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APLICATION OF THEORY OF GRAPHS TO RELIABILITY ANALYSIS OF NETWORK OF LIFELINE ENGINEERING

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ABSTRACT

The analysis of reliability of network of lifeline engineering can be finished using the calculation of road matrix, two motheds for calculating road matrix are presented: Warshall algorithm and coloring method. For a network with n nodes, the calculation of the road matrix of which can be realized using the check of the connectivity among nodes, in fact, the Warshall algorithm is a recurrent logical operation, from the beginning of the road matrix $A, A^2, A^3 \ldots A^l$ are respectively calculated step by step. This corresponds to the calculation of the number of roads from v_i to v_j respectively passing through nodes $v_1, v_1-v_2, \ldots, v_1-v_2\ldots-v_l$ or by 2,3....(l+1)steps(there is one road at least). Obviously, the estimation of lth step from v_l to v_l is based on the one of(l-1)th step from v_i to $v_{(l-1)}$. The basic idea of the coroling method is as follows: for an effective network with n nodes (here only discuss single source), assume that every node of the network is uncolored before suffering damage, as soon as a damage mode of the network is assigined, a coloring material would flow from the source and make the adjacent nodes colored, these colored nodes are called the secondary sources and have same function as the one the source have, and all of the colored nodes should connect the source, in fact, the coloring method is a way " the analysis of connectivity of an effective network is relied upon instead of the analysis of a definite one". By the effective network we refer to the network that consists of good or basic good elements. As far as the calculating operation time concerd, for a network with n nodes, the operation time is directly proportional to 3th(or incomplete 3th)power of n for the Warshall algorithm and 2th (or incomplete 2th) power of n for the coloring method. By comparsion with a general arithmetic operation or logical one of road matrix that needs the operation time in proportion to 4th (or incomplete 4th)power of n, two motheds presented in this paper can save the calculating operation time and are adaped for analysing large network of lifeline engineering.

KEYWORD

Theory of graphs; lifeline engineering; damage probability; random sampling; reliability analysis; Warshall algorithm; coloring method; adjacent matrix; road mattrix; coloring material.

INTRODUCTION

Under the action of earthquqke, the reliability analysis of network of lifeline engineering deals with knowlogys of seisogeology, earthquake resistance of structure and network analysis etc, which is implemented using the Monte Carlo's method to need to solve two aspects of problems: Firstly, Set up a model of damage distribution on which to base the estimation of damage probability of every segment in the network ,(there may be a lot of segments between two nodes), hence, the earthquake circumstance at the network site and its surroundings are to be studied, and the earthquake risk of the site is to be estimated. At same time, the damage mechanism of single element in the network should be studied (one segment may consist of a lot of elements), and then the damage probabilty of every segment in the network is to be estimated. Secondly, make a analysis of the reliability of the network basing on the distribution of damage probability of the network gotten in the previous steps, and then the subsamples of damage mode of every segment in the network are randomly sampled to simulate one kind of damage mode of the network and a record of the damage mode of every node is made in this simulation. Finally, find an occurrence frequence of damage mode of every node that is taken as the estimation of an occurrence probability of damage mode of every node in the network.

Warshall algorithm

The Warshall algorithm belongs to a category of the theory of graphs, in the theory of graphs (Kaicheng Lu 1981), the algorithm relating to graphs is implemented by computer, and the computer would identify a graph by matrix, so the computer is an useful tool for studing graphs.

1. Construct a matrix for a directed graph G(V,E)

$$A = (a_{ij})_{n\times m}$$

in which

$$a_{ij} = \begin{cases} 1 & (v_i, v_j) \in E \\ 0 & \text{other} \end{cases} \qquad (1)$$

the matrix A is called the adjacent one of graph G, V is a set of nodes and E a set of sides, knowing an ajacent matrix A is knowing all information of G.

