11 WCEE

Copyright © 1996 Elsevier Science Ltd
Paper No. 1534. (quote when citing this article)
Eleventh World Conference on Earthquake Engineering
ISBN: 0 08 042822 3

NONLINEAR ANALYSIS OF BASE-ISOLATED MDOF STRUCTURES

TSO-CHIEN PAN and GUICHANG YANG

School of Civil and Structural Engineering, Nanyang Technological University
Nanyang Avenue, Singapore 639798, Republic of Singapore

ABSTRACT

A generalized analytical method and an efficient numerical solution procedure are developed for the nonlinear seismic analysis of base-isolated multistory buildings. In the analytical development, while the superstructure is modeled as a linearly elastic system with a rigid diaphragm at each floor level, the seismic isolation system of laminated high damping rubber bearings is characterized by a nonlinear mathematical function. Taking advantage of the localized nonlinear behavior of a base-isolated multistory building, the numerical solution procedure can be largely simplified by applying the modal analysis to the superstructure to obtain a composite mass matrix for attachment to the base slab. This process results in a set of uncoupled nonlinear equations of motion with an integral term involving only the base slab displacements. The set of integral-differential equations can be efficiently solved using the cubic B-spline collocation method.

KEY WORDS

Base-isolation; Nonlinear seismic analysis; Multistory building

INTRODUCTION

Base isolation has increasingly become an accepted way for reducing damage in buildings and their contents in earthquake prone areas. One of the most commonly used seismic isolation system comprises the laminated natural rubber bearings which are positioned at the base level of a multistory building so as to reduce the transmission of damaging earthquake ground acceleration into the superstructure. The laminated high damping rubber bearings (LHDRBs) in an isolation system exhibit strong nonlinear behavior. Their force-displacement properties depend on the axial load, bilateral load and rate of loading. In order to predict the seismic response of a base-isolated structure using LHDRB isolation system, it is necessary to find a suitable mathematical model of the isolation devices as well as an efficient numerical algorithm for carrying out the nonlinear dynamic analysis.

For nonlinear dynamic analysis, there are general purpose computer programs such as DRAIN-2D (Kannan and Powell, 1975) which can also be used for seismic analysis of base-isolated structures. Elements available in these programs are however limited to those exhibiting general hysteretic behavior and cannot accurately model LHDRBs. Special purpose computer programs have thus been developed for the nonlinear dynamic analysis of base-isolated structures. NPAD (Way and Jeng, 1988), for example, has a plasticity based bilinear element that can be used to model elastomeric bearings. Recently, 3D-BASIS (Nagarajaiah *et al.*, 1989,

1991) was developed with more sophisticated nonlinear elements for dynamic analysis of base-isolated structures in 3D.

It is generally difficult to model the force-displacement properties of LHDRBs using only a few parameters, as usually is done for the plasticity based elements. The authors thus propose the macromodel shown in (1) and (2) with a set of undetermined parameters to simulate the force-displacement characteristics of LHDRBs:

$$F(u,u) = K(u,\dot{u})u + C(u,\dot{u})\dot{u} \tag{1}$$

where

$$\begin{aligned} K(u,\dot{u}) &= \frac{a_1 + a_2u^2 + a_3u^4}{a_4 + a_5u^2 + a_6u^4} + \frac{a_7}{\cosh^2(a_8\dot{u})} \\ C(u,\dot{u}) &= \frac{a_9 + a_{10}u^2}{\sqrt{a_{11}^2 + \dot{u}^2}} \end{aligned} \tag{2}$$

in which the parameters a_i with i = 1 to 11 can be determined using the least squares method based on the experimental data of either a coupon test or a full-scale bearing test. The proposed macromodel of (1) and (2) represents the shear force as a nonlinear function of the shear deformation and the deformation velocity. Results of a case study applying the proposed macromodel to coupon test data are shown in Fig. 1. It shows that the proposed macromodel simulates closely the experimental data. Equations (1) and (2) are therefore used in the following work to describe the mechanical behavior of LHDRBs.

