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OPTIMAL DESIGN OF ACTIVE CONTROL ALGORITHMS FOR TRANSIENT UNKNOWN FORCING FUNCTION

G. ZUCCARO[2] - G. VOIELLO[1] - F. PAPA[1] - C. CENNAMO[1] - A. BARATTA[1]

[1] Department of "Scienza delle Costruzioni", University of Naples "Federico II"
Piazzale V. Tecchio, 80 (Fuorigrotta) - 80125 Napoli (Italy)
[2] Centre Interdepartmental of Research, University of Naples "Federico II"
Via Toledo, 402 - 80134 Napoli (Italy)

ABSTRACT

The paper summarizes the basic approaches and the results obtained by the group of researchers listed above, regarding the active control of the structures. Optimal design of linear active control algorithms, unknown and stochastic forcing functions, delayed and non-delayed systems, SDOF, MDOF and tolerance of the results are some of the problems studied; see Baratta et al., 1992 thro.1995, for details.

KEYWORDS

Optimal; linear; control; norm; solution; delay; stability; stochastic; forcing.

INTRODUCTION

The problem to keep into admissible limits the entity of the response of a MDOF structure subject to seismic action can be approached by the application of active control techniques, based on the application of contrast forces able to limit the effects of dynamic shaking on the structure strength. When a control device is to be designed, a basic task is the choice of the algorithm, and the calibration of the relevant parameters, able to yeld optimal performance of the system.
In the classical optimal control notations (Soong, 1988), the control optimization is usually based on the minimization of a quadratic performance index

$$J = \int_0^t \left[\mathbf{z}^T(t)\mathbf{V}\mathbf{z}(t) + \mathbf{q}^T(t)\mathbf{R}\mathbf{q}(t) \right] dt \tag{1}$$

with **z**(t) the state vector of the system, **q**(t) control force vector, **V** and **R** weighting matrices that balance the quality of the control system and the exercise freight.
If the loading history is not known a-priori, it is not possible to solve the motion equation in the classic form; so, for the definition of the control algorithm it is possible to refer to the "norm" (Baratta et al., 1993, 1994, 1995[1], 1995[2]) solution. It is assumed that the forcing function is known only through an "intensity" parameter, namely its "energy" in the quadratic sense.
The controlled response function is expressed through the "norm" of the response parameters, independently of the details concerning the seismic accelerogram. The response operator norm is

minimized by the condition that the maximal entity of control force cannot exceed a fraction of the maximal stress which could be experienced by uncontrolled stucture.
Recent works have verified the possibility to apply this procedure both to S.D.O.F. (Baratta et al., 1993) systems and to M.D.O.F. (Baratta et al., 1994, 1995[1], 1995[2]), remaining into the field of the linear control algorithm (closed loop) for linear structures.
Moreover one of the major problems in the design of the control algorithm is due to the delay in the action of the servomechanism, which may lead to undesired effects (Baratta 1992, Baratta et al 1994[3], 1995[3], 1995[4]) especially considering that the external force is not known a-priori in detail. In this case the solution in norm has been pursued by a step rational procedure either for the SDOF or for the MDOF systems.
The possibility to compensate the effect of delay has been investigated too, assuming the forcing function, that is being realized symultaneously to the control force process, to be a sample from a random function, with unknown - or partially unknown - characters, that can be recognized in real time while the motion is going on.
Detailed reference to literature can be found in the Baratta et al papers (1992 through 1995).

NORM SOLUTIONS FOR LINEAR STRUCTURES

For structures with only one degree of freedom effects (Baratta et al. 1993), the equation of controlled motion is

$$\ddot{u}(t)+2\zeta_0\omega_0\dot{u}(t)+\omega_0^2 u(t)=f(t)+q(t) \tag{2}$$

with

$$q(t)=-2\zeta'\omega'\dot{u}(t)-\omega'^2 u(t) \tag{3}$$

q(t) being the control force, f(t) the external force and $\omega_c=\sqrt{\omega_0^2+\omega'^2}$, $\zeta_c=\dfrac{\zeta_0\,\omega_0+\zeta'\,\omega'}{\omega_c}$, respectively the pulsation and the damping of the controlled system. Assuming the response operator norm of the structure $L(\omega,\zeta)$ (valid both for $\omega=\omega_0$ and $\zeta=\zeta_0$ and for $\omega=\omega_C$ and $\zeta=\zeta_C$) and the norm operator $H(\omega_c,\zeta_c)$ that produces the control force, the problem is developed minimizing the quantity