Changing arrangement order of nodes of graph G, the corresponding rows or columns of the adjacent matrix A are to be made to interchang each other. Generally speaking, If the lth and mth rows of the matrix A are to be made to interchang each other, this can be realized by the matrix A to be left-multiplied by the matrix P, and that the matrix P has such a feature that the mth element in the lth row is 1, and the lth element in the mth row is 1, and the elements in the other rows in the matrix A are only 1 on the principal diagonal and the others are 0. If the lth and mth columns of the matrix A are to be made to interchang each other, then the matrix A is to be right-multiplied by the Q in which the mth element in the lth column is 1, and the lth element in the mth column is 1, the elements in the other columns in the matrix A are only 1 on the principal diagonal and the others are 0.

In addition, two important properties of the graph G can be also obtained from an adjacent matrix A (see fig. 1):

1) the number of the nonzero elements in the ith row of the adjacent matrix A (a_{ij}) equals the one of sides taking v_i as a begining point. 2) the number of nonzero elements in the jth column equals the one of sides taking v_j as an endpoint. For example, the adjacent matrix A of the graph in the figure 1 is

$$A = \begin{vmatrix} 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{vmatrix}$$

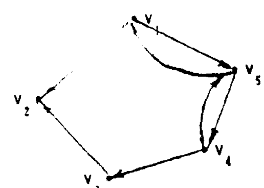

Fig. 1. A graph with 5 vertexes

In the first row, the sides taking v_1 as a bigining point are 1-2 and 1-5 ones, in the second column, the sides taking v_2 as an endpoint are 1-2 and 3-2 ones.

2) $B = A^2$

$$B = A^2 = \begin{vmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 2 \end{vmatrix} = (a_{ij}^{(2)})$$

In which

$$b_{ij} = a_{ij}^{(2)} = \sum_{k=1}^{5} a_{ik} b_{kj} \qquad (i,j=1,2,\ldots,5)$$
$$= a_{i1}b_{1j}+a_{i2}b_{2j}+a_{i3}b_{3j}+a_{i4}b_{4j}+a_{i5}b_{5j}$$

b_{ij} or $(a_{ij}^{(2)})$ denotes the number of roads from vi to vj by two steps, each term of right side above the second equality shows the number of roads from v_i to vj passing through 1,2,...5 nodes respectively.

Gerenally speaking, for the graph with n nodes, if $a_{il} \times a_{lj} \neq 0$ $(i,j=1,2,\ldots n)$, then if and only if $a_{il}=a_{lj}=1$, this is a description of the roads from vi to vl and v_l to v_j to be connected, and $a_{ij}^{(2)}=1$ shows one road from v_i to v_j by two steps.

As analogy, the element $a_{ij}^{(k)}$ in the matrix $A = (a_{ij}^{(k)})_{n\times n}$ shows the number of roads from v_i to v_j by k steps. So we get

$$B = (b_{ij}) = A + A^2 + \ldots\ldots + A^k = \sum_{l=1}^{k} A^l \qquad (2)$$

In which

$$b_{ij} = \sum_{l=1}^{k} a_{ij}^{(l)} \qquad i=1,2\ldots\ldots n$$

b_{ij} shows the numbe of roads from v_i to v_j not to exceed k steps.
For the graph $G=(V,E)$, $|V|=n$, it should be judged that through what nodes would be passed from vi to vj, the answer can be obtained from a road matrix Bn

$$B_n = A^1 + A^2 + \ldots\ldots + A^n = (b_{ij}^{(n)}) \qquad (3)$$

and so a road matrix and its algorithm are introduced in what follows

2. the concept of a road matrix

For a directed graph $G=(V,E)$, let

$$P = (p_{ij})_{n\times n} \qquad (4)$$

in which

$$p_{ij} = \begin{cases} 1 & b_{ij}^{(n)} \neq 0 \\ 0 & b_{ij}^{(n)} = 0 \end{cases}$$

P is called a road matrix of graph G

The above is the arithmetic operation of a road matrix to be found from the adjacent matrix. The logical operation of a road matrix is given below

Assume $A = (a_{ij})_{n\times n}$ and $B = (b_{ij})_{n\times n}$ are the matrixes consisting of element 1 or 0.