In this paper, a generalized analytical model is developed, which considers a multistory base-isolated structure as a combined system of an elastic superstructure and a nonlinear base isolation system. Since the nonlinearity is confined to the base isolation system, the nonlinear equations of motion of the base slab can be decoupled from the superstructure displacements by attaching a composite mass matrix to the base slab. However, this process introduces an integral term into the set of uncoupled nonlinear equations which involves only the base slab displacements. The B-spline collocation method can then be employed in the solution algorithm to deal with the integral-differential equations resulting from the decoupling process.

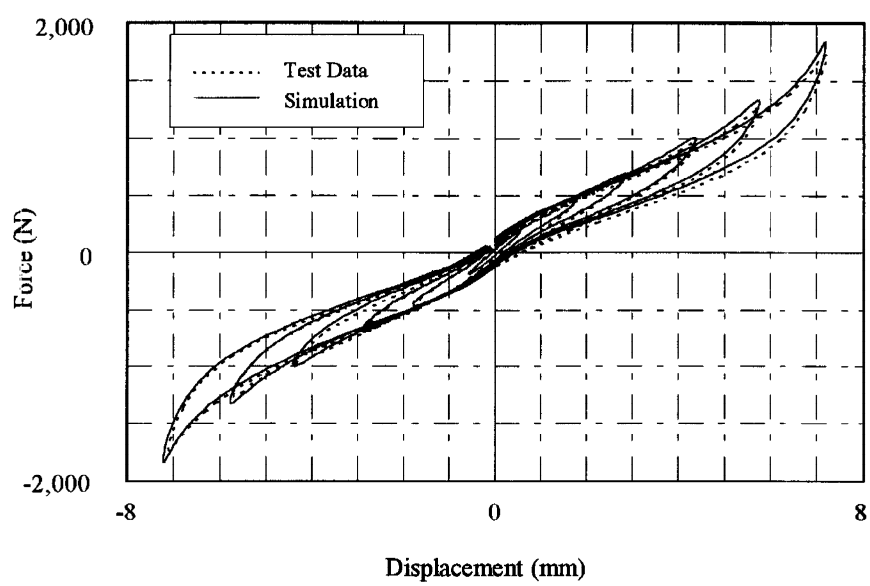

Fig. 1. Comparison of coupon test data and simulation using (1) and (2)

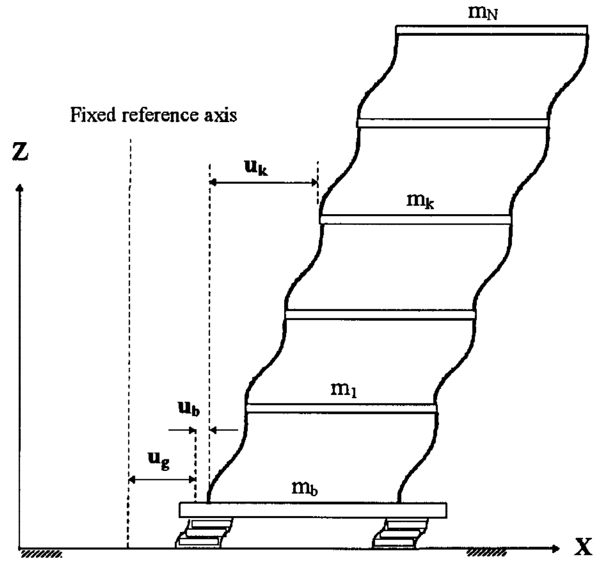

Fig. 2. Displacement coordinates of a base-isolated multistory structure

EQUATIONS OF MOTION OF A BASE-ISOLATED MULTISTORY STRUCTURE

The equations of motion of the N-story based-isolated structure shown in Fig. 2 take the form

