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STATISTICAL ESTIIMATION OF EARTHQUAKE GROUND MOTION CHARACTERISITCS

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ABSTRACT

Statistics of earthquake ground motion characteristics are estimated on the basis of a seismologically consistent source model and a proposed estimation approach. In this paper, the source model is combined with an earth model consisting of a layered half-space in order to generate synthetic ground motions, and their statistics are estimated within each time-space window that is much small compared with the entire time-space domain. Specifically, the frequency-wave number spectrum of the ground motion, or simply the F-K spectrum, is firstly estimated under the assumption of ergodicity of the ground motion in both time and space in the window, which is reasonable from the engineering point of view. The auto- and cross-spectral density functions as well as coherence functions are then obtained based on the F-K spectrum using the Wiener-Khintchine theorem. Verification of the proposed estimation approach with others is presented for a specific case. As an example, statistical estimation of the 1989 Loma Prieta earthquake ground motion characteristics is carried out.

KEYWORDS

Earthquake Motion Synthetics, Statistical Estimation, F-K Spectrum, Loma Prieta Earthquake

INTRODUCTION

The spatial variation of earthquake ground motion has non-negligible effects on the elongated structures such as bridges, underground pipelines etc. While the ground motion is non-stationary in time and non-homogeneous in space, the essential feature of the spatial variation are usually captured by idealizing it as stationary and homogeneous functions of time and space at least within an appropriate time-space window (composed of a space window and a time window), which may be seen schematically in Fig. 1. Only then, it is possible to examine the effects of the spatial variation of the ground motion on the structural responses by taking advantage of such quantities as the frequency-wave number (F-K) spectrum, cross-spectral density function matrix, coherence function and the like.

In the present study, the ground motion $f(x, y, t)$ is expressed in terms of infinite number of plane waves having the integral representation satisfying the specified boundary conditions, originating from prescribed seismic source. The solution takes the following form

$$f(x,y,t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \tilde{f}(\kappa_x,\kappa_y,\omega)\ exp[i\kappa_x x + i\kappa_y y + i\omega t]\ d\kappa_x d\kappa_y d\omega \tag{1}$$

where $f(x,y,t)$ is the ground motion of displacement, velocity or acceleration, and $\tilde{f}(\kappa_x,\kappa_y,\ \omega)$ is the Fourier coefficient of the ground motion. In this procedure, $\tilde{f}$ may be found first and the three-fold Fourier transform is then performed to obtain the ground motion in the time-space domain (cf. equation (1)). As asserted earlier, the ground motion $f(x,y,t)$ is neither homogeneous in space (x,y) nor stationary in time t. However, a time-space window may be properly selected so that the corresponding ground motion in the window can be considered as both homogeneous and stationary function of (x,y,t) in approximation, as shown in Fig. 1. The F-K spectrum of the ground motion can then be found and the auto- and cross-spectral density functions and other degenerate statistical quantities are obtained therefrom.

GROUND MOTION REPRESENTATION IN A TIME-SPACE WINDOW

Considering a time-space window centered at (x_0,y_0,t_0) or equivalently $(r_0,\ \theta_0,t_0)$ with window lengths being (X_W,Y_W,T_W), as shown in Fig. 2, the corresponding ground motion in the window f_W may have an integral representation similar to equation (1). With a properly mathematical manipulation, one may find the following relationship between $\tilde{f}_W$ and $\tilde{f}$;

$$\tilde{f}_W(\kappa_x,\kappa_y,\omega) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \tilde{f}(\kappa_x',\kappa_y',\omega')w(\kappa_x'-\kappa_x;x_0,X_W)w(\kappa_y'-\kappa_y;y_0,Y_W)$$
$$w(\omega'-\omega;t_0,T_W)d\kappa_x' d\kappa_y' d\omega' \tag{2}$$

where window function $w(\Delta;c,W)= sin(\Delta W/2)exp[i\Delta c]/\pi/\Delta$. If Δ is frequency, then W and c are the time parameters, and if Δ is wave number, then W and c are the distance parameters. It can be seen clearly from equation (2) that the representation of $\tilde{f}_W$ is a three-fold convolution of $\tilde{f}$ involving three window functions w. However, the three-fold convolution can be actually carried out quickly in numerical computation, since W is selected to be not too short and thus w dies down quickly as Δ increases. When W approaches the infinity, w behaves as a Delta function. In the case of T_W, X_W and Y_W being very large, the selected window is actually equivalent to the original entire time-space domain, and equation (2) will of course result in $\tilde{f}_W \simeq \tilde{f}$.

