



Fig. 4 Tension stiffning of cracked concrete

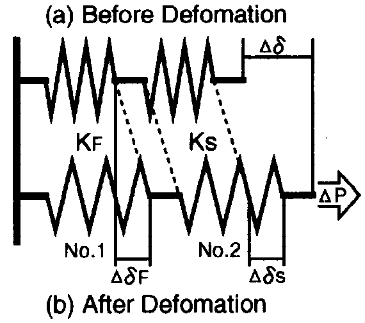



Fig. 5 Illustration of two-spring system

Coupled Model

The flexure and shear models were formulated independently in the above, and these models shall be coupled in series in this section. For simplicity, let us consider 2-spring system as shown in Fig. 5 which modeled the R/C column. The spring of No.1 expresses the flexural stiffness $K_F$ and the spring of No.2 the shear stiffness $K_S$. Since these springs are coupled in series, the following equilibrium of forces and compatibility of deformations are satisfied.

$$\Delta P = \Delta P_F = \Delta P_S, \quad \Delta\delta = \Delta\delta_F + \Delta\delta_S \tag{14}$$

where $\Delta P$, $\Delta P_F$ and $\Delta P_S$ indicate the incremental forces in the total system, the flexural spring and the shear spring, $\Delta\delta$, $\Delta\delta_F$ and $\Delta\delta_S$ the corresponding deformation increments. The internal force increments in each spring can be related to the deformation increments by the following equation.

$$\Delta P_F + \Delta P_F'' = K_F \Delta\delta_F, \; \Delta P_S + \Delta P_S'' = K_S \Delta\delta_S \tag{15}$$

where $\Delta P_F''$ and $\Delta P_S''$ indicate the unbalanced force increments due to nonlinearities of the flexural and shear springs. Substituting Eq.(15) into Eq.(14), the stiffness equation for the spring system can be derived as follows;

$$\Delta P = K\Delta\delta - \Delta P'', \; K = (K_F^{-1} + K_S^{-1})^{-1}, \; \Delta P'' = K(K_F^{-1}\Delta P_F'' + K_S^{-1}\Delta P_S'') \tag{16}$$

Up to now, the macroscopic model has been formulated by assuming that the resistance mechanisms for flexure and shear can be dealt with indepentently. However, there must be an interaction between them. In this study, it is assumed that cross sectional area of the R/C column, which is effective for the shear transfer, is reduced due to the existence of flexural cracks. Thus, the following "interaction factor $\Omega$ " shall be introduced to evaluate an effective area $A_e = \Omega A$.

$$\Omega = (V - V_{cr})/V \tag{17}$$

where $V$ indicates the total volume of R/C column and $V_{cr}$ the sum of volumes of fiber elements in which flexural cracks are open.

## ANALYSIS RESULTS

Verification of Proposed Model

In order to study validity of the proposed model, the R/C column specimens tested in the past are analyzed. The detail of bar arrangement and dimension of specimens are shown in Fig. 6 and the structural parameters are listed in Table 1(Lejano, 1995). C-2 specimen was tested under the cyclic horizontal loading as well as high constant axial force, and C-3 specimen was tested under the cyclic horizontal loading as well as fluctuating axial force. The shearing force–horizontal displacement response calculated under the monotonic horizontal load for C-2 specimen is compared with the test result in Fig. 7. The coupled model A in the figure indicates the result calculated by taking the interaction be-

Table 1 Structural properties of specimens

| Series | Main Reinf. Ratio Pg(%) | Shear Reinf. Ratio Ps(%) | Yield Strength fy (kgf/cm²) | Concrete Strength f'c (kgf/cm²) | Applied Axial Force | | |
|---|---|---|---|---|---|---|---|
| | | | | | Type | N(tf) | N/Nu(%) |
| C-2 | 3.94 | 0.89 | 5060 (D13) | 404 | Const. | 244.6 | 58 *a) |
| C-3 | (24-D13) | (4-D16 @52) | 3790 (D6) | 419 | Fluc. | -108.0 ~257.7 | -70~60 *b) *a) |