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} = -\mathbf{M}\mathbf{r}(\ddot{\mathbf{u}}_b + \ddot{\mathbf{u}}_g) \tag{3}$$

$$\mathbf{r}^T\mathbf{M}\ddot{\mathbf{u}} + (\mathbf{r}^T\mathbf{M}\mathbf{r} + \mathbf{M}_b)\ddot{\mathbf{u}}_b + \mathbf{C}_b\dot{\mathbf{u}}_b + \mathbf{K}_b\mathbf{u}_b = -(\mathbf{r}^T\mathbf{M}\mathbf{r} + \mathbf{M}_b)\ddot{\mathbf{u}}_g \tag{4}$$

where $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are the mass, damping and stiffness matrices of the superstructure, respectively; $\mathbf{M}_b$, $\mathbf{C}_b$ and $\mathbf{K}_b$ are the mass, damping and stiffness matrices of the base slab, respectively; $\mathbf{u}$ is the displacement of superstructure relative to the base slab; $\mathbf{u}_b$ is the displacement of the base slab relative to the ground; $\mathbf{r}$ is a matrix which couples each degree of freedom (DOF) of the superstructure displacement to ground displacement; and $\ddot{\mathbf{u}}_g = [\ddot{u}_{gx}, \ddot{u}_{gy}, 0]^T$ is the vector of horizontal ground accelerations along the x and y axes.

In order to simplify the analysis of multistory base-isolated structures in 3D, it is assumed that the floors are rigid diaphragms and that the structural masses are lumped at the floor levels. Based on these assumptions, **M** and **K** matrices of the N-story superstructure can be shown as

$$\mathbf{M} = \mathrm{diag}\left[\mathbf{M}_k\right] \qquad k = 1, 2, ..., N \tag{5}$$

$$\mathbf{K} = \begin{bmatrix} \mathbf{K}_1 + \mathbf{K}_2 & -\mathbf{K}_2 & & & \\ -\mathbf{K}_2 & \mathbf{K}_2 + \mathbf{K}_3 & -\mathbf{K}_3 & & \\ & \cdots & \cdots & \cdots & \\ & & -\mathbf{K}_{N-1} & \mathbf{K}_{N-1} + \mathbf{K}_N & -\mathbf{K}_N \\ & & & -\mathbf{K}_N & \mathbf{K}_N \end{bmatrix} \tag{6}$$

where the mass matrix $\mathbf{M}_k$ and stiffness matrix $\mathbf{K}_k$ of the k-th floor are in the following form:

$$\mathbf{M_k} = \begin{bmatrix} m_k & 0 & -y_{kc}m_k \\ 0 & m_k & x_{kc}m_k \\ -y_{kc}m_k & x_{kc}m_k & J_{kc} + (x_{kc}^2 + y_{kc}^2)m_k \end{bmatrix} \tag{7}$$

$$\mathbf{K_k} = \sum_{\mathrm{i}} \mathbf{T_{ki}^T} \begin{bmatrix} k_{kix} & 0 \\ 0 & k_{kiy} \end{bmatrix} \mathbf{T_{ki}} \tag{8}$$

In (7) for the k-th floor, m_k is the floor mass, x_{kc} and y_{kc} are the x and y coordinates of the center of mass, and J_{kc} is the rotational inertia about the vertical axis at the center of mass. In (8) for the i-th column between the (k-1)-th and k-th floors, k_{kix} and k_{kiy} are the lateral stiffness along the x and y axes, respectively, and T_{ki} is the transformation matrix.

While the mass matrix $\mathbf{M_b}$ of the base slab has the same format as in (7), the stiffness and damping matrices are as follows:

$$\mathbf{K_b} = \sum_i \mathbf{T_{bi}^T} \begin{bmatrix} K(u_{bi}, \dot{u}_{bi}) & 0 \\ 0 & K(u_{bi}, \dot{u}_{bi}) \end{bmatrix} \mathbf{T_{bi}} \tag{9}$$

$$\mathbf{C_b} = \sum_i \mathbf{T_{bi}^T} \begin{bmatrix} C(u_{bi}, \dot{u}_{bi}) & 0 \\ 0 & C(u_{bi}, \dot{u}_{bi}) \end{bmatrix} \mathbf{T_{bi}} \tag{10}$$

where $K(u_{bi}, \dot{u}_{bi})$ and $C(u_{bi}, \dot{u}_{bi})$ follow the proposed macromodel of (1) and (2), and $u_{bi}^2 = u_{bix}^2 + u_{biy}^2$ is the resultant displacement of the i-th bearing of the base isolation system.

