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**Paper No. 1288. (quote when citing this article)**
Eleventh World Conference on Earthquake Engineering
ISBN: 0 08 042822 3

# ON THE USE OF STRAIN SOFTENING FOR PREDICTION OF POST-PEAK SEISMIC RESPONSE OF STRUCTURES

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## ABSTRACT

Analytical prediction of response after strain localization or cracking for static problems is being increasingly investigated by employing strain softening plasticity. In this study the authors explore the possibility of using strain softening elasto-plasticity for the prediction of post-peak seismic response. Numerical implementation of strain softening has been known to cause problems of convergence, load step sensitivity and discretization sensitivity (or mesh sensitivity). Many of these difficulties have been surmounted for static analysis. This study highlights the numerical problems associated with the use of strain softening in the solution of dynamic problems and suggests methods of overcoming them. The results indicate that dynamic response does not become unbounded due to strain softening. Strain softening, however, introduces a larger zero frequency component as compared to strain hardening or perfect plasticity. The frequency content at frequencies other than zero is not significantly altered.

## KEYWORDS

Seismic response; strain softening; elastoplastic; post-peak; localization; load step sensitivity.

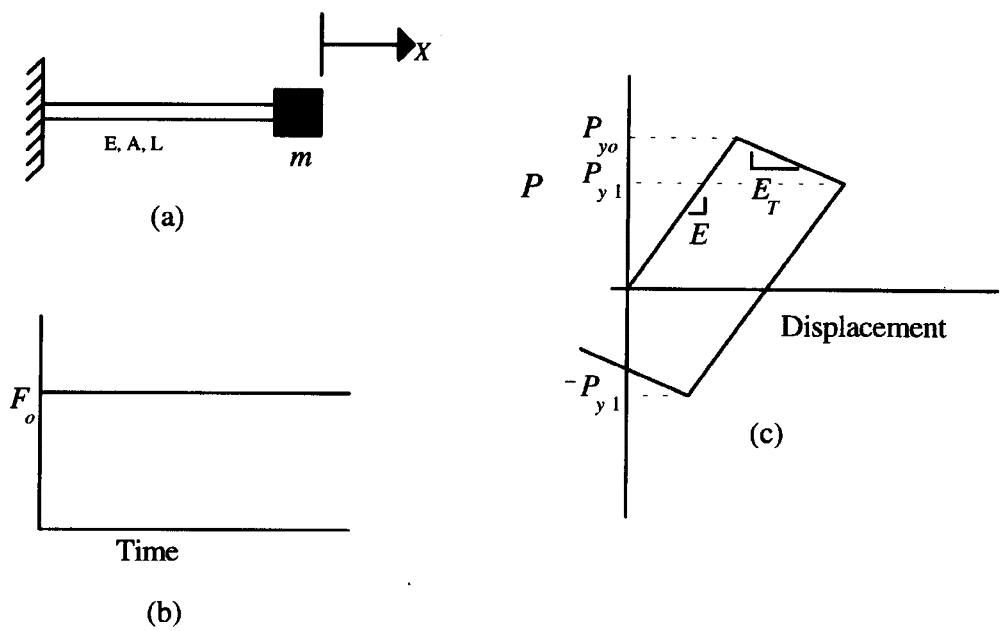
## LOAD STEP SENSITIVITY

The strategy adopted in the past for incremental/iterative stress updating (Owen and Hinton, 1980) led to accumulation of spurious plastic strains. This was earlier termed as path dependent behaviour (Mondkar and Powell, 1978; Marques, 1984) and later, perhaps more appropriately, load step sensitivity (Bicanic and Pankaj, 1990) or problem of spurious plastic strain (Ramm and Matzenmiller, 1988; Crisfield, 1991). While the use of a wrong strategy can lead to spurious plastic strains in all plasticity problems the effect is more pronounced for softening problems. Except for some investigations (Moin and Pankaj, 1994; Pankaj, Moin and Barthwal, 1994) research in this regard has generally been confined to static problems where two methodologies have emerged.

In the first conventional method (Strategy A), stresses are updated at the end of each iteration based on the strain increment computed for that iteration. In the second (Strategy B), stress increment is computed for all strain increments accumulated upto that iteration and the stresses are updated only after the iteration process has converged.

These methodologies were incorporated for dynamic analysis in a computer program that used Newmark's unconditionally stable direct integration algorithm for the solution of elastoplastic problems (Pankaj, Moin and Barthwal, 1994). The initial stiffness approach was employed. It was seen that the presence of acceleration dependent inertia forces and velocity dependent damping forces do not impose any additional complexity when Strategy B called the dynamic total residual strategy is employed. A simple problem was numerically solved to study the difference between the two strategies.