$$L(\omega',\zeta')=S_{max}/\|f\|=\omega_0^2\,L(\omega_c,\zeta_c)=\min$$

with

$$Q(\omega',\zeta')=\|q_{max}\|/\|f\|=H(\omega_c,\zeta_c)\le Q_0 \tag{4}$$

being Q_0 the limit on control force and S_{max} the maximum stress in the structure.
The control strategy can be identified in the search of the optimal values, [in the sense of the problem (4)] ζ'_0 and ω'_0 for assigned value of damping and pulsation of the structure.
To this aim, the adopted procedure is a random-walk technique for the search of constrained minimum (4). The procedure of the optimization is applied for different value of ζ'_0 and ω'_0 and with the control force $Q_0=\beta L(0,0)$, L(0,0) being the value of maximal stress $S_{max,0}/\|f\|$ on the uncontrolled structure.
The results obtained through the numerical experiments are tested with simulation procedures and they proved the efficiency of control procedure. In fact, assuming $\omega_0=30\ sec^{-1}$ and $\zeta_0=5\%$, with $\beta=0,3$, the response is reduced at 25% of uncontrolled structure, and with $\beta=0,5$, about 12%.
Analogously, for MDOF systems (Baratta et al. 1994), the possibility of minimizing the response operator norm is approached under the condition that the entity of forcing function doesn't exceed a fraction of the maximal stress which could be experienced by the structure without control. From the equation of the motion

$$\mathbf{M}\ddot{\mathbf{u}}(t)+\mathbf{C}\dot{\mathbf{u}}(t)+\mathbf{K}\mathbf{u}(t)=\mathbf{f}(t)+\mathbf{q}(t) \tag{5}$$

where $\mathbf{q}(t) = -\mathbf{C}'\dot{\mathbf{u}}(t) - \mathbf{K}'\mathbf{u}(t)$ is the vector of control force; the matrices **C'** and **K'** depend on the location of the actuators and of the sensors and on the value attributed to the elements (linear control algorithm coefficients) and they may be noth simmetric. So, the equation of controlled motion in the standard form, can be diagonalized by decomposing the damping matrix in a diagonal component and in a "remains" matrix with the principal diagonal made by zero elements.
Putting $s_{\max_0}$ as the maximum instantaneous value of the modulus of the stress vector, the inequality can be written

$$|q|_{\max} \leq \beta s_{\max_0} \tag{6}$$

where $|q|_{\max}$, the maximum instantaneous value of the modulus of the control force, depends on the control function $G_c(\mathbf{K}',\mathbf{C}')$ and from the forcing energy norm $\|\varphi\|_T$, $\varphi(t)$ being the instantaneous modulus of forcing vector $\mathbf{f}(t)$

$$|q|_{\max} \leq G_c(K',C') \cdot \|\varphi\|_T \tag{7}$$

and $s_{\max_c}$, maximum instantaneous value of modulus of stress vector in the controlled structure, depends on maximum shear in the controlled structure S_c and from the forcing norm $\|\varphi\|_T$.

$$s_{\max} \leq S_c(\mathbf{K}',\mathbf{C}') \cdot \|\varphi\|_T \tag{8}$$

Putting $S_0 = S_c(0,0)$, the problem is reduced to the following optimization

$$\min_{\mathbf{K}',\mathbf{C}'} S_c(\mathbf{K}',\mathbf{C}') \;\; sub \;\; G_c(\mathbf{K}',\mathbf{C}') \leq \beta\, S_0 \tag{9}$$