*a)Nu=0.85BD*fc+As*fy (ultimate compressive force) *b)Nu=As*fy (ultimate tensileforce) where As=PsBD

tween flexure and shear into consideration, and the coupled model B indicates the result calculated by taking no interaction into consideration. The calculated result by the model A agrees well with the test result. Fig. 8(a) shows the comparison between the calculated and observed shearing force–flexural deformation responses and Fig. 8(b) shows the comparison between the calculated and observed shearing force–shear deformation responses. The deformation components calculated by the model A agree well with the test results. On the other hand, the model B provides the different responses of deformation components from the test results. Figs. 9 and 10 show the comparisons between the calculated and observed hysteresis curves for C-2 and C-3 specimens. It can be said that the predicted results by the proposed model with interaction can simulate overall behaviors with practically reasonable accuracy, although the predicted hysteresis responses at large deformation level are slightly different from the test results.

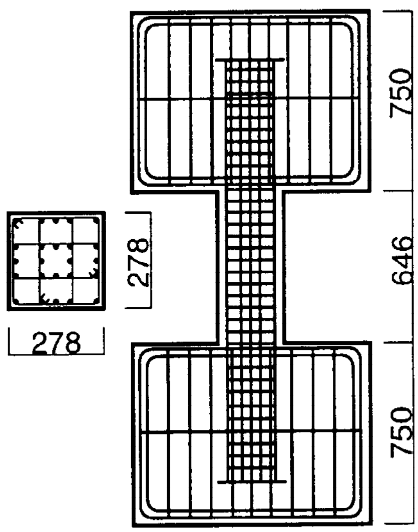



Fig. 6 Detail of specimens

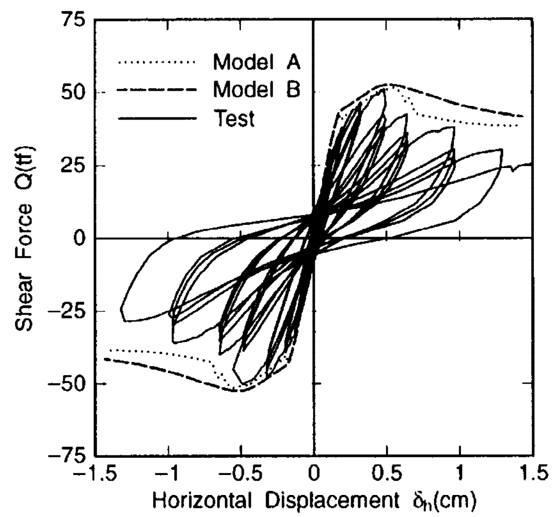
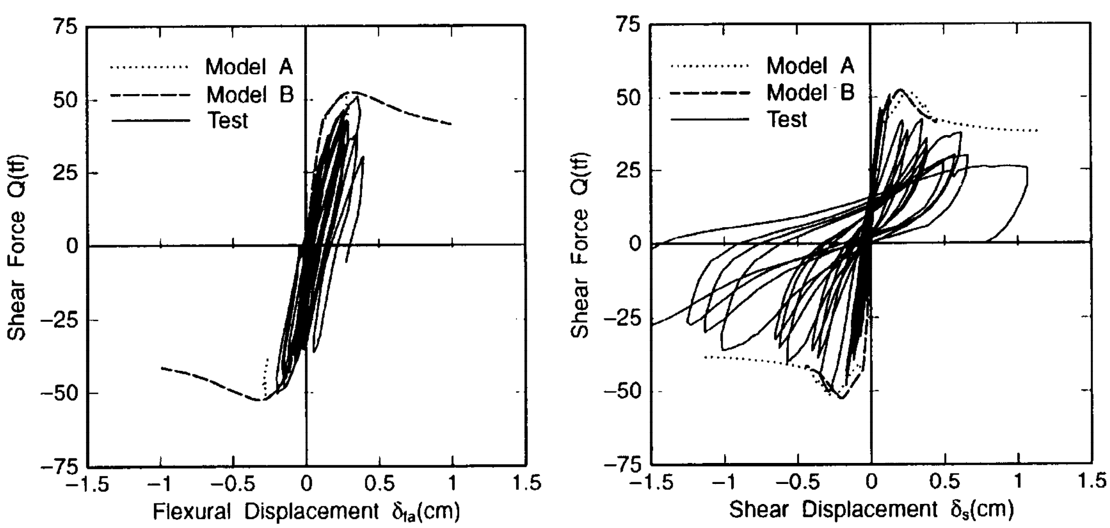