A bar element with a concentrated mass at one end as shown in Fig. 1a assumed to be undergoing axial vibration was assumed to constitute an undamped single degree freedom system. A step function load as shown in Fig. 1b was assumed to act on the mass of the system. The idealized elastic strain softening plastic load displacement material behaviour is shown in Fig. 1c. The exact solution for this problem is available (Moin and Pankaj, 1994).




$m = 1,\ A = 1,\ L = 1,\ E = 1,\ E_T = 0.1,\ P_{yo} = 15,\ F_0 = 10$

Fig. 1 (a) Undamped SDF system
(b) Step function load and
(c) Idealized load displacement behavior of the spring.

The displacement response comparison using the two strategies is shown in Fig. 2. It can be seen that for the smallest time step ($\Delta t = 0.01$sec) the numerical procedures match well with the exact solution. The difference between the exact solution and the numerical solutions becomes larger with the increase in step size. In each case the results from the dynamic total residual strategy are closer to the exact solution. A study of the variation of plastic strain with time shows that greater spurious plastic strain results when Strategy A is adopted.

A single degree freedom system would yield the same results for static problems irrespective of the choice of solution strategy. In fact for static problems the difference in the results from the two procedures emerges due to inter element stress adjustments. At the first sight it is surprising to see a difference of such magnitude for a single degree of freedom system in case of dynamic problems. This apparently is due to the inter dependence of displacement, velocity and acceleration vectors. Thus the importance of the choice of the total residual strategy for dynamic analysis cannot be overemphasized.

## SEISMIC RESPONSE

### *SDF System*

The base of a single degree of freedom (SDF) system of Fig. 1a with $K$=4100 N/mm and $m$=100 Kg was subjected to an actual earthquake acceleration history. The corrected accelerogram of the Uttarkashi earthquake of October 20, 1991 obtained at 30.738N and 78.792E (Earthquake Engineering Studies, 1993) was used for the purpose. A constant viscous damping of 5% of the critical was considered and the bar was assumed to be (a) Elastic, (b) Perfectly plastic and (c) Strain softening. For the strain softening case $E_T$= 410 N/mm was assumed. The response was computed using the total residual strategy and Newmark algorithm with $\Delta t$= 0.02 sec. The yield force value $P_{yo} = 1.23\times10^5$ N was assumed. The displacement response is shown in Fig. 3. It can be seen that for nonlinear cases the mass does not vibrate about the zero displacement position. The Fourier analysis (Fig. 4) of the response indicates that in general, the predominant frequency content does not change with the change in post-elastic constitutive behaviour. Some low frequency components, however, appear to have been added to the response. Moreover, due to the mass finding new mean position to vibrate about, a zero frequency component is also seen in the response (Fig. 4). These changes of the mean position take place with the accumulation of the plastic displacements, which happens in a short time and remains constant thereafter (Fig. 5). The maximum plastic displacement for this example was found to be 48.56 mm and 54.94 mm for perfectly plastic and softening cases respectively.

It is interesting to see that elastoplasticity does not alter the predominant frequency response of the system. It, however, introduces a zero frequency and some low frequency components in the response. It is also seen that strain softening plasticity does not lead to an unbounded response and can be used with dynamic problems.

### *Koyna Dam Analysis*

Strain softening has been utilized in static analysis for prediction of strain localization or cracking. In order to explore the possibility of using strain softening for prediction of cracking in a continuum under dynamic loads the non overflow section of the Koyna dam which experienced an earthquake on Dec. 11, 1967 was analysed. The structure was idealized using 136 eight noded isoparametric elements. The dam section was assumed to be homogeneous with $E = 31.005\times10^6\,\mathrm{KN/m^2}$, unit weight $\rho = 2.442\ \mathrm{KN\,sec^2/m^4}$ and Poisson's ratio $\nu = 0.2$ (Chopra and Chakrabarty, 1971). Damping was assumed to be 5% of critical. Isotropic strain softening plasticity using Mohr Coulomb yield function was employed to represent post-yield material behaviour. The cohesion $c = 7071\ \mathrm{KN/m^2}$ and friction $\phi = 62.73^\circ$ were assumed (Owen and Hinton, 1980). A linear post-yield softening modulus of 10% of $E$ was also assumed. The dam was subjected to the horizontal component of Koyna earthquake (Krishna, Chandrasekaran and Saini, 1969). The principal strain plot at an instant when maximum principal strain (anywhere in the dam) is observed is shown in Fig. 6a. Tensile strains are shown using double lines and compressive strains using single lines. Large strains can be seen to be confined to a localized region. Figure 6b shows the regions that have undergone some amount of permanent plastic strain at the end of the excitation. Localization is seen to be confined to small regions on upstream and downstream faces. These simulations match well with the actual cracks that were observed after the earthquake (Chopra and Chakrabarty, 1971).