Definition $C = A \vee B = (c_{ij})_{n\times n}$ then $c_{ij} = a_{ij} \vee b_{ij}$ (5)

Definition $D = A \wedge B = (d_{ij})_{n\times n}$ then $d_{ij} = \bigvee_{k=1}^{n} (a_{ik} \wedge b_{kj})$ (6)

If A is written for $A^{(1)}$, let

$$A^{(2)} = A^{(1)} \wedge A^{(1)}, A^{(3)} = A^{(2)} \wedge A^{(1)} \ldots\ldots$$

then the formular for calucating road matrix by logical operation can obtained from above,i.e.,

$$P = A^{(1)} \vee A^{(2)} \vee \ldots\ldots \vee A^{(n)} = \bigvee_{k=1}^{n} A^{(k)} \quad (7)$$

in which

$$A^{(1)} = (a_{ij})$$

$$A^{(2)} = A^{(1)} \wedge A^{(1)} = (a_{ij}) \wedge (a_{ij}) = \bigvee_{k=1}^{n} (a_{im} \wedge a_{mj})$$

$$\vdots$$

$$A^{(n)} = A^{(n-1)} \wedge A^{(1)} = (a_{ij}^{(n-1)}) \wedge (a_{ij}) = \bigvee_{k=1}^{n} (a_{im}^{(n-1)} \wedge a_{mj})$$

It is easily shown that, whether a logical operation or arithmetic one, the calculation of a road matrix needs $n^3(n-1)$multiplies and $n^2(n-1)^2$additions.

3.The Warshall algorithm of a road matrix

In fact,the Warshall algorithm is a recurrent logical one,in order to introduce it,we still cite the Fig. 1. as example.

In the Fig. 1, for finding out the nodes to be reached from v_1 by one, two, ...k steps, we only need to calculate the 1th row elements in the matrixes $A^{(k)}$,k=1,2,...that are shown by $A_1^{(k)}$.

It can be seen from the Fig. 2 that $a^{(15)}$ in $A_1^{(1)}$ shows v_5 is to be reached from $v_1$by one step,i.e.,there is a v_1-v_5 road. $a_{15}^{(3)}$=2 in $A_1^{(3)}$ denotes $v_5$to be reached from v1 passing along two roads by 3 steps, in which first road can be searched out as the following proceed:First two steps are to be finished by $a_{15}^{(1)}$ in $A_1^{(1)}$ times $a_{51}^{(1)}$ in $A_1^{(1)}$, i.e., $a_{11}^{(2)}=a_{15}^{(1)} \wedge a_{51}^{(1)}$ or v_1-v_5-v_1,and third step to reach v5 can be finished by $a_{11}^{(2)}$ in $A_1^{(2)}$ times $a_{15}^{(1)}$in $A_1^{(1)}$, i.e., $a_{15}^{(3)}=a_{11}^{(2)}a_{15}^{(1)}$, so there is a v_1-v_5-v_1-v_5 road,the 2nd road fron v_1 to v_5 by steps is a v_1-v_5-v_4-v_5 one ,as shown Fig. 2. Obviously,it is impossible for v_1 to reach v_5 by two steps.

$$A_1^{(1)} = (0\ 1\ 0\ 0\ \textcircled{1}_{15})$$

$$A_1^{(2)} = A_1^{(1)} \begin{pmatrix} 0&1&0&0&1 \\ 0&0&0&0&0 \\ 0&1&0&0&1 \\ 0&0&1&0&1 \\ \textcircled{1}_{51}&0&0&\textcircled{1}_{54}&0 \end{pmatrix} = (0\ 1\ 0\ 0\ \textcircled{1}_{15}) \begin{pmatrix} 0&1&0&0&1 \\ 0&0&0&0&0 \\ 0&1&0&0&1 \\ 0&0&1&0&1 \\ \textcircled{1}_{51}&0&0&\textcircled{1}_{54}&0 \end{pmatrix} = (\textcircled{1}_{151}\ 0\ 0\ \textcircled{1}_{154}\ 0)$$

$$A_1^{(3)} = A_1^{(2)} \begin{pmatrix} 0&1&0&0&\textcircled{1}_{15} \\ 0&0&0&0&0 \\ 0&1&0&0&1 \\ 0&0&1&0&\textcircled{1}_{45} \\ 1&0&0&1&0 \end{pmatrix} = (\textcircled{1}_{151}\ 0\ 0\ \textcircled{1}_{154}\ 0) \begin{pmatrix} 0&1&0&0&\textcircled{1}_{15} \\ 0&0&0&0&0 \\ 0&1&0&0&1 \\ 0&0&1&0&\textcircled{1}_{45} \\ 1&0&0&1&0 \end{pmatrix} = (0\ 1\ 1\ 0\ \textcircled{2}_{1515,1545})$$