If the displacement of superstructure relative to the base slab is expanded in terms of the eigenvectors ϕ_i of the corresponding fixed-base superstructure, it takes the form

$$\mathbf{u} = \sum_{i=1}^{3N} q_i \phi_i \tag{11}$$

where q_i is the normal coordinate. The damping matrix of the superstructure $\mathbf{C}$, which satisfies the orthogonal condition $\phi_i^T \mathbf{C} \phi_i = 0$ for $i \neq j$, can be determined using the following formula:

$$\mathbf{C} = \sum_{i=1}^{3N} \frac{2\beta_i \omega_i}{\phi_i^T \mathbf{M} \phi_i} (\mathbf{M} \phi_i)(\phi_i^T \mathbf{M}) \tag{12}$$

Substituting (11) and (12) into (3) and (4) results in the following two equations:

$$\ddot{q}_i + 2\beta_i \omega_i \dot{q}_i + \omega_i^2 q_i = -\mathbf{L_i}(\mathbf{\ddot{u}_b} + \mathbf{\ddot{u}_g}) \tag{13}$$

$$\sum_{i=1}^{3N} \mathbf{r^T M} \phi_i \ddot{q}_i + (\mathbf{r^T M r} + \mathbf{M_b})\mathbf{\ddot{u}_b} + \mathbf{C_b \dot{u}_b} + \mathbf{K_b u_b} = -(\mathbf{r^T M r} + \mathbf{M_b})\mathbf{\ddot{u}_g} \tag{14}$$

in which $\mathbf{L_i} = (\phi_i^T \mathbf{M r}) / (\phi_i^T \mathbf{M} \phi_i)$ is the modal participation factor of the corresponding fixed-base superstructure. From (13), the normal coordinates q_i can be determined in terms of $\mathbf{\ddot{u}_b}$ and $\mathbf{\ddot{u}_g}$ through the

Duhamel integral,

$$q_i = -\frac{\mathbf{L_i}}{\omega_i}\int_0^t \left[\ddot{\mathbf{u}}_\mathbf{b}(t-\tau) + \ddot{\mathbf{u}}_\mathbf{g}(t-\tau)\right]\exp(-\omega_i\beta_i\tau)\sin\omega_i\tau d\tau \tag{15}$$

Assuming $\ddot{\mathbf{u}}_\mathbf{b}(0) = 0$ and $\ddot{\mathbf{u}}_\mathbf{g}(0) = 0$ and integrating by parts, one obtains

$$\dot{q}_i = -\mathbf{L_i}\int_0^t \left[\ddot{\mathbf{u}}_\mathbf{b}(t-\tau) + \ddot{\mathbf{u}}_\mathbf{g}(t-\tau)\right]\exp(-\omega_i\beta_i\tau)[-\beta_i\sin\omega_i\tau + \cos\omega_i\tau]d\tau \tag{16}$$

which, together with (15), can be substituted into (13) to arrive at the following equation:

$$\begin{aligned}\ddot{q}_i &= -\mathbf{L_i}\left(\ddot{\mathbf{u}}_\mathbf{b} + \ddot{\mathbf{u}}_\mathbf{g}\right) \\ &\quad + \mathbf{L_i}\omega_i\int_0^t \left[\ddot{\mathbf{u}}_\mathbf{b}(t-\tau) + \ddot{\mathbf{u}}_\mathbf{g}(t-\tau)\right]\exp(-\omega_i\beta_i\tau)\left[\left(1-2\beta_i^2\right)\sin\omega_i\tau + 2\beta_i\cos\omega_i\tau\right]d\tau\end{aligned} \tag{17}$$