Fig. 7 Monotonic force-displacement curve of C-2




a) Flexural displacement b) Shear dsplacement

Fig. 8 Defomation components of C-2

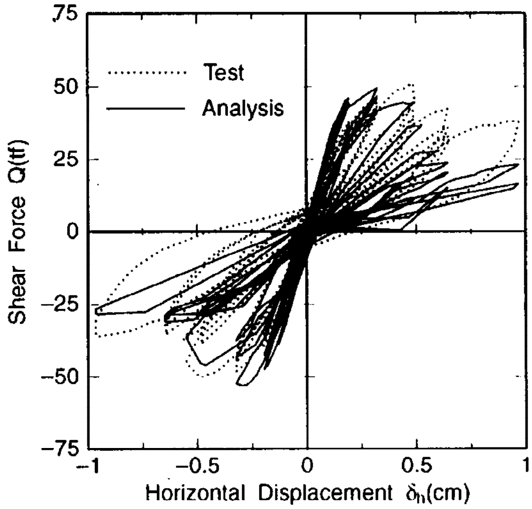



Fig. 9 Hysteresis curve of C-2

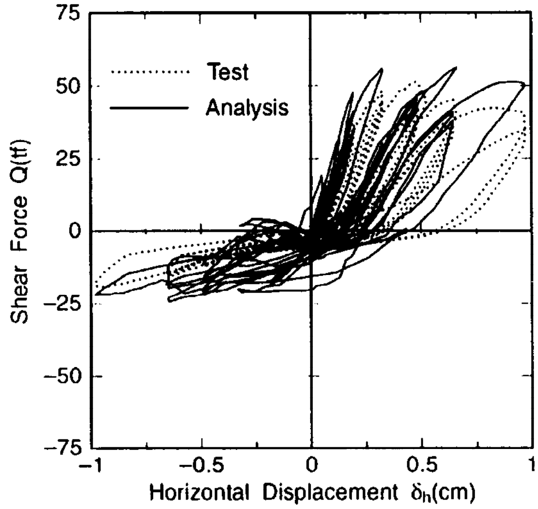
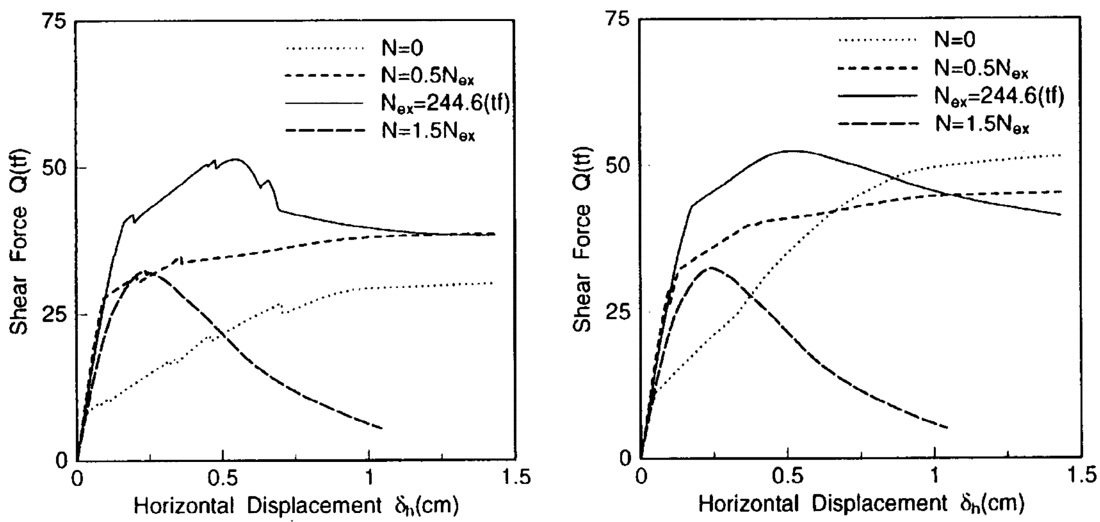