Fig. 2 A breif accound of analysis proceed

This algorithm is to calculate in how many steps $v_1$to reach v_5,but just what nodes are to be passed through, which would depend on the configuration of the Fig. 1.

As mentioned above, for the graph G with n nodes, if we would like to know what nodes are reached from v1 by k=1 ,2, ..n steps, then we only need to calculate $A_1^{(k)}$, and that k is sequentially arranged from 1 to n. Obviously,the calculating result of the (1+1) th step is to be based on the one of the 1th step. Therefore this algorithm is a recurrent operation.

Let the intitial value of a road matrix be $A=(a_{jk})$, which is shown by $P^{(0)}=(p^{(0)}_{jk})$, and $p^{(0)}_{jk}=a_{jk}$, if l cycles from 1 to n, then we can judge whether v_k are reached from v_j passing through nodes v_l or v_1, v_2 or $v_1, v_2, \ldots, v_n$.

Assume $l=1$, we get $P^{(l)}=(p^{(l)}_{jk})$, in which

$$p^{(l)}_{jk}=p^{(l-1)}_{jk} \vee (p^{(l1)}_{jl} \wedge p^{(l2)}_{lk}), \qquad j,k=1,2,\ldots\ldots n$$

$$l_1=\begin{cases} l-1 & k\leq l \\ l & k>l \end{cases} \qquad l_2=\begin{cases} l-1 & j\leq l \\ l & j>l \end{cases}$$

$p^{(l)}_{jk}=1$ shows $p^{(l-1)}_{jk}$ or $p^{(l1)}_{jl}=p^{(l2)}_{lk}=1$, i.e., there is at least a road from v_j to v_k in $(v_j, v_1, v_2, \ldots\ldots v_l, v_k)$. When $l=n$, the Warshall algorithm will be proven to be true.

For the graph with n nodes, the steps for Warshall algorithm to calculate a road matrix are as follows:

(1) $P \leftarrow A$
(2) $i \leftarrow 1$
(3) $j \leftarrow 1$
(4) $p_{jk} \leftarrow p_{jk} \vee (p_{ji} \wedge p_{ik})$ $\quad i,j=1,2,\ldots\ldots n$
(5) $j \leftarrow j+1$ if $j \leq n$ then go to (4)
(6) $i \leftarrow i+1$ if $i \leq n$ then go to (3), otherwise stop.

Obviously, the calculation of road matrix by Warshall algorithm needs n^3 times logical operation in all.

Coloring method

Up to now, the reliability analysis of network, whether to adopt the accurate mode enumeration method or approximate Monte Carto's method, is based on the check of connectivity of a definite network. For a network in which the connecting relation among the node and element has been entered in a computer memory, Wang kaisuan etc (1992) adopt the cut set method, but we adopt the Warshall algorithm and coloring method for judging the connectivity of the network. the later needs 2n and $2n^2$ times less the calculating times than the former does. In fact, the coloring method only deals with the connectivity of an effective network, by the effective network refer to the network that consists of good or basic good elements, which depends on the result of simulating sampling, therefore, the effective network is a random one. the coloring method and Warshall algorithm make an analysis of reliability of a given network, their results should be very same except for the difference between the calculating speeds.

