



## **EARTHQUAKE ISOLATION OF EQUIPMENTS INSTALLED ON FLOORS OF TALL BUILDINGS**

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### **ABSTRACT**

Two analysis models are presented for earthquake-isolation systems of equipments employed in the tall buildings. The response of the isolated equipments installed on floors of tall building to earthquakes is analysed by random vibration approach. Aiming at the optimization of limiting the absolute acceleration and/or relative displacement of equipments in tall buildings induced by earthquakes, numerical studies of 1 degree-of-freedom(DOF) and 2 DOF equipment-isolator systems are conducted. Useful results are obtained for the design of earthquake-isolation of tall building equipments.

### **KEYWORDS**

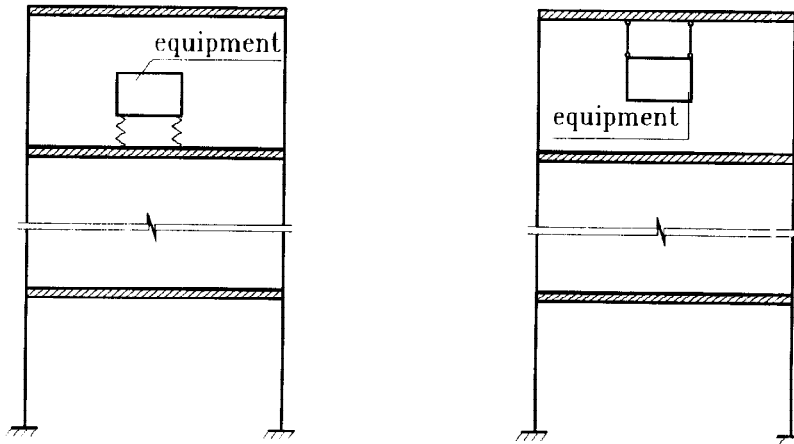
Earthquake isolation, equipments in tall buildings, random vibration.

### **INTRODUCTION**

The more developed the society, the more important and expensive the equipments employed in the tall buildings. That is the reason why more and more attention is paid to earthquake-induced destruction control of tall building equipments over the world. Although it is convenient to install equipments in rigid or simple supports, they may suffer damage or stopping work due to sever absolute accelerations or relative displacements caused by earthquakes. One effective way to reduce and limit the seismic response of equipments installed on floors of tall buildings is isolation.

The earthquake energy from foundation of tall buildings is first transmitted to floors, then to equipments. So, there are two means of earthquake isolation of equipments used for the service of tall buildings. One is the base or suspension isolation of tall buildings. The other is the base or suspension isolation of equipments (see Fig. 1 ), interfering the transmission route of vibration energy from floors to equipments. Obviously, the easier mean is the latter (Inaudi, 1993).

The seismic behaviour of equipments installed on floors of tall buildings is not only affected by the characteristics of equipments and ground movements of earthquakes, but also affected by the structural dynamic characteristics of tall buildings and the location of the floor on which equipments are supported. It is clear that there are specialties in earthquake isolation of equipments in tall buildings.



**Fig. 1 Earthquake-isolation of the equipment in tall buildings**

This paper is going to discuss the modeling of the equipment-isolation system. Formulations for predicting the random seismic response of isolated equipments mounted on or suspended from floors of tall buildings are presented. The parameter investigations are conducted for the optimum design of the isolation systems.

### **MODELING FOR ANALYSIS**

In general, the mass of tall buildings is much greater than that of equipments, and the mass and stiffness of equipments are much larger than those of isolators (e.g. , steel springs, steel rollers and rubber blocks). So, it is reasonable to assume that (a) the isolated equipment is a rigid body, (b) the mass of the isolator employed is negligible, and (c) the vibration of equipments has no effect on the natural vibration modes of tall buildings.

A rigid body has 6 degrees of freedom, i.e., 3 translational movements and 3 rotational movements. Generally, these 6 movements are coupled. In this case, the movements of the body representing a equipment are very complicated since an arbitrary movement in one direction will cause the movements in all other directions. For the easiness of controlling the vibration movements of equipments induced by earthquakes, the uncoupling of the movements in multi-directions is favourable. It is possible to do this by proper layout of isolators and restrainers. If a equipment-isolator system is completely uncoupled, the movements of the equipment in all 6 directions are independent. The system can then be simplified by a number of single-degree-of-freedom system

and analysed by the model shown in Fig. 2a .