STATISTICAL ESTIMATION OF GROUND MOTION IN WINDOW

In many engineering applications, W can be appropriately selected so that the ground motion may be assumed to be both stationary process and homogeneous within this window (e.g. Fig. 1). Further, it is usually assumed that the ground motion in the window is ergodic in both time t and space (x,y). Therefore, the correlation function of the ground motion in the window can be found to be

$$R_{f_W}(\xi_x,\xi_y,\tau) = \frac{1}{T_W X_W Y_W}\int_{t_0-T_W'/2}^{t_0+T_W'/2}\int_{y_0-Y_W'/2}^{y_0+Y_W'/2}\int_{x_0-X_W'/2}^{x_0+X_W'/2} f_W(x,y,t)f_W(x+\xi_x,y+\xi_y,t+\tau)dxdydt \tag{3}$$

where ξ_x and ξ_y are respectively the separation distances in the x- and y-directions, τ is the time lag, $X_W' = X_W - 2|\xi_x|$, $Y_W' = Y_W - 2|\xi_y|$ and $T_W' = T_W - 2|\tau|$.

Substituting integral representation of f_W into equation (3) and using the three-dimensional version of Wiener-Khintchine transform theorem, one may obtain F-K spectrum P_{f_W} of the ground motion in the window as

$$P_{fw}(\kappa_x,\kappa_y,\omega) = \frac{(2\pi)^3}{T_W X_W Y_W} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \tilde{f}_W^*(\kappa_x',\kappa_y',\omega')\,\tilde{f}_W(\kappa_x,\kappa_y,\omega)$$
$$w(\kappa_x-\kappa_x';x_0,X_W')\,w(\kappa_y-\kappa_y';y_0,Y_W')\,w(\omega-\omega';t_0,T_W')\,d\kappa_x' d\kappa_y' d\omega' \tag{4}$$

where the asterisk stands for the complex conjugate. As indicated before, the three-fold convolution can be carried out quickly in numerical computation as long as the window lengths are not selected too short. When the window lengths, T_W, X_W and Y_W, are selected to be very long, the window function may be approximated as a Delta function, resulting in the F-K spectrum as

$$P_{fw}(\kappa_x,\kappa_y,\omega) \simeq (2\pi)^3\ |\tilde{f}_W(\kappa_x,\kappa_y,\omega)|^2/(T_W X_W Y_W) \tag{5}$$

which is consistent with the common practice in general and with the approximation used by Theoharis (1991) in particular. However, the assumption of stationarity and homogeneity of the ground motion for a much large window is not appropriate for the present purpose.

An F-K spectrum essentially contains all the information of the temporal and spatial statistics of the ground motion in the selected window. Other degenerate statistics of the ground motion can be obtained mathematically in terms of the F-K spectrum. For example, the cross-spectral density function can be obtained by

$$C_{fw}(\xi_x,\xi_y,\omega) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} P_{fw}(\kappa_x,\kappa_y,\omega)\ exp[i\kappa_x\xi_x + i\kappa_y\xi_y]\,d\kappa_x\,d\kappa_y \tag{6}$$

The power spectral density function is obtained from the cross-spectral density function, i.e.

$$S_{fw}(\omega) = C_{fw}(\xi_x=0,\xi_y=0,\omega) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} P_{fw}(\kappa_x,\kappa_y,\omega)\,d\kappa_x\,d\kappa_y \tag{7}$$

Finally, the frequency-dependent coherence function is obtained as

$$\gamma_{fw}(\xi_x,\xi_y,\omega) = |\ C_{fw}(\xi_x,\xi_y,\omega)\ |/S_{fw}(\omega) \tag{8}$$