## CONCLUSIONS

Strain softening in the context of elastoplasticity appears to have the potential for predicting strain localization or cracking in seismic problems. The results do not become unbounded and generally the frequency content is not significantly altered. It is, however, important to use the correct stress updating strategy to prevent spurious plastic strains.

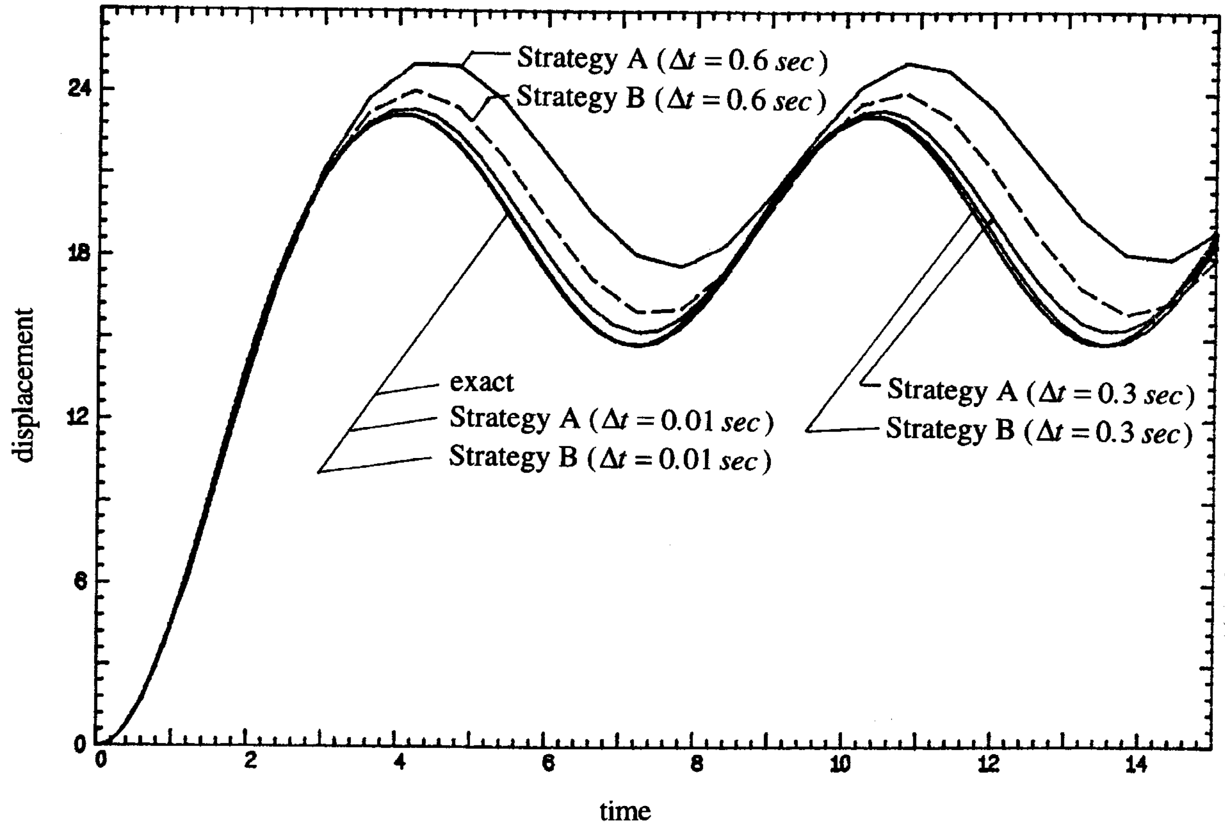



Fig. 2. A comparison of two stress updating strategies.