If it is hard to meet the complete uncoupling conditions, the partial uncoupling of the system is relatively easy to achieve in practice. The typical partial uncoupling situation is that the translational movement in one horizontal direction is only coupled with the rotational movement in the vertical plane containing the traslational direction. In this case, the system can be modeled by a 2-DOF system, which is shown in Fig. 2b .

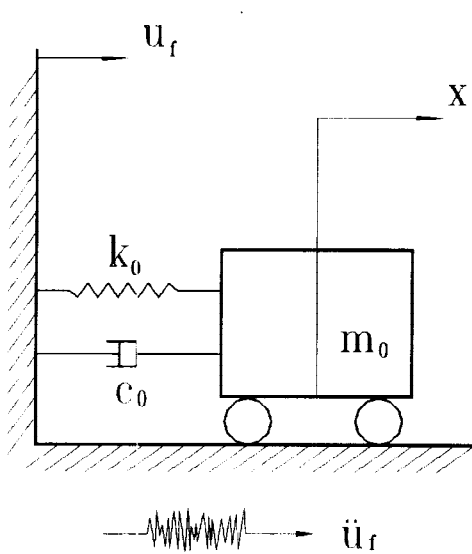
The motion equation of the single DOF equipment-isolator system is

$$\ddot{\delta} + 2\xi_0\omega_0\dot{\delta} + \omega_0^2\delta = -\ddot{u}_f \quad (1)$$

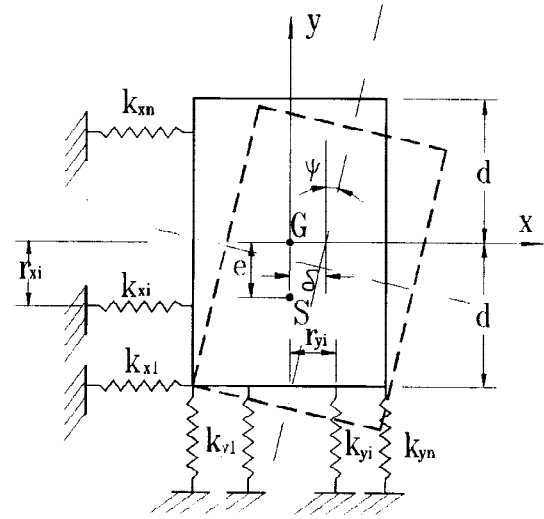
in which

$$\omega_0 = \sqrt{\frac{k_0}{m_0}} \quad , \quad \xi_0 = \frac{c_0}{2\sqrt{k_0 m_0}}$$

where  $m_0$  is the mass of the equipment;  $k_0$  ,  $c_0$  are the stiffness and damping coefficient of the isolator respectively;  $\ddot{u}_f$  is the absolute acceleration of the floor on which the equipment is installed, and  $\delta$  is the relative displacement of the equipment to the floor.



(a) Single DOF system



(b) Two DOF system

**Fig. 2 Analysis model of the equipment-isolator system**

The motion of the 2-DOF equipment-isolator system is governed by

$$[M] \begin{Bmatrix} \ddot{\delta} \\ \ddot{\psi} \end{Bmatrix} + [C] \begin{Bmatrix} \dot{\delta} \\ \dot{\psi} \end{Bmatrix} + [K] \begin{Bmatrix} \delta \\ \psi \end{Bmatrix} = -[M] \begin{Bmatrix} \ddot{u}_f \\ 0 \end{Bmatrix} \quad (2)$$

in which

$$[M] = \begin{bmatrix} m_0 & 0 \\ 0 & J_0 \end{bmatrix}$$

$$[C] = \begin{bmatrix} c_x & c_{x\psi} \\ c_{\psi x} & c_\psi \end{bmatrix}$$

$$[K] = \begin{bmatrix} k_x & k_{x\psi} \\ k_{\psi x} & k_\psi \end{bmatrix}$$

$$k_x = \sum_{i=1}^n k_{xi} \quad , \quad c_x = \sum_{i=1}^n c_{xi}$$

$$k_{x\psi} = -\sum_{i=1}^n k_{xi} r_{xi} = k_{\psi x} \quad , \quad c_{x\psi} = -\sum_{i=1}^n c_{xi} r_{xi} = c_{\psi x}$$