STATISTICS OF LOMA PRIETA EARTHQUAKE MOTION IN WINDOW

As an example, the statistics of the Loma Prieta earthquake ground motion in a local area (a time-space window) are presently calculated and discussed in this section. The model for the Loma Prieta earthquake developed by Zeng et al. (1991, 1993), Zhang and Deodatis (1994) is used. Specifically, the earth model is based on geological profiles of the Santa Cruz mountain area and consists of three layers overlaying a half-space. The seismic source mechanism was suggested by Zeng et al. (1993) and consists of a bilaterally propagating shear slip over a rectangular fault. The fault has a length of 40 km along the strike direction (130° clockwise from the north direction) and a width of 14 km along the dip direction (70° down from the horizontal). The total seismic moment is $2.9 \times 10^{19}\ N\cdot m$. With the aid of the compute code "SEISMO" at Princeton University which uses a discrete wave number method to solve wave propagation and scattering in a layered half-space, the Fourier amplitude of ground motion, specifically $\tilde{f}(\kappa_x,\kappa_y,\ \omega)$, can be obtained.

Although statistics of ground motion in any given time-space window may be obtained using the proposed method, only the strong ground motion that is related to a certain time window for a given space window is useful for engineering applications. For example, the ground velocity and acceleration are zero when all the seismic waves have not reached or have passed over a given space window in some time windows, which is obviously useless information to a practical engineering. This may be confirmed in Fig. 1. Therefore, it is important to select properly a time window for a given space window so that the strong ground motion in the time-space window can be captured.

It is well-known that the earthquake ground motion consists of various seismic waves that are body waves (P and S waves) and surface waves (e.g. Rayleigh and Love waves). The primary energy of seismic waves are carried out by the S and surface waves, which implies that the strong ground motion is contributed mostly by the S and surface waves. Therefore, the center of a time window (t_0) for the strong ground motion in a given space window should be no earlier than the time the first S wave propagates directly from the hypocenter (coordinate origin) to the space window center (x_0,y_0) or r_0 (see Fig. 2). On the other hand, the center of a time window may also not be too much later than the time the last S wave propagates directly from the source to the given space window. Otherwise, most of S and possibly surface waves pass over the given space window. The preceding observations may be summaried as follows:

(1) identify a space window (center location and window lengths), where the statistics of the ground motion are needed (e.g. general location of a large span bridge);

(2) calculate the time required for an S wave to propagate from the hypocenter to the space window center, denoted as t_s;

(3) calculate the least time required for an S wave to propagate from one side to the other of the space window, denoted as t_w;

(4) calculate the time difference between the first and last wave signals generated in the seismic source, denoted as t_d;

(5) obtain a proper length of the time window on the basis of $T_W \simeq t_d + t_w$ so that the obtained ground motion in such a selected window is not only strong (including most of S waves and surface waves) but also stationary and homogeneous, and finally

(6) obtain a proper time window center in terms of $t_0 \simeq t_s + T_W/2 + t_w/2$.

As an example, the following window is selected, which is centered at (t_0=20 *sec*, x_0=17 *km*, y_0=10 *km*) with window lengths being (T_W=15 *sec*, X_W=Y_W=15 *km*). The F-K spectra and their coherence functions of the ground acceleration in the selected window at ω=5, 10, 20 and 30 *rad/sec* are computed, which are displayed in Figs. 3 and 4. As seen in Fig. 3, the maximum value of the F-K spectra is much small at both low ($\omega = 5$ *rad/sec*) and high ($\omega = 30$ *rad/sec*) frequencies, compared with that at ω=10 and 20 *rad/sec*, which indicates the dominant energy carried by the acceleration in the window is around ω=10 or 20 *rad/sec*. Fig. 4 shows that as the separation distance gets large, the coherence decays both exponentially in an oscillatory fashion at a given frequency and more quickly at the high frequency than at the low frequency as expected. The coherence characteristics observed from the present numerical examples are basically consistent with those obtained using actually earthquake records observed at SMART 1 array, as seen in Loh and Yeh (1988) and Loh, C.H. (1991).