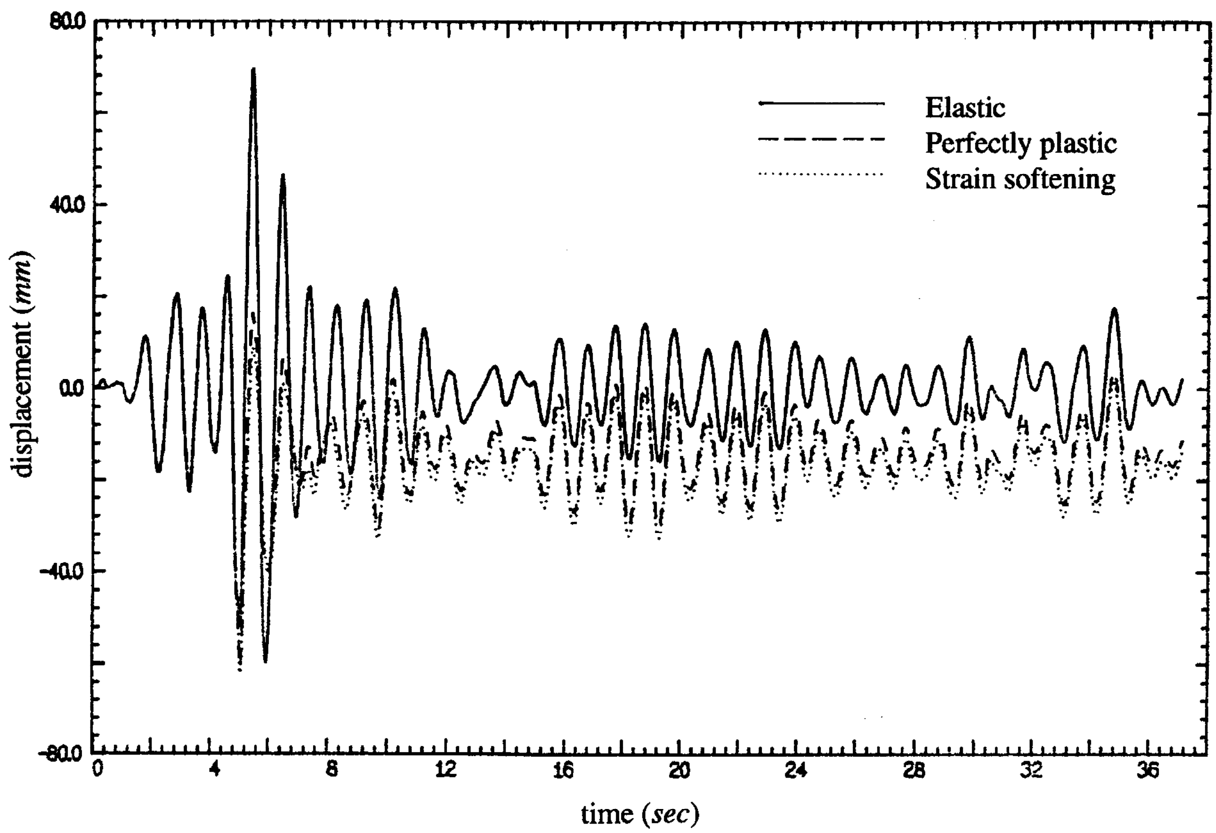



Fig. 3. Displacement response to Uttarkashi earthquake.