$$k_\psi = \sum_{i=1}^n k_{xi} r_{xi}^2 + \sum_{i=1}^m k_{yi} r_{yi}^2 \quad , \quad c_\psi = \sum_{i=1}^n c_{xi} r_{xi}^2 + \sum_{i=1}^m c_{yi} r_{yi}^2$$

where  $\delta$  is the relative displacement of the equipment at its mass center to the floor;  $\psi$  is the rotation of the equipment around its mass center;  $k_{xi}$ ,  $c_{xi}$  are the stiffness and damping coefficient of an isolator in x-direction respectively;  $k_{yi}$ ,  $c_{yi}$  are the stiffness and damping coefficient of an isolator in y-direction respectively;  $r_{xi}$ ,  $r_{yi}$  represent the positions of the isolators in x-direction and y-direction respectively (see Fig. 2b); and  $n$ ,  $m$  are the number of the isolators in x and y-direction respectively.

## RANDOM VIBRATION ANALYSIS

The ground movement of earthquakes is very irregular, which may appropriately be characterized as a stochastic process. Subjected to random seismic motions, the vibration response of equipments in tall buildings will also be stochastic. Since the variance of a random vibration indicates the average intensity of oscillation, variance of the random seismic response of a isolated equipment can be suitably used as the index of the goodness of the isolators employed. So, the aim of isolation is to reduce the variance of the random seismic response of equipments in tall building as much as possible by choosing the most favourable parameters of isolators.

The random vibration theory gives

$$S_e(\omega) = |H_e(\omega)|^2 S_{\ddot{u}_f}(\omega) \quad (3)$$

in which  $S_e(\omega)$  is the power spectral density of the equipment;  $H_e(\omega)$  is the frequency response function of the equipment; and  $S_{\ddot{u}_f}(\omega)$  is the power spectral density of the absolute acceleration of the floor, on which the equipment supported, given by

$$S_{\ddot{u}_f}(\omega) = [1 + \sum_{L=1}^N \phi_{iL} H_L(\omega)]^* [1 + \sum_{L=1}^N \phi_{iL} H_L(\omega)] S_{\ddot{u}_g}(\omega) \quad (4)$$

where

$$H_L = \frac{-\gamma_L \omega^2}{(\omega_L^2 - \omega^2) + j2\xi_L \omega_L \omega} \quad , \quad \gamma_L = -\frac{\sum_{i=1}^N \phi_{iL} m_i}{\sum_{i=1}^N \phi_{iL}^2 m_i}$$

the subscript \* denotes conjugation;  $j = \sqrt{-1}$ ;  $\omega_L$ ,  $\xi_L$  are the  $L$ th natural circular frequency and damping ratio of the tall building respectively;  $\phi_{iL}$  is the component of  $L$ th mode of the building at position of the  $i$ th floor;  $m_i$  is the mass of the  $i$ th floor; and  $S_{\ddot{u}_g}(\omega)$  is the power spectral density of the ground acceleration, the formula of which proposed by Kanai(1957) and Tajimi(1960) can be employed.

Conducting Florier transform of Eq. 1 , the frequency response function of the absolute acceleration and relative displacement of the equipment to the absolute acceleration of the floor can be obtained by

$$H_{\ddot{x}}(\omega) = \frac{\omega_0^2 + j2\xi_0\omega_0\omega}{(\omega_0^2 - \omega^2) + j2\xi_0\omega_0\omega} \quad (5a)$$

$$H_y(\omega) = \frac{-1}{(\omega_0^2 - \omega^2) + j2\xi_0\omega_0\omega} \quad (5b)$$

For the 2-DOF equipment-isolator system shown in Fig. 2b, the frequency response function the traslational absolute acceleration and relative displacement of the equipment at the mass center and the frequency response function of the rotational acceleration and displacement of the equipment around the mass center to the translational absolute acceleration of the floor can also be obtained by Fourier transform of Eq. 2 , given by

$$H_{\ddot{x}}(\omega) = \frac{1}{|A(\omega)|} \{ [-(c_x c_\psi + k_x J_0 - c_{x\psi} c_{\psi x})\omega^2 + k_\psi k_x - k_{x\psi} k_{\psi x}] + j[-c_x J_0 \omega^3 + (c_x k_\psi + c_\psi k_x - c_{x\psi} k_{\psi x} - c_{\psi x} k_{x\psi})\omega] \} \quad (6a)$$

$$H_\delta(\omega) = \frac{1}{|A(\omega)|} [-m_0 J_0 \omega^2 + m_0 k_\psi + j m_0 c_\psi \omega] \quad (6b)$$