The basic ideal of the coloring method is as follows: Assume there is an active network with n nodes(single source and multiple ones, here only discuss single one), the source of the network has a "coloring material" in store, all nodes are uncolored before suffering damage, as soon as a damage mode of the network is simulated using Monte Carlo's method, a "coloring material" would flow from the source and pass through the effective elements and make the adjacent nodes "coloured", these "coloured nodes" are called as the secondary sources and have same function as the one the source has, and the "coloring material" would flow from the secondary source, and the outer nodes of which are continuously made to colourate along the effective elements. When the " coloring material" has no way of colourating more nodes, all "coloured" nodes and the source are connected. Obviously, all uncoloured nodes and the source are unconnected.

The steps for the Coloring method to check the connectitity of a network are as follows:

1. Based on a damage mode of a network to be assiqined by the Monte-Carto's method or mode enumeration method, Make a list of an effective element and an effective source;

2. From the begining of the first effective element, Sequentially judge the effective elements as the following steps:

(1) If the number of an end node has already been recored in the list of effective source, then the effective

element with the end node number is to be deleted and go to(3).

(2) If the number of a beginning node has been recorded in the list of effective source, then the corresponding number of the end node is added in the list of effective source,and then the effective element with the begining node and end one is to be deleted and go to(3).

(3) If the effective elements in the list of effective element have not been examined, then the following effective element is examined and go to(1),otherwise go to(4).

(4)If the number of effective sources has not increased in this round,go to 3,otherwise go to 2.

3. All nodes in the list of effective source are connected with the source,the others are unconnected with the one.

Obviously, the coloring method is an utmost incomplete mixed twofold cycle. Under general conditions, for the network with n nodes, the calculating operation time is directly proportional 2nd (or incomplete 2nd) power of n.

Example 1

Taking the water-supplied pipeline system in town of Mianlin county as example, which is of a a branch-network connection type (see Fig.2.), and combining the Monte Carlo's method with the Warshall algorithm,we make an analysis of reliability of the system under action of earthquake.

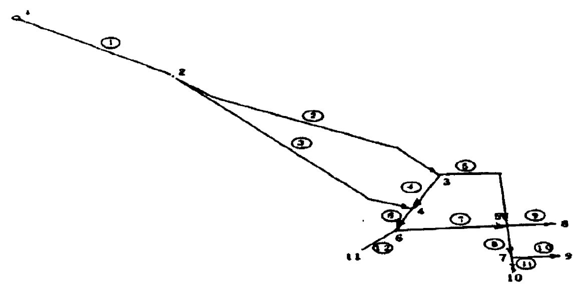

Fig.2. Sketch of water supplied pipeline system
in town of Mianlin county

The prediction of reliability probability of the water-supplied pipeline system is based on the damage probability of every segment of pipeline, its process is as follows: Sample randomly subsamples of damage mode of every segment of pipeline,and simulate a damage mode of the system,and check its connectivity,and record the mode of every node,finally find the times of occurrence of damage mode of every segment that are taken as an evaluation of occurrence probabilities of damage modes of every node in the system.

For the network structure of the water-supplied pipeline system in the town of Mainlin county,its adjacent matrix A is follows:

$$A=\left|\begin{matrix}
0&1&0&0&0&0&0&0&0&0&0\\
0&0&1&1&0&0&0&0&0&0&0\\
0&0&0&1&1&0&0&0&0&0&0\\
0&0&1&0&0&1&0&0&0&0&0\\
0&0&0&0&0&0&1&1&0&0&0\\
0&0&0&0&1&0&0&0&0&0&1\\
0&0&0&0&0&0&0&0&1&1&0\\
0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0
\end{matrix}\right|$$

The damage modes of the system are simulated using the Monte Carto method,and then the matrix $B = (b_{ij}) = \sum_{l=1}^{k} A^{l}$ is calculated using the Warshall algorithm for checking the connectivity among nodes, in which, b_{ij}

denotes the number of roads from node v_i not to exceed k steps to reach v_j.

The results are listed in table 2.

Table 1. The forecast of reliability probability water-supplied pipeline system (earthquake intensity Ⅱ)

number of segment	2	3	4	5	6	7	8	9	10	11
basic good	0.10000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
moderate reliability	0.23720	0.00260	0.00200	0.00060	0.00080	0.00000	0.00000	0.00000	0.00000	0.00040
severe unreliability	0.75280	0.99740	0.99800	0.99940	0.99920	1.00000	1.00000	1.00000	1.00000	0.99960

Example 2

A primary network is shown as the figure 3. after assigining one kind of damage mode of network using Monte Carto's method(see figure 4), the effective elements and source are listed in table 2.