Equation (17) can then be substituted into (14) to obtain the following equation of integral-differential type:

$$\begin{aligned}&\left(\mathbf{r^TMr} + \mathbf{M_b} - \sum_{i=1}^{3N}\mathbf{r^TM}\boldsymbol{\phi}_i\mathbf{L_i}\right)\ddot{\mathbf{u}}_\mathbf{b} + \mathbf{C_b}\dot{\mathbf{u}}_\mathbf{b} + \mathbf{K_b}\mathbf{u_b} + \sum_{i=1}^{3N}\mathbf{r^TM}\boldsymbol{\phi}_i\mathbf{L_i}\omega_i I_{bi} \\ &= -\left(\mathbf{r^TMr} + \mathbf{M_b} - \sum_{i=1}^{3N}\mathbf{r^TM}\boldsymbol{\phi}_i\mathbf{L_i}\right)\ddot{\mathbf{u}}_\mathbf{g} - \sum_{i=1}^{3N}\mathbf{r^TM}\boldsymbol{\phi}_i\mathbf{L_i}\omega_i I_{gi}\end{aligned} \tag{18}$$

where

$$I_{bi} = \exp(-\omega_i\beta_i t)\left\{\left[\left(1-2\beta_i^2\right)\sin\omega_i t + 2\beta_i\cos\omega_i t\right]I_{bic} - \left[\left(1-2\beta_i^2\right)\cos\omega_i t - 2\beta_i\sin\omega_i t\right]I_{bis}\right\} \tag{19}$$

$$I_{gi} = \exp(-\omega_i\beta_i t)\left\{\left[\left(1-2\beta_i^2\right)\sin\omega_i t + 2\beta_i\cos\omega_i t\right]I_{gic} - \left[\left(1-2\beta_i^2\right)\cos\omega_i t - 2\beta_i\sin\omega_i t\right]I_{gis}\right\} \tag{20}$$

in which

$$I_{bic} = \int_0^t \ddot{u}_b(\tau)\exp(\omega_i\beta_i\tau)\cos\omega_i\tau d\tau \tag{21}$$

$$I_{bis} = \int_0^t \ddot{u}_b(\tau)\exp(\omega_i\beta_i\tau)\sin\omega_i\tau d\tau \tag{22}$$

$$I_{gic} = \int_0^t \ddot{u}_g(\tau)\exp(\omega_i\beta_i\tau)\cos\omega_i\tau d\tau \tag{23}$$

$$I_{gis} = \int_0^t \ddot{u}_g(\tau)\exp(\omega_i\beta_i\tau)\sin\omega_i\tau d\tau \tag{24}$$

Equation (18) is an uncoupled equation which involves only the displacement, velocity and acceleration of the base slab as the unknowns. Substituting (5) and (6) into (18) results in the following equation in a more compact form:

$$\overline{\mathbf{M}}\ddot{\mathbf{u}}_\mathbf{b} + \mathbf{C_b}\dot{\mathbf{u}}_\mathbf{b} + \mathbf{K_b}\mathbf{u_b} + \sum_{i=1}^{3N}\mathbf{M_i^*}\omega_i\mathbf{L_i}I_{bi} = -\overline{\mathbf{M}}\ddot{\mathbf{u}}_\mathbf{g} - \sum_{i=1}^{3N}\mathbf{M_i^*}\omega_i\mathbf{L_i}I_{gi} \tag{25}$$

where $\overline{\mathbf{M}}$ is a composite mass matrix for the uncoupled equation,

$$\overline{\mathbf{M}} = \sum_{k=1}^{N} \mathbf{M_k} + \mathbf{M_b} - \sum_{i=1}^{3N} \mathbf{M_i^*} \tag{26}$$

$$\mathbf{M_i^*} = \frac{\sum_{j=1}^{N}\sum_{k=1}^{N} \mathbf{M_j} \psi^{i}_{jk} \mathbf{M_k}}{\sum_{k=1}^{N} \varphi_k^{iT} \mathbf{M_k} \varphi_k^i} \tag{27}$$

in which

$$\psi^{i}_{jk} = \begin{bmatrix} \phi_{(3j-2),i}\phi_{(3k-2),i} & \phi_{(3j-2),i}\phi_{(3k-1),i} & \phi_{(3j-2),i}\phi_{3k,i} \\ \phi_{(3j-1),i}\phi_{(3k-2),i} & \phi_{(3j-1),i}\phi_{(3k-1),i} & \phi_{(3j-1),i}\phi_{3k,i} \\ \phi_{3j,i}\phi_{(3k-2),i} & \phi_{3j,i}\phi_{(3k-1),i} & \phi_{3j,i}\phi_{3k,i} \end{bmatrix} \tag{28}$$

$$\varphi_k^i = \begin{Bmatrix} \phi_{(3k-2),i} \\ \phi_{(3k-1),i} \\ \phi_{3k,i} \end{Bmatrix} \tag{29}$$