CONCLUDING REMARKS

A method for estimating the temporal and spatial statistics of a strong earthquake ground motion in a properly-selected time-space window is proposed on the basis of a theoretical model of earthquake motion. Compared with the conventional estimate which is based on the earthquake records, the proposed method provides an alternative way to obtain the statistics of the ground motion where few or no earthquake records are available. The computational results of a special case with the use of

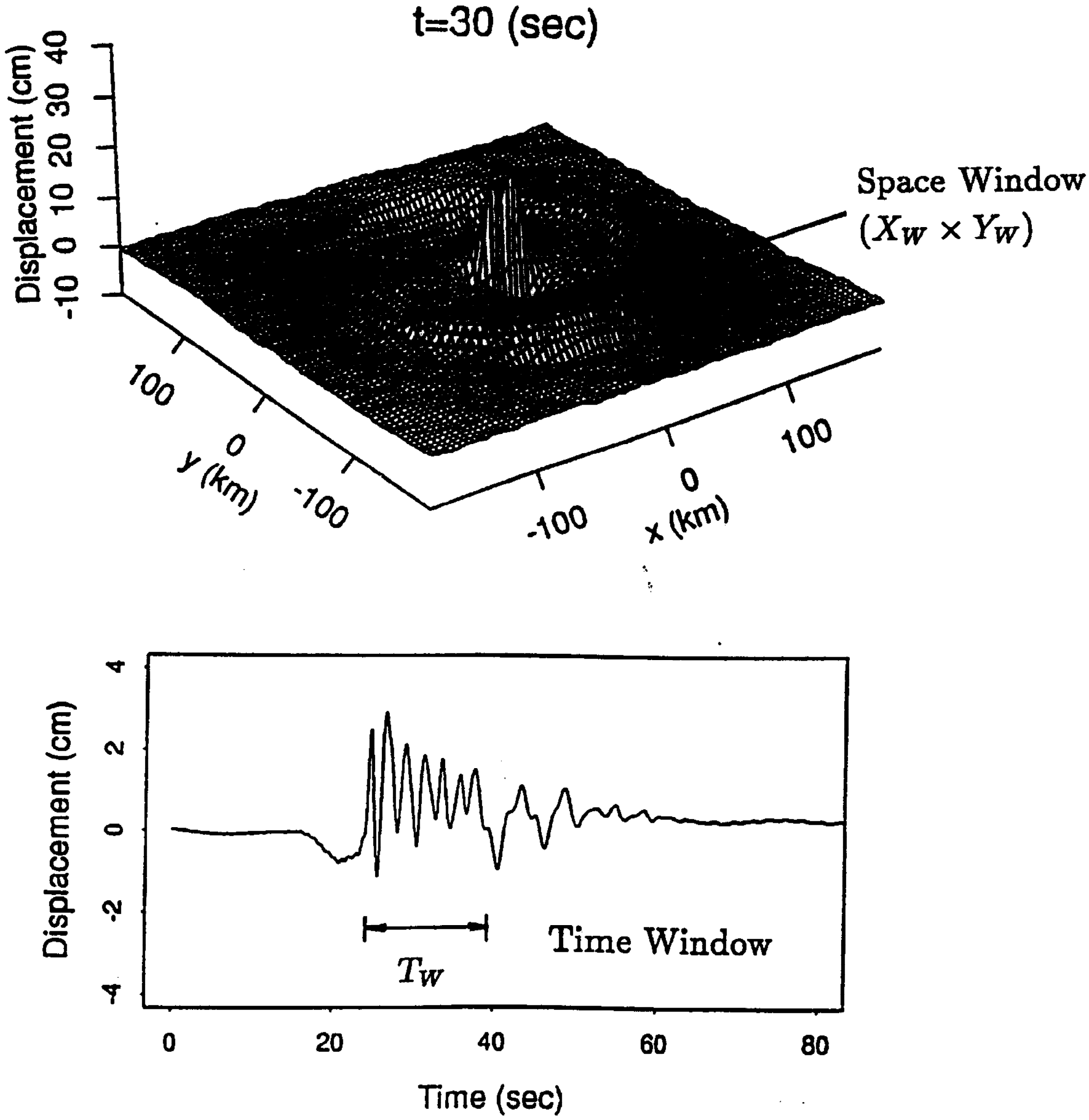

Figure 1 A strong earthquake ground motion in a time-space window: (upper) ground displacement response at time instant t=30 *sec* in a 360 *km* $\times$ 360 *km* area and (lower) ground motion