For example, for a structure with three floors, putting actuators and sensors on every floor and considering the action of a ground shaking modeled by a shot noise motion, the results for β=0.1 are that the maximum control force experienced by the frame turns out be equal to s_{max}/7.0 and the maximum controlled stress is equal to s_{max}/1.8. So the controlled response is reduced about 55% of uncontrolled structure.
Later studies have proved that the results can be improved effects (Baratta et al 1995[1], 1995[2]).
Using the technique of modal control, both the possibility of checking all the modes to realize an "optimal control", and the possibility of checking only one, however realizing an efficient control but with a smaller expense in terms of energy, can be verified. As generally known, the "independent modal space control" (IMSC) is based upon the possibility of transforming the equation of the motion with *n* degrees of freedom of MDOF system, into *n* uncoupled equations in modal coordinates of the type, so that the control force can be introduced directly in the modal equation, as modal control force

$$\ddot{\phi}_i + 2\zeta_i\omega_i\dot{\phi}_i + \omega_i^2\phi_i = w_i(t) + g(t) \tag{10}$$

with $g_i(t) = -(2\zeta_i'\omega_i'\dot{\phi}_i + \omega_i'^2\phi_i)$. This process results in a very simple procedure, allowing also, in case it is requred, to apply the control force to a reduced number of degrees of freedom.
In other words, the coefficients ζ_i' and ω_i' are deduced from optimal control of eq. (4) (Baratta et al. 1993), in correspondence of the i-th mode damping and pulsation parametres ζ_i and ω_i.
So, the optimally controlled equation of the i-th mode can be written as follows

$$\ddot{\phi}_{ic} + 2\zeta_i\omega_i\dot{\phi}_{ic} + \omega_i^2\phi_{ic} + 2\zeta'_i\omega'_i\dot{\phi}_{ic} + \omega'^2_i\phi_{ic} = w(t) \tag{11}$$

and, assembling again into the matrix form, one gets

$$\ddot{\boldsymbol{\phi}}_c + \mathbf{D}\dot{\boldsymbol{\phi}}_c + \boldsymbol{\Lambda}\boldsymbol{\phi}_c + \mathbf{D}'\dot{\boldsymbol{\phi}}_c + \boldsymbol{\Lambda}'\boldsymbol{\phi}_c = \mathbf{w}(t) \tag{12}$$

in which $\mathbf{D}'$and $\Lambda'$are diagonal and positive defined matrices that contain the control coefficients in modal terms. Now it is possible to go back to nodal displacement components **u** and to mechanical characters of control system, building up motion of control coefficients in the **u**-space $\mathbf{K}_c$ and $\mathbf{C}_c$.
The theory presented in the above is applied to three floor frame (Norm Optimized Modal Control), stressed from shot-noise-type ground shaking and with control devices applied at every floor.
The results are tested for values of the control force equal to 1/20 S_0 (β=0.05) and the controlled response is reduced about 45% of uncontrolled response.
This procedure also allows to controll only a reduced number of degree of freedom and so to save the economical and the energetical engagement (pseudo-optimized control).
In that case, the principle of the design of the control coefficients can be identified in the requirement that the answer of the partial control system should be as closer as possible to the one obtained with the Norm Optimized Modal Control.
So, to obtain with the partial control an effect as close as possible to the one obtained with the "optimal" control, one aims at

$$\|\Delta\mathbf{C}_c\| = \min; \quad \|\Delta\mathbf{K}_c\| = \min \tag{13}$$

where $\Delta\mathbf{C}_c$ and $\Delta\mathbf{K}_c$ are matrices obtained from the difference of total and partial control matrices.
The performance can be applied to provide only a small number of actuators despite sensors remain placed on every degree of freedom.
For example, putting only one actuator in a three floor structure on the 2nd floor, for a value of the maximum allowed control force equal to 1/4 S_0 (β=0.25), the controlled response is reduced to about 23% of the uncontrolled maximum stress at the first floor.
Based on the assumption that the original structure exhibits uncoupled modes, the design procedure of the nodal control parameters through the in-norm optimization of the response of the single modes, appears efficient enough, so that one obtains a significant reduction of the structural strenght with a rather poor control action.
If a smaller number of d.o.f. are controlled, the effectiveness is obviously reduced, but it still exhibits considerable advantages because a significant attenuation of the structural response is obtained without requiring hard performances by the control device.