$$H_{\ddot{\psi}}(\omega) = \frac{1}{|A(\omega)|} [-m_0 k_{\psi x} \omega^2 - j m_0 c_{\psi x} \omega^3] \quad (6c)$$

$$H_\psi(\omega) = \frac{1}{|A(\omega)|} [m_0 k_{\psi x} + j m_0 c_{\psi x} \omega] \quad (6d)$$

in which

$$|A(\omega)| = [m_0 J_0 \omega^4 - (m_0 k_\psi + c_x c_\psi + k_x J_0 - c_{x\psi}^2)\omega^2 - k_x k_\psi - k_{x\psi}^2] + j[-(m_0 c_\psi + J_0 c_x)\omega^3 + (c_x k_x + c_\psi k_x - 2c_{x\psi} k_{x\psi})\omega] \quad (7)$$

The variance of the absolute translational and rotational acceleration and the relative translational and rotational displacement of the equipment at the mass center are then easily obtained by

$$\sigma_{\ddot{x}}^2 = \int_{-\infty}^{+\infty} |H_{\ddot{x}}(\omega)| S_{\ddot{u}_f}(\omega) d\omega \quad (8a)$$

$$\sigma_\delta^2 = \int_{-\infty}^{+\infty} |H_\delta(\omega)| S_{\ddot{u}_f}(\omega) d\omega \quad (8b)$$

$$\sigma_{\ddot{\psi}}^2 = \int_{-\infty}^{+\infty} |H_{\ddot{\psi}}(\omega)| S_{\ddot{u}_f}(\omega) d\omega \quad (8c)$$

$$\sigma_\psi^2 = \int_{-\infty}^{+\infty} |H_\psi(\omega)| S_{\ddot{u}_f}(\omega) d\omega \quad (8d)$$

Because of the rotation, the seismic response of the translation-rotation coupled equipment at the top or bottom is more intense than that at the mass center. The variance of the maximum traslational absolute

acceleration and relative displacement of the equipment at the top or bottom can be evaluated by

$$\sigma_{\ddot{x}_{\max}}^2 = \sigma_{\ddot{x}}^2 + d^2 \sigma_{\ddot{\psi}}^2 \quad (9a)$$

$$\sigma_{\delta_{\max}}^2 = \sigma_{\delta}^2 + d^2 \sigma_{\psi}^2 \quad (9b)$$

where  $d$  is the maximum distance of the top or bottom of the equipment to its mass center.

## NUMERICAL STUDIES

Two numerical examples are given here. Fig. 3 and Fig. 4 show, respectively, the variation of the variance of the absolute acceleration and relative displacement of a class of single-DOF equipments and translation-rotation coupled 2-DOF equipments at the top of a tall building with the period of the equipments. In these figures,  $\sigma_{\ddot{y}_f}$  is the variance of the absolute acceleration of the floor where the equipments locate;  $T_s$  is the fundamental natural period of the tall building;  $T_x, T_\psi$  are the translational and rotational period of the equipments respectively, given by  $T_x = 2\pi\sqrt{m_0/k_0}$ ,  $T_\psi = 2\pi\sqrt{m_0/k_\psi}$ ,  $\xi_x, \xi_\psi$  are the damping ratio of the equipment in translational and rotational direction respectively, given by  $\xi_x = 0.5c_x/\sqrt{k_x m_0}$ ,  $\xi_\psi = 0.5c_\psi/\sqrt{k_\psi m_0}$ .