The steps for the coloring method to analyse the network is as follows:

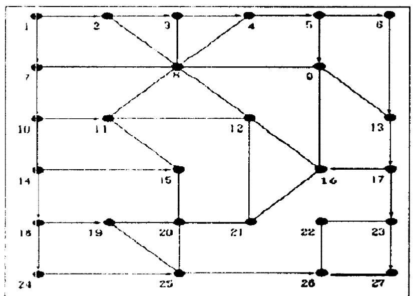

Fig.3. primary network

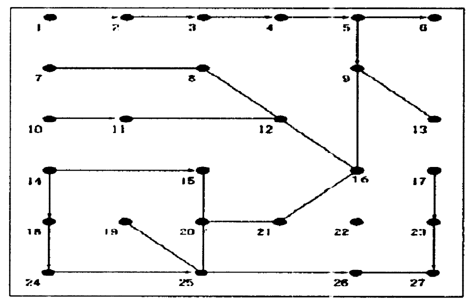

Fig.4. one kind of damage mode of primary network

step 1: Make a list of effective elements and source. (see table 2)

Table 2. A list of effective elements and source of network(see fig.4.)

1-2V	5-9 V	9-16	11-12	14-18V	17-23V	25-19	25-26V
2-3V	7-8	16-9	12-11	15-20	20-21	20-25	26-27
3-4V	8-7	9-13	12-16	20-15	21-20	25-20	27-26
4-5V	8-12	13-9	6-12	16-21	18-24V	23-27 V	
5-6V	12-8	10-11	14-15V	21-16	19-25	24-25V	
the effective source: 1							

Note: V shows an directed element.

step 2: After first round ends, the effective elements and sources are listed in table 3.

Table 3. A list of effective elements and secondary ones for a damage mode of the network in the first round (see fig.4.)

--	--	--	11-12	14-18 V	17-23 V	--	--
--	7-8	--	12-11	15-20	--	--	--
--	8-7	--	--	20-15	--	--	--
--	8-12	--	--	--	18-24 V	23-27 V	
--	12-8	10-11	14-15V	--	19-25	--	
the effective sources or secondary ones:							
1, 2, 3, 4, 5, 6, 9, 16, 13, 12, 21, 20, 25, 26, 27							

Step 3.the results in the second round and third one are listed in table 4 and table 5 respectively.

Table 4. A list of effective elements and secondary ones for a damage mode of the network in the second round . (see fig.4.)

--	--	--	--	14-18V	17-23V	--	--
--	7-8	--	--	--	--	--	--
--	8-7	--	--	--	--	--	--
--	--	--	--	--	18-24V	--	
--	--	10-11	14-15V	--	--	--	

the effective sources or secondary ones:
1, 2, 3, 4, 5, 6, 9, 16, 13, 12, 21, 20, 25, 26, 27, 8, 11, 15, 19

Table 5. A list of effective elements and secondary ones for a damage mode of the network in the third round(see fig.4.)

--	--	--	--	14-18V	17-23V	--	--
--	--	--	--	--	--	--	--
--	--	--	--	--	------		
--	--	--	--	--	18-24 V	--	
--	--	--	--	--	--	--	

the effective sources or secondary ones:
1, 2, 3, 4, 5, 6, 9, 16, 13, 12, 21, 20, 25, 26, 27, 8, 11, 15, 19, 7

The result in the fourth round (omited) is as same as the one in the 3rd round, so after fourth round is finished,the operation ends(i.e. step 3.)

CONCLUSION

In the analysis of reliability of network of lifeline engineering,combing the Monte Carlo's method with the coloring method and Warshall algorithm, the former having a feature of flexible simulation and the later having an advantage over general arithmetic operation or logical one in saving calculating time, the reliability analysis of lorge network of lifeline engineering including water supplied system, circuit network,electric system and convention mechanism operation system etc can be implemented.

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