NORM SOLUTION FOR A DELAYED LINEAR CONTROL. NUMERICAL PROCEDURE

In the next paragraphs one summarizes the results obtained for a MDOF linear system described above when the delay of the response of servomechanism deputed to the active control is taken into account; in this case the control force **q**(t) in the eq. (5) becomes

$$\mathbf{q}(t) = \mathbf{C}_1\dot{\mathbf{s}}(t-\tau) + \mathbf{K}_1\mathbf{s}(t-\tau) \tag{14}$$

where $\mathbf{C}_1$ and $\mathbf{K}_1$ are the matrices of control coefficients, *not necessarily neither positive definite nor symmetric* as for **C** and **K**.

Not-Dimensional Time Equation

Assuming that ω_0^2 is the minimum eigenvalue of the standard stiffness matrix **S** obtained as $\mathbf{M}^{-1/2}\,\mathbf{K}\,\mathbf{M}^{-1/2}$, it is possible to introduce the non-dimensional time parameter $\vartheta = \omega_0 t$ ($\bar{\vartheta} = \omega_0 \tau$), whence the variable in eq (5) becomes $\mathbf{y}(\vartheta) = \mathbf{M}^{1/2}\,\mathbf{u}(\vartheta/\omega_0)$ and the non dimensional delayed equation, with the position

$(*)' = d(*)/d\vartheta$, is

$$\begin{gathered}\mathbf{y}''(\vartheta)+2\mathbf{E}\,\mathbf{y}'(\vartheta)+\boldsymbol{\Omega}\mathbf{y}(\vartheta)+\mathbf{c}(\vartheta)=\boldsymbol{\varphi}(\vartheta)\\ \mathbf{y}(0)=\mathbf{y}_0\ ;\ \mathbf{y}'(0)=\mathbf{y}'_0\end{gathered} \tag{15}$$

where

$$\begin{gathered}\mathbf{E}=\mathbf{M}^{-1/2}\mathbf{C}\mathbf{M}^{-1/2}/2\,\omega_0\ ;\quad \boldsymbol{\Omega}=\mathbf{M}^{-1/2}\mathbf{K}\,\mathbf{M}^{-1/2}/\,\omega_0^2;\\ \mathbf{A}=\mathbf{M}^{-1/2}\mathbf{K}_1\,\mathbf{M}^{-1/2}/\,\omega_0^2\ ;\quad \mathbf{H}=\mathbf{M}^{-1/2}\mathbf{C}_1\,\mathbf{M}^{-1/2}/2\,\omega_0\\ \mathbf{c}(\vartheta)=\mathbf{q}(\vartheta/\omega_0)=2\mathbf{H}\,\mathbf{y}'(\vartheta-\bar{\vartheta})+\mathbf{A}\,\mathbf{y}(\vartheta-\bar{\vartheta});\quad \varphi(\vartheta)=\mathbf{M}^{-1/2}\,\mathbf{f}(\vartheta\ \omega_0)/\omega_0^2\end{gathered} \tag{16}$$

A Numerical procedure

Let consider the 2n dimensional state vector

$$\mathbf{z}(\vartheta)=\begin{bmatrix}\mathbf{y}(\vartheta)\\ \mathbf{y}'(\vartheta)\end{bmatrix};\ \mathbf{z}_i=\begin{bmatrix}\mathbf{y}(\vartheta_i)\\ \mathbf{y}'(\vartheta_i)\end{bmatrix}=\begin{bmatrix}\mathbf{y}_i\\ \mathbf{y}'_i\end{bmatrix};\ \mathbf{z}_0=\begin{bmatrix}\mathbf{y}_0\\ \mathbf{y}'_0\end{bmatrix} \tag{17}$$

with θ_i an element of a discrete sequence of time instants spaced by $\Delta\vartheta$ on the duration of the external disturbance $\varphi(\vartheta)$ Assuming for the solution a step rationale form, and using compact matrix notation, with $\mathbf{z}_i = \mathbf{z}(t_i)$, one obtains

$$\mathbf{z}_{i+1}=\mathbf{P}\,\mathbf{z}_i+\mathbf{Q}\,\mathbf{z}_{i-k}+\mathbf{W}\varphi_i\cdot\Delta\vartheta \tag{18}$$

where $\varphi_i = \varphi(\theta_i)$ and denoted with $\mathbf{I}_n$ the *n-dimensional unit matrix*