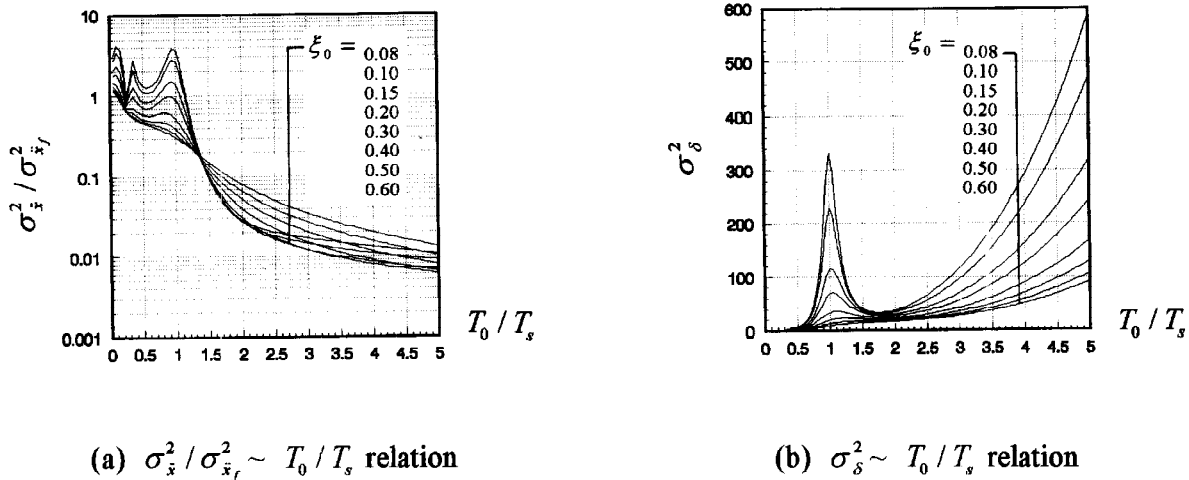


Fig. 3 Seismic response of single-DOF equipment-isolator system  
( $T_s = 5.0$  sec)

Some remarks can be summarized from the numerical studies as follows:

- (1) The damp of single degree-of freedom isolator-equipment system can suppress the relative displacement response of equipments to earthquakes. The higher the damp, the less the response.
- (2) There is a critical period  $T_c$  for the isolation system of single degree-of freedom equipments employed in tall buildings. When the period of the system  $T_0 < T_c$ , the damp of the system can suppress the absolute acceleration response of equipments to earthquakes; and when  $T_0 > T_c$ , an optimum damp ratio  $\xi_{opt}$  is

existed to make the absolute acceleration response variance of equipments minimum.

- (3) When the ratio of the period of isolator-equipment system to the fundamental period of tall building  $T_0 / T_s$ , lies between 1.5 and 1.8, both absolute acceleration response and relative displacement response are relative small. That can be chosen as ideal range of period for the purpose of isolation design.
- (4) If the equipment can rotate beside move translationally, the increasing of rotative damp and rotative stiffness is found to be favourable to decreasing both the maximum relative displacement response and the maximum absolute acceleration response of equipments to earthquakes.

Similar remarks to above are found by conducting many other numerical studies (Huang, 1994).

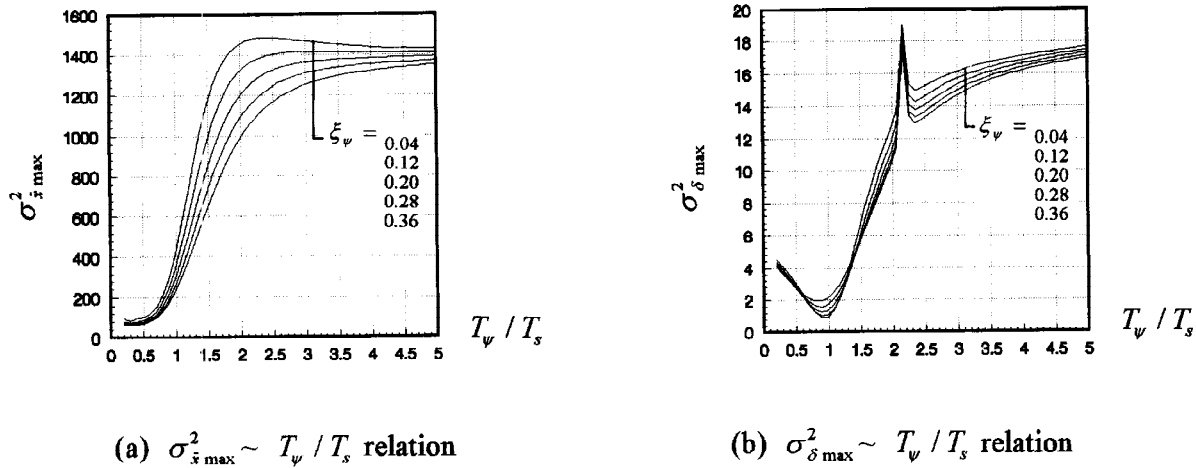


Fig. 4 Seismic response of translation-rotation coupled equipment-isolator system  
 $(T_s = 1.05 \text{ sec} ; T_x / T_s = 3.17 ; \xi_x = 0.01 ; d = 1.0 \text{ m} )$

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