SOLUTION ALGORITHM

Based upon the above derivations, computation procedures for numerical solutions can be summarized as: (i) Assemble the mass, damping and stiffness matrices of the superstructure and the base isolation system, (ii) Solve the eigenvalue problem for ϕ_i of the corresponding fixed-base superstructure, (iii) Solve (25) for base slab acceleration $\ddot{\mathbf{u}}_\mathbf{b}$, (iv) Calculate the normal coordinate q_i using (15), and (v) Calculate the relative displacement of superstructure $\mathbf{u}$ using (11). Note that (25) is an equation of integral-differential type, for which the spline collocation method is a suitable solution technique. The dynamic response is therefore obtained by numerical integration of (25) using the cubic B-spline collocation method.

The displacements of the base slab interpolated using the cubic B-spline function Ω take the form

$$\mathbf{u_b} = \left[u_{bx}, u_{by}, u_{b\theta}\right]^T = \sum_{j=-1}^{p+1} \mathbf{c_j} \Omega\left(\frac{t-t_j}{\Delta t}\right) \qquad j = -1, 0, 1, \ldots, p+1 \tag{30}$$

where $\mathbf{c_j} = \left[c_{jx}, c_{jy}, c_{j\theta}\right]^T$, Δt is the time step, p is the total number of collocation points, and

$$\Omega(s) = \frac{1}{6}\begin{cases} (s+2)^3 & s \in [-2,-1] \\ (s+2)^3 - 4(s+1)^3 & s \in [-1,\ 0] \\ (2-s)^3 - 4(1-s)^3 & s \in [\ 0,\ 1] \\ (3-s)^3 & s \in [\ 1,\ 2] \\ 0 & |s| > 2 \end{cases} \tag{31}$$

At the collocation point $t_k = k\Delta t$, the displacement, velocity and acceleration of the base slab can be expressed as

$$\begin{aligned}\mathbf{u}_b(t_k) &= (\mathbf{c}_{k-1} + 4\mathbf{c}_k + \mathbf{c}_{k+1})/6 \\ \dot{\mathbf{u}}_b(t_k) &= (\mathbf{c}_{k+1} - \mathbf{c}_{k-1})/(2\Delta t) \qquad k = 0, 1, 2, \ldots, p+1 \\ \ddot{\mathbf{u}}_b(t_k) &= (\mathbf{c}_{k-1} - 2\mathbf{c}_k + \mathbf{c}_{k+1})/(\Delta t)^2\end{aligned} \tag{32}$$

Substituting (32) into (25) and setting $t = t_k$, the following equation can be obtained:

$$\eta_1\mathbf{c}_{k-1} + \eta_2\mathbf{c}_k + \eta_3\mathbf{c}_{k+1} + \sum_{i=1}^{3N}\mathbf{M}_i^*\omega_i\mathbf{L}_i I_{bi}(t_k) = \mathbf{F}_k \tag{33}$$

where

$$\begin{aligned}\eta_1 &= \overline{\mathbf{M}}/(\Delta t)^2 - \mathbf{C}_b/(2\Delta t) + \mathbf{K}_b/6 \\ \eta_2 &= -2\overline{\mathbf{M}}/(\Delta t)^2 + \mathbf{K}_b/3 \\ \eta_3 &= \overline{\mathbf{M}}/(\Delta t)^2 + \mathbf{C}_b/(2\Delta t) + \mathbf{K}_b/6 \\ \mathbf{F}_k &= -\overline{\mathbf{M}}\ddot{\mathbf{u}}_g(t_k) + \sum_{i=1}^{3N}\mathbf{M}_i^*\omega_i\mathbf{L}_i I_{gi}(t_k)\end{aligned} \tag{34}$$