$$\mathbf{P}=\begin{bmatrix}\mathbf{I}_n-\boldsymbol{\Omega}\dfrac{\Delta\vartheta^2}{2} & \Delta\vartheta\left(\mathbf{I}_n-\mathbf{E}\Delta\vartheta\right)\\ -\boldsymbol{\Omega}\Delta\vartheta & \mathbf{I}_n-2\mathbf{E}\Delta\vartheta\end{bmatrix};\ \mathbf{Q}=\begin{bmatrix}\dfrac{-\mathbf{A}\,\Delta\vartheta^2}{2} & -\mathbf{H}\Delta\vartheta^2\\ -\mathbf{A}\,\Delta\vartheta & -2\mathbf{H}\Delta\vartheta\end{bmatrix};\mathbf{W}=\begin{bmatrix}\mathbf{I}_n\cdot\Delta\vartheta/2\\ \mathbf{I}_n\end{bmatrix}\Delta\vartheta \tag{19}$$

The solution of (18) is searched in the form

$$\mathbf{z}_i=\mathbf{V}_i\,\mathbf{z}_0+\sum_{j=1}^{i}\mathbf{R}_{ij}\,\boldsymbol{\phi}_{j-1}\cdot\Delta\vartheta \tag{20}$$

where $\mathbf{R}_{ij}$ are matrices [2n x n] that are built up sequentially by a recursive procedure such that all matrices $\mathbf{R}_{mj}$ are known for $m \le i$.

From eq. (20) one derives the vector $\mathbf{z}_{i\text{-}k}$ and substituting in eq. (18) and rearranging one gets

$$\mathbf{z}_{i+1}=\mathbf{V}_{i+1}\,\mathbf{z}_0+\sum_{j=1}^{i+1}\mathbf{R}_{i+1,j}\,\mathrm{j}_{j-1}\cdot\Delta\vartheta \tag{21}$$

with

$$\mathbf{V}_1=\mathbf{P}\ ;\ \mathbf{R}_{ii}=\mathbf{W}\ ;\ \mathbf{R}_{ij}=\mathbf{0}\ \text{ for } i<j\ ;\ \mathbf{R}_{i,j+1}=\mathbf{R}_{i-1,j} \tag{22}$$

$$\mathbf{V}_{i+1}=\begin{cases}\mathbf{P}\,\mathbf{V}_i & \mathit{for}\ i\le k\\ \mathbf{P}\,\mathbf{V}_i+\mathbf{Q}\,\mathbf{V}_{i-k} & \mathit{for}\ i>k\end{cases};\ \mathbf{R}_{i+1,j}=\begin{cases}\mathbf{P}\,\mathbf{R}_{ij} & \mathit{for}\ j>i-k\\ \mathbf{P}\,\mathbf{R}_{ij}+\mathbf{Q}\,\mathbf{R}_{i-k,j} & \mathit{for}\ j\le i-k\end{cases} \tag{23}$$

Equation (20) can be rearranged in a closed form, assuming that $\mathbf{z}_0 = \mathbf{0}$

$$\mathbf{z}_i=\mathcal{R}_i\,\boldsymbol{\phi}_i\,\Delta\vartheta \tag{24}$$

where $\mathcal{R}_i$ is a matrix $\left[2n\times ni\right]$, expressed in the form

$$\mathcal{R}_i=\left[\mathbf{R}_{i1}\vdots\mathbf{R}_{i2}\vdots\mathbf{R}_{i3}\vdots\ldots\ldots\mathbf{R}_{ii}\right] \tag{25}$$

and $\boldsymbol{\phi}_i$ is the external force vector $\left[ni\times 1\right]$, i denoting the step-time considered.