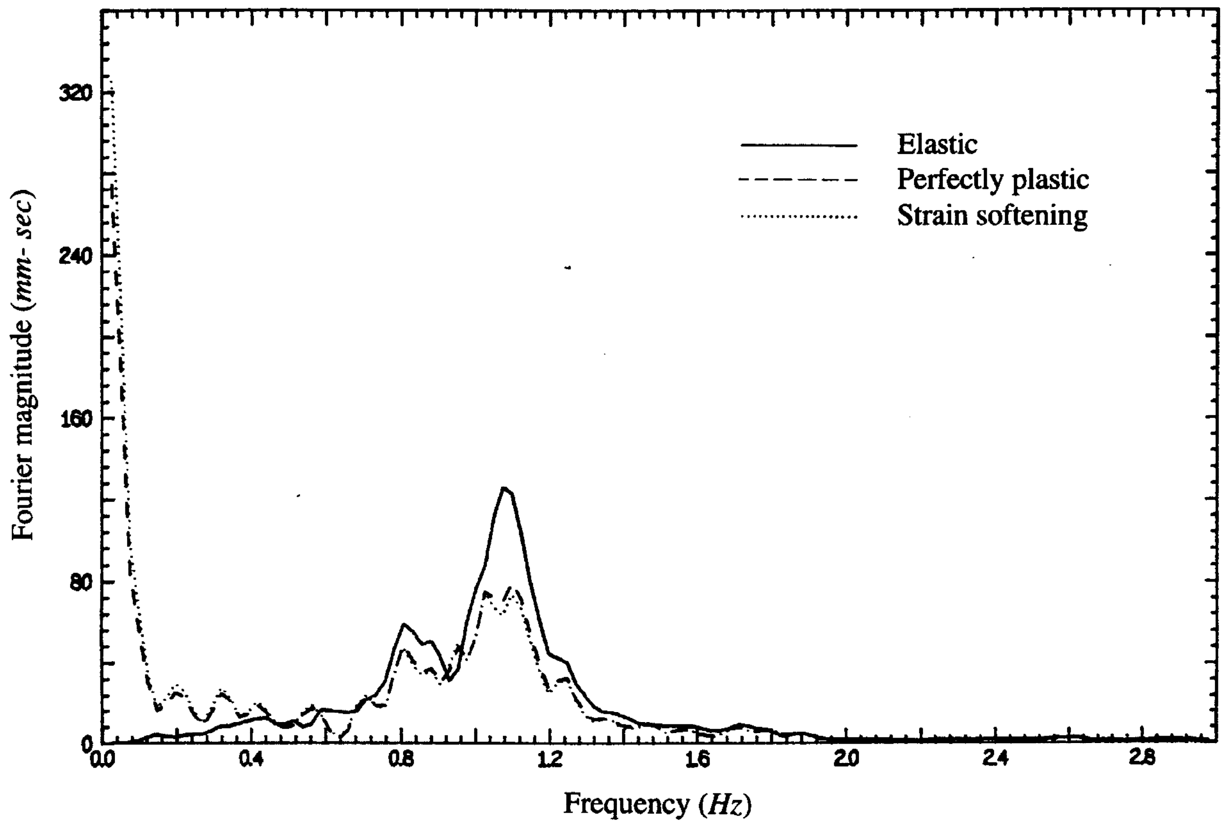



Fig. 4. Fourier magnitude plot of displacement response.

