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ANALYSIS AND DESIGN OF VISCOELASTIC DAMPER FOR EARTHQUAKE-RESISTANT STRUCTURE

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ABSTRACT

Recent Kobe earthquake has warranted the need of passive control devices. These devices can be used for retrofitting of the bridges as well as in new bridges. In this study new viscoelastic damper has been fabricated, which is simple and robust in construction. Assuming a uniaxial shear type deformation, a 5-parameter fractional-derivative hysteretic model has been proposed to simulate the frequency dependent behaviour of VE damper. The development of the model is based on experimentally observed dynamic characteristics of the VE dampers. The proposed model is validated by dynamic tests on this new damper and a good agreement between predicted and experimental results is obtained. Numerical algorithms for the solution of the force-displacement relationship in both the frequency domain and the time domain are presented. Currently the design stiffness and damping coefficent of VE damper is based on single frequency which corresponds to damped natural frequency of the structure. This approach of design may not satisfy the robustness criteria of passive control scheme, keeping this point in view a rigorous method which takes into account the frequency dependent behaviour of VE damper has been developed. This optimal design procedure uses the concept of active control system which is modified for passive control system with some additional technique of optimal control theory. Using this procedure the optimal variation of frequency dependent behaviour can be obtained without many assumptions and simplification. This new approach of design can be applied to any passive control system.

KEYWORDS

Fractional derivative hysteretic model, Optimal frequency dependent design

INTRODUCTION

Viscoelastic dampers are used for energy dissipation. Now a days very reliable and durable viscoelastic material are available, some of the VE material are even temperature independent. This makes the VE damper very attractive for vibration control. Since energy dissipation in the such damper is due to deformation of the VE material, this mechanism of energy dissipation minimizes the mechanical wear and tear of damper. This makes the VE damper almost maintenance free as compared to conventional oil damper. Keeping these points in mind a new VE damper has been fabricated, which is simple and robust in construction as compared to other VE damper, used by other reseachers, such as (Markis et al., 1990) and (Kasai et al., 1993) . The device configuration, presented in this paper exploits the energy dissipation capability of semi-solid type viscoelastic material, in a better way then the configuration studied by (Markis et al., 1990) and (Kasai et al., 1993). This new damper can be used for retrofitting of bridges against earthquake, as well as discrete energy dissipation device for any structure. It has been recognized that the mechanical properties of viscoelastic material depends strongly on the excitation frequency. Therefore the mechanical properties of damper are dependent on the natural frequency of the structure as well as on the frequency content of the excitation. This frequency dependent property of the viscoelastic damper provides an interface between the external excitation and dynamic system. Furthermore the frequency dependent stiffness and damping adds, a extra parameter to calculate the damped natural frequency of the structure (Zhang et al., 1992). The robustness of passive and active control scheme depends on the amount uncertainties involved in the estimation of the structural parameter. This warrants the need of accurate mechanical model from discrete energy dissipation devices. To construct a model for the frequency dependent mechanical property of viscoelastic material, a fractional derivative model (FDM) has a advantage, over the commonly used linear integer models. Currently the design stiffness and damping coefficent of VE damper is based on single frequency which corresponds to damped natural frequency of the structure. This approach of design may not satisfy the robustness criteria of passive control scheme, keeping this point in view a rigorous method which takes into account the frequency dependent behaviour of VE damper has been developed. The frequency dependent behaviour of VE damper has been modeled 4-parameter model, then these four parameter has been optimized for the performance index using optimal control theory. This optimal design procedure uses the concept of active control system which is modified for passive control system with some additional technique of optimal control theory. Using this procedure the optimal variation of frequency dependent behaviour can be obtained without many assumptions and simplification.

FRACTIONAL DERIVATIVE MODEL

In this study five parameter(G, b, c, α, β) fractional derivative model has been used, to express the time dependent stress $\tau(t)$-strain $\gamma(t)$ which is as follows:

$$\tau(t) + bD^{\beta}\tau(t) = G\gamma(t) + cD^{\alpha}\gamma(t) \tag{1}$$

where G, b, c, α, β are empirical parameters. Fractional differintegration is an operator that generalizes the differentiation or intregation to non integal order. A commonly used definitions due