Norm Bounds for the control parameters

In order to control the system one can take under observation some of the response parameters by evaluating their norm or the norm of some linear combination of them.
Consider that a m-vector $\mathbf{g}(\vartheta)$, assembling m response parameters to be observed, is expressed, using a suitable matrix $\mathbf{G}$ [m x 2n], by any linear transform of $\mathbf{z}$

$$\mathbf{g}(\vartheta)=\mathbf{G}\,\mathbf{z}(\vartheta) \tag{26}$$

a numerical procedure to evaluate the norm of $\mathbf{g}(\vartheta)$ is exposed
Remembering the eq. (24)

$$\mathbf{g}_i=\mathbf{g}(\vartheta_i)=\mathbf{G}\,R_i\,\boldsymbol{\phi}_i\,\Delta\vartheta = D_{Gi}\,\boldsymbol{\phi}_i\,\Delta\vartheta \tag{27}$$

therefore the norm of $\mathbf{g}_i$ is searched in the form

$$\mathbf{g}_i^2=\mathbf{g}_i^T\,\mathbf{g}_i=\boldsymbol{\phi}_i^T\,\mathbf{D}_{Gi}^T\,\mathbf{D}_{Gi}\boldsymbol{\phi}_i\;\;\Delta\vartheta^2 \tag{28}$$

after some algebra one can obtain

$$g_i=\left\|\mathbf{g}_i\right\|\leq r_{Gi}\,\left\|\varphi\right\|_{\vartheta_i}\,\sqrt{\Delta\vartheta} \tag{29}$$

where the calculation of the values r_{Gi}^2 proceeds alongside with the iteration yielding the sequence of matrices $\mathbf{R}_{ij}$. In fact, firstly one obtains that

$$r_{G1}^2=\mathrm{Tr}\left[\mathbf{D}_{G1}^T\,\mathbf{D}_{G1}\right] \tag{30}$$

and then it is possible to establish a rule to calculate $r_{G,i+1}^2$ after r_{Gi}^2 is known.
Remembering the eq. (22), one can write

$$r_{G,i+1}^2=\mathrm{Tr}\left[\mathbf{D}_{G,i+1}^T\,\mathbf{D}_{G,i+1}\right]=\mathrm{Tr}\left[\mathbf{R}_{i+1,1}^T\,\mathbf{G}^T\mathbf{G}\mathbf{R}_{i+1,1}\right]+\mathrm{Tr}\left[\mathbf{D}_{Gi}^T\mathbf{D}_{Gi}\right]=\mathrm{Tr}\left[\mathbf{R}_{i+1,1}^T\,\mathbf{G}^T\mathbf{G}\mathbf{R}_{i+1,1}\right]+r_{Gi}^2 \tag{31}$$

From the above equation, one observes that r_{Gi}^2 is a monotone increasing function of the index i. So, one can write

$$g_{max}\leq r_{GN}\,\left\|\varphi\right\|\sqrt{\Delta\vartheta} \tag{32}$$

Optimal design of the control algorithm

If one wants, for instance, to control the displacement vector $\mathbf{y}(\vartheta)$, one can define the matrix $\mathbf{G}$ in the form

$$\mathbf{G}_y=\left[\mathbf{I}_n\;\;\vdots\;\;\mathbf{0}_n\right] \tag{33}$$

So, from eq. (32), one gets

$$y_{max}\leq r_{G_yN}\,\left\|\varphi\right\|\sqrt{\Delta\vartheta} \tag{34}$$

The main problem in control design is to verify that the control action has the maximum efficiency, while saving the control force $\mathbf{c}(\vartheta)$ under a prefixed threshold; therefore one needs to evaluate the norm of the current control force too.
From eq. (26), one can write

$$\mathbf{c}(\vartheta)=\mathbf{G}_c\,\mathbf{z}(\vartheta-\overline{\vartheta}) \qquad \text{with } \mathbf{G}_c=\left[2\mathbf{H}\;\;\vdots\;\;\mathbf{A}\right] \tag{35}$$

so the norm bound on the control force $\mathbf{c}(\vartheta)$ is given by

$$c_{max} \le r_{G_cN} \|\varphi\| \sqrt{\Delta\vartheta} \tag{36}$$

The problem of the optimal design can be set as follows

$$\textit{Find } y_{max} = \textit{minimum} \qquad \textit{sub } c_{max} \le c_0 \tag{37}$$

Putting $C_0 = c_0 / \left(\|\varphi\| \sqrt{\Delta\vartheta} \right)$ and remembering the procedure exposed in the previous paragraphs the problem can be expressed in the form

$$\textit{Find } r_{G_yN} \textit{ minimum} \qquad \textit{sub } r_{G_cN} \le C_0 \tag{38}$$