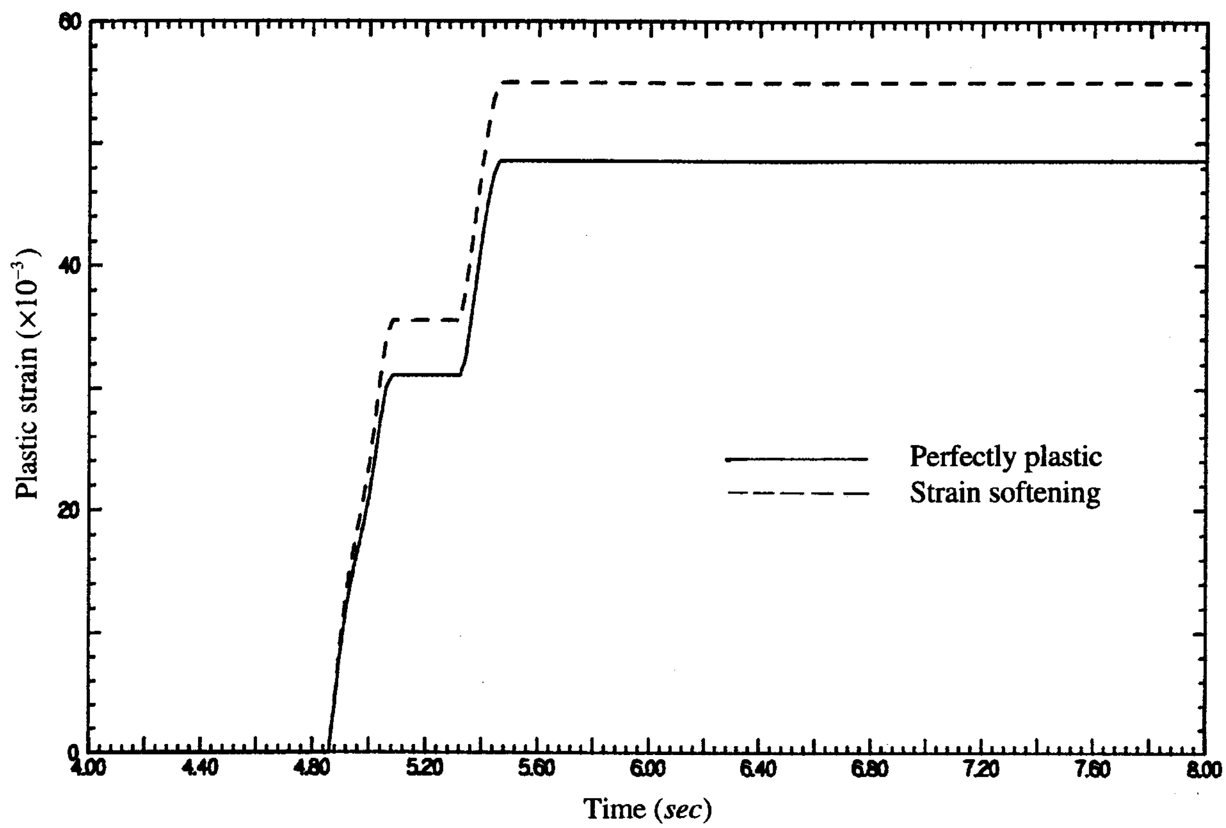



Fig. 5. Plastic strain variation with time.

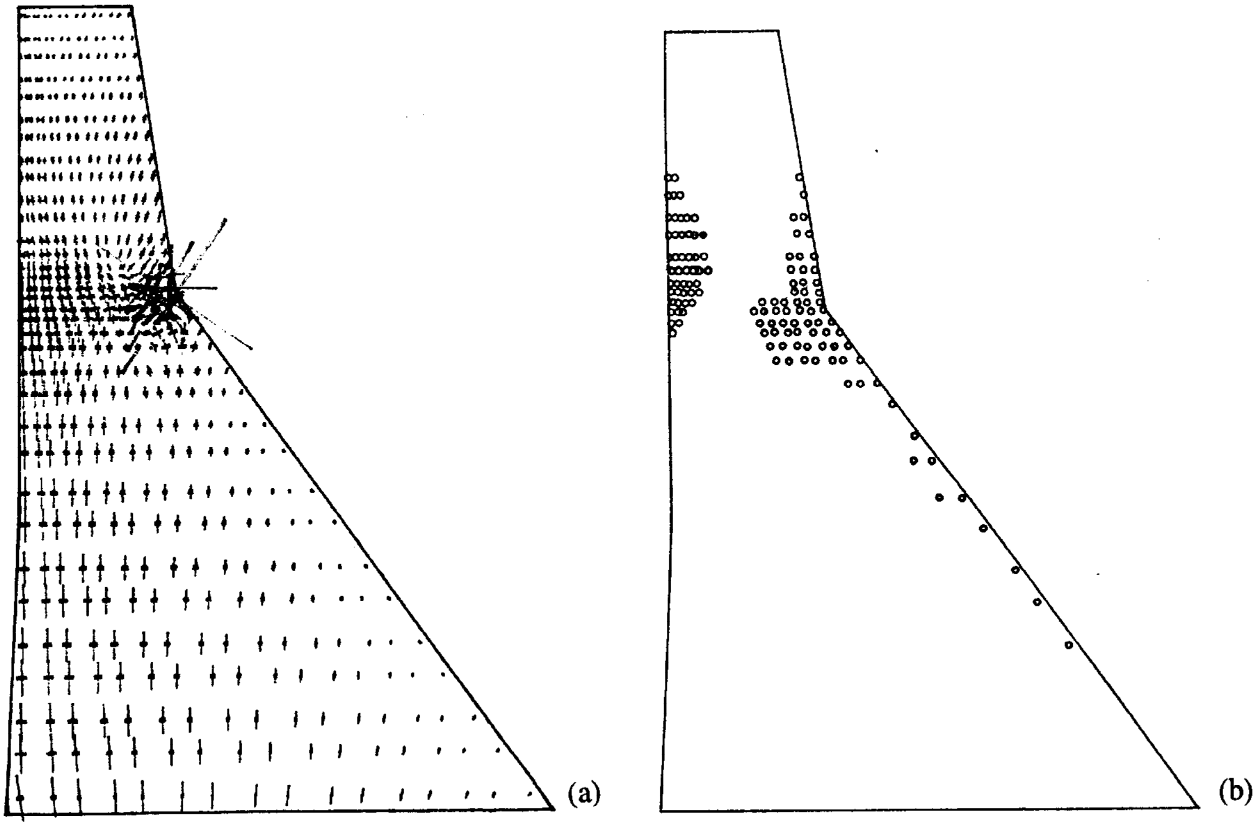



Fig. 6. (a) Principal strain plot, (b) Yielded Gauss points.

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