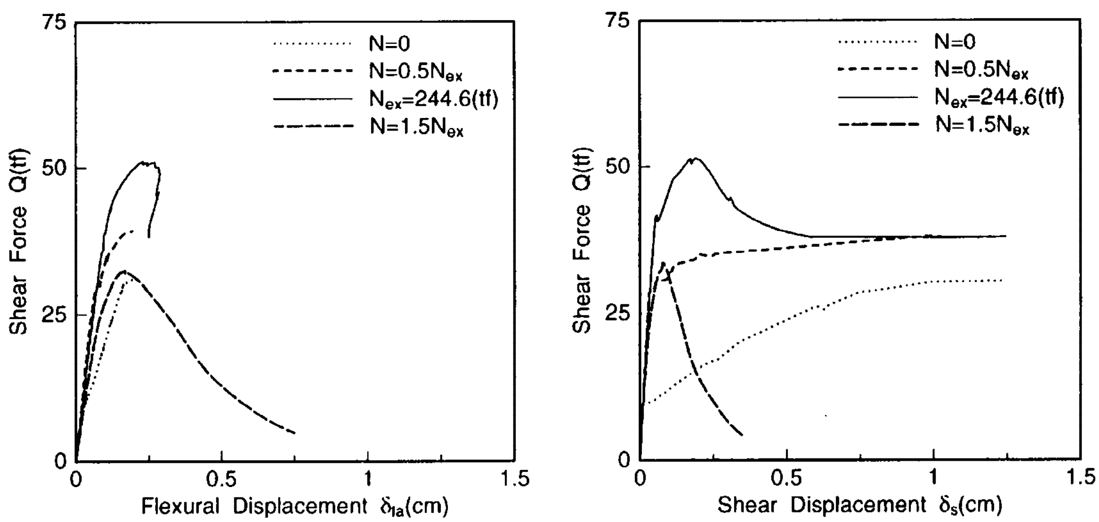

Fig. 10 Hysteresis curve of C-3




a) Interaction considered

b) Interaction not considered

Fig. 11 Effect of axial force on Q-$\delta_h$ curve




a) Flexural displacement

b) Shear displacement

Fig. 12 Effect of axial force on defomation components

Parametric Analyses

In this section, parametric analyses by the proposed model shall be conducted using C-2 as the reference specimen. The parameters to be studied are (1) magnitude of axial force, (2) amount of shear reinforcement and (3) concrete strength. Fig. 11 shows the comparison between the calculated and observed shearing force–horizontal deformation responses when the magnitude of applied axial force is varied. Note that $N_{ex}$=22.6tf indicates the magnitude of axial force applied in the test. Fig. 12(a) compares the calculated shearing force–flexural deformation responses and Fig. 12(b) the calculated shearing force–shear deformation responses. It is seen that the axial force has significant influence on strength, ductility and fracture mode of R/C columns. Next, the analysis was conducted on the R/C column in which amount of shear reinforcement was varied. The results indicated that lesser shear reinforcement caused brittle behaviors and on the other hand with more reinforcement the shear strength was increased and also the ductility was improved. Finally, effect of concrete strength on behaviors of R/C columns was studied. It was found that concrete stength had significant influence on the resistance mechanism of R/C columns.

## CONCLUSIONS

The following findings were obtained through the numerical studies on R/C columns by the proposed model.

(1) The proposed model can simulate load–deformation responses of R/C columns under high or fluctuating axial force as well as horizontal cyclic load with reasonable accuracy.
(2) The interaction between flexure and shear must be considered to simulate both behaviors adeqately and the concept of interaction factor proposed is effective.
(3) Magnitude of applied axial force has significant influence on strength, ductility and fracture mode of R/C columns.
(4) Although there is the upper limit, increase in shear reinforcement increases strength and ductility of R/C columns.
(5) Making concrete strength higher, increase in strength of R/C column is expected. However, amount of shear reinforcement must be increased at the same time to guarantee flexural fracture mode with sufficient ductility.

## ACKNOWLEDGEMENT

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