displacement time history at one location in the space window the proposed approach are verified to be consistent with those obtained using Liu's approach. As an example of estimating statistics of the 1989 Loma Prieta earthquake at local area, a procedure for the proper selection of a time-space window is given so that the strong ground motion in the window can be obtained. The characteristics of the statistics of the ground motion are also investigated in this study, which is fundamentally useful to the aseismic structural design and dynamic analysis for the long-span structural systems subjected to earthquakes.

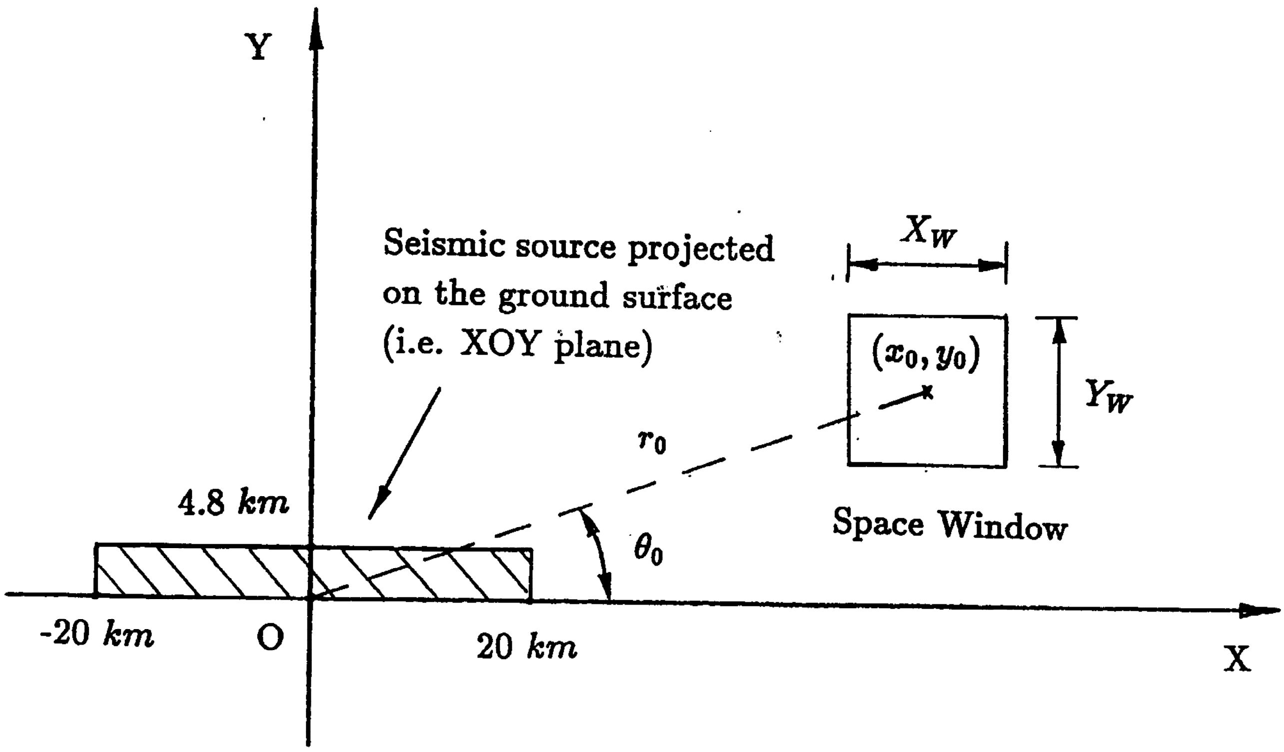

Figure 2 A space window and seismic source projected on the ground surface (XOY plane)