Since one recognizes that, apart from $\Delta\vartheta$

$$r_{G_yN} = Y(\mathbf{H},\mathbf{A} \mid \mathbf{E},\Omega) \quad ; \quad r_{G_cN} = C(\mathbf{H},\mathbf{A} \mid \mathbf{E},\Omega) \tag{39}$$

finally the problem is set as follows

$$\begin{cases} \text{Find } (\mathbf{H},\mathbf{A}): & Y_0 = \min\limits_{\mathbf{H},\mathbf{A}} Y(\mathbf{H},\mathbf{A} \mid \mathbf{E},\Omega) \\ \text{Sub} & C(\mathbf{H},\mathbf{A} \mid \mathbf{E},\Omega) \le C_0 \end{cases} \tag{40}$$

whence one concludes that the value of y_{max}, expressed in eq.(34) depends on the choice of the control parameters **H** and **A**, governing the control force in eq. (15).

NUMERICAL APPLICATION: THE SDOF SYSTEM

The procedure described above has been applied to a SDOF structure under the action of a seismic ground shaking and for a number of values assigned to the expected delay with a more or less strict upper bound on the control force, choosen as percentage of the maximum shear force $S_{0,max}$ of the uncontrolled structure. The results have proved that the presence of the delay is very critical for the efficiency of linear control algorithms

Table 1

$\overline{\vartheta}$	η_{min}	α_{min}	Y_0	Y_{min}
3	-0.246	-0.139	3.243	1.373
9	-0.073	-0.075	3.243	2.172
15	-0.03	-0.059	3.243	2.560
30	0.002	0.037	3.243	2.985

$c_0 \le S_{0,max}$

Table 2

η_{min}	α_{min}	Y_0	Y_{min}
-0.098	-0.076	3.243	1.6064
-0.073	-0.075	3.243	2.172
-0.03	-0.059	3.243	2.560
0.002	0.037	3.243	2.985

$c_0 \le 0.1\ S_{0,max.}$

Table I and II report the optimal values of η and α, found for different values of the non-dimensional delay ($\overline{\vartheta}$=3, $\overline{\vartheta}$=9, $\overline{\vartheta}$=15, $\overline{\vartheta}$=30) and the relevant values of the norm of the uncontrolled response Y_0 $(0,0|\zeta)$ and the minimum norm of the response of the controlled structure Y_{min} $(\eta, \alpha|\zeta)$, respectively for $C_0 \le S_{0,max.}$ and for $C_0 \le 0.1\ S_{0,max}$. The given structure is assumed to have a damping coefficient ζ= 0.03. The results have been verified through the application of simulation processes. The seismic action has been simulated by assuming the forcing function equal to a Gaussian shot-noise; with ϑ varying between 0 and 150.

The simulations effects (Baratta et al. 1995[3]) showed encouraging attenuation of the response only for values of the delay in the range $0 \le \overline{\vartheta} \le 10$; values of delay above this threshold need algorithms more sophisticated than the linear one. Moreover statistic analysis carried on 100 samples of the forcing function, for any value of the delay $\overline{\vartheta}$, showed that the mean of the uncontrolled response, μ_x, is obviously unaffected by $\overline{\vartheta}$, while the mean μ_y of the optimally delayed-controlled response is significantly

influenced by the delay $\overline{\vartheta}$; it is lower for small delays and tends to μ_x with increasing delay.
On the other side, the effect of control turns out to be rather stable as the delay increases: the absolute value of tolerance in the estimation of delay seems to increase with increasing the nominal delay; this effect is easily explained by the reduction in the control force allowed in the optimal situation. Nevertheless, one should note that the relative tolerance (Baratta et al. 1995[3]) remains larger for smaller delay.
As mentioned in the introduction stochastic prediction of the forcing function has been investigated too, and its general characters have been outlined (Baratta et al. 1995[3], 1995[4]). As a temporary conclusion, it can be assessed that the performance of this approach depends mainly on the correlation structure of the forcing function. Generally speaking, it is expected that the stochastic prediction can successfully work only if the correlation length of the forcing function is larger (better, *much* larger) than the delay time τ.

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