Equation (33) is therefore a nonlinear system with $\mathbf{c}_k$ being the unknowns and can be solved using an iterative method at each time step t_k, according to the following formula:

$$\mathbf{c}_{k+1} = \frac{1}{\eta_3}\left[\mathbf{F}_k - \eta_1\mathbf{c}_{k-1} - \eta_2\mathbf{c}_k - \sum_{i=1}^{3N}\mathbf{M}_i^*\omega_i\mathbf{L}_i I_{bi}(t_k)\right] \qquad k = 0, 1, 2, \ldots, p+1 \tag{35}$$

The starting vectors $\mathbf{c}_{-1}$, $\mathbf{c}_0$, and $\mathbf{c}_1$ for (35) take the form

$$\begin{aligned}\mathbf{c}_{-1} &= \mathbf{u}_{b0} - \dot{\mathbf{u}}_{b0}\Delta t + \ddot{\mathbf{u}}_{b0}(\Delta t)^2/3 \\ \mathbf{c}_0 &= \mathbf{u}_{b0} - \ddot{\mathbf{u}}_{b0}(\Delta t)^2/6 \\ \mathbf{c}_1 &= \mathbf{u}_{b0} + \dot{\mathbf{u}}_{b0}\Delta t + \ddot{\mathbf{u}}_{b0}(\Delta t)^2/3\end{aligned} \tag{36}$$

In the above equations, $\mathbf{u}_{b0}$ and $\dot{\mathbf{u}}_{b0}$ are the initial displacement and velocity, and $\ddot{\mathbf{u}}_{b0}$ can be determined by substituting $\mathbf{u}_{b0}$ and $\dot{\mathbf{u}}_{b0}$ into the dynamic equilibrium equation (25) with $t = 0$.

Having obtained $\mathbf{c}_k$ as shown in (35), the base slab displacement $\mathbf{u}_b$, velocity $\dot{\mathbf{u}}_b$, and acceleration $\ddot{\mathbf{u}}_b$ can be computed using (32). The normal coordinate q_i and subsequently the relative displacement of superstructure $\mathbf{u}$ can be calculated according to (15) and (11), respectively. The numerical algorithm for computing the nonlinear seismic response of base-isolated multistory structures incorporating LHDRBs is thus complete.

CONCLUSIONS

A generalized analytical method which results in an efficient computation algorithm has been developed to address the combined system of a linear superstructure and nonlinear LHDRB seismic isolators. In order to reduce the computation effort in the iterative procedures of nonlinear dynamics, the eigenvalue problem of the corresponding fixed-base superstructure is solved first. The displacements of the superstructure are then eliminated from the equations of motion of the base slab. The system of nonlinear equations so obtained is uncoupled and has only as unknowns the three DOFs associated with the base slab motions. This makes the iterative solution of a base-isolated multistory structure in 3D much more efficient than the conventional method where the equations of motion involve all DOFs of the superstructure plus those of the isolation system. The computation effort for the localized nonlinear system is thus largely reduced.

REFERENCES

Kannan, A. M. and Powell, G. H. (1975). DRAIN-2D – A general purpose computer program for dynamic analysis of inelastic plane structures with user guide. *Report No. UCB/EERC 73/22* (Revised Aug. 1975), Earthquake Engineering Research Center, University of California, Berkeley, California.

Nagarajaiah, S., Reinhorn, A. M. and Constantinou, M. C. (1989). Nonlinear dynamic analysis of three dimensional base isolated structures (3D-BASIS). *Proc.*, Fourth U. S. National Conf. on Earthquake Engineering., Vol. **3**, 579-588.

Nagarajaiah, S., Reinhorn, A. M. and Constantinou, M. C. (1991). Nonlinear dynamic analysis of 3-D base-isolated structures, *J. Struct. Engng*, ASCE, **117**, 2035-2054.

Way, D. and Jeng, V. (1988). NPAD – A computer program for the analysis of base-isolated structures. *Proc.,* ASME Pressure Vessels and Piping Conf., PVP - Vol. **147**, 65-69.