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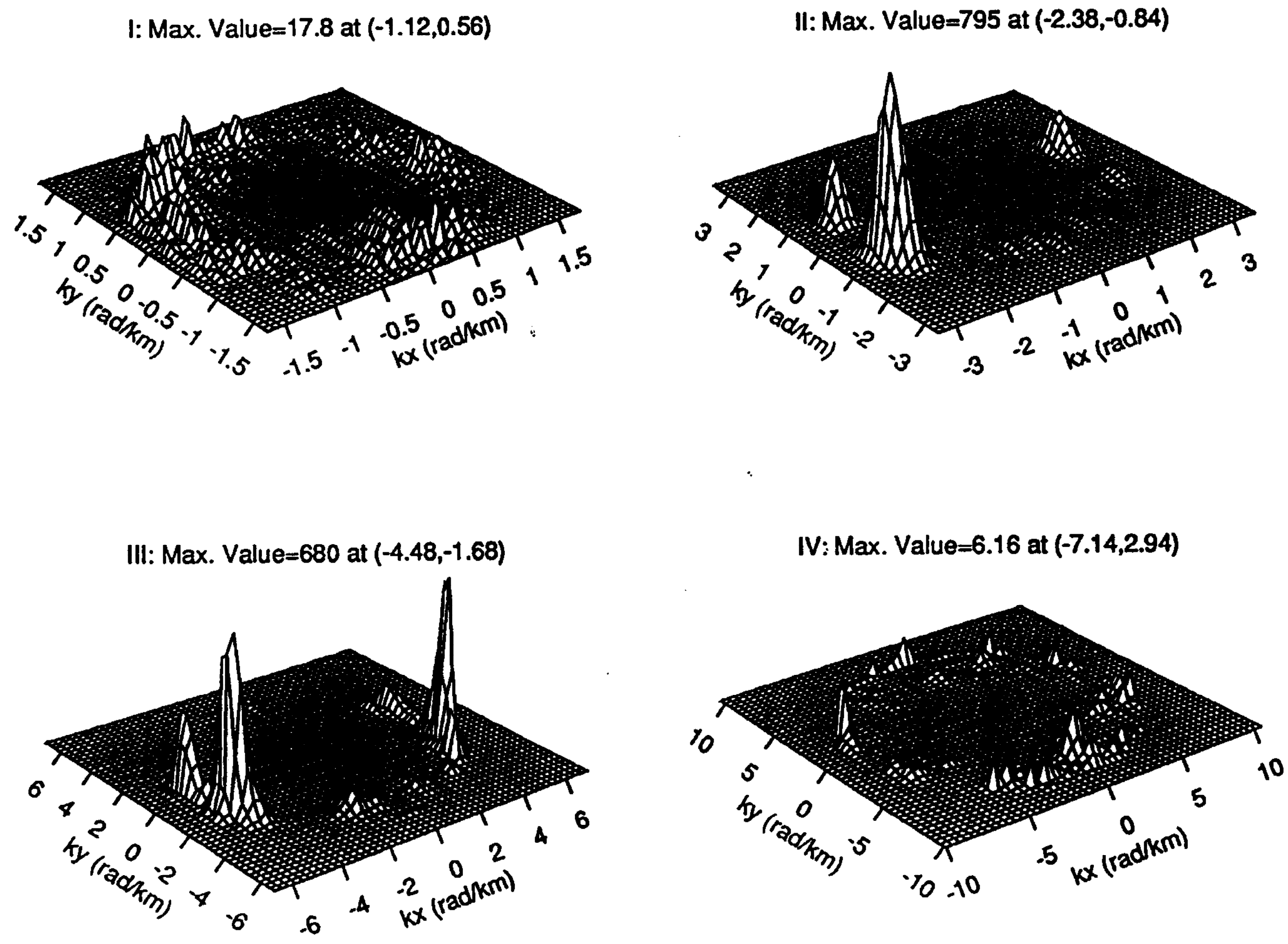

Figure 3 F-K spectra of Loma Prieta earthquake ground acceleration in the strike direction at ω=5,10, 20 and 30 rad/sec for plots I, II, III and IV, respectively, which is in the window centered at (t_0=20 sec, x_0=17 km and y_0=10 km) with window lengths T_W=15 sec and X_W=Y_W=15 km)

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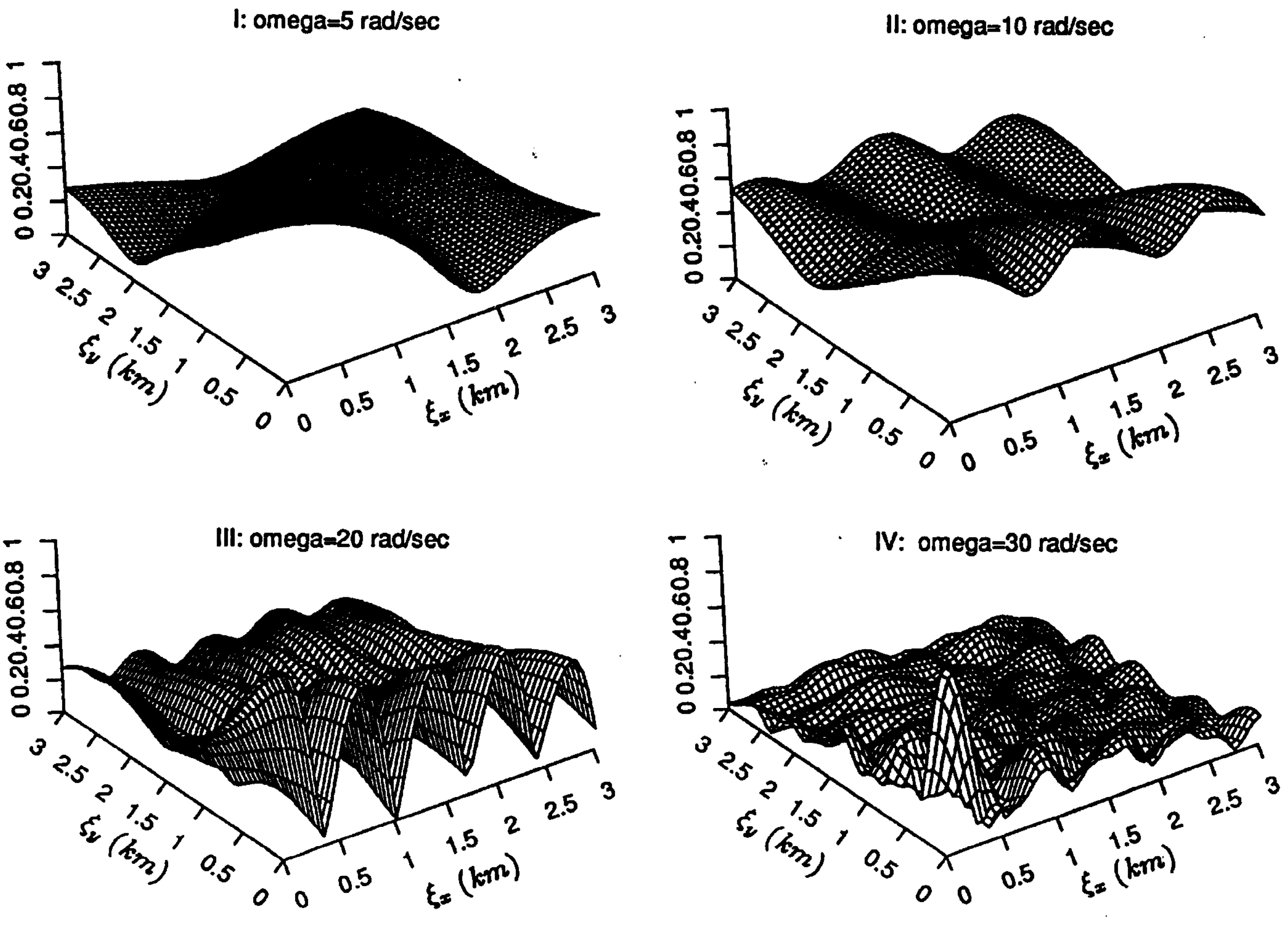

Figure 4 Coherencies of Loma Prieta earthquake ground acceleration in the strike direction at ω=5,10, 20 and 30 *rad/sec* for plots I, II, III and IV, respectively, which is in the window centered at (t_0=20 *sec*, x_0=17 *km* and y_0=10 *km*) with window lengths T_W=15 *sec* and X_W=Y_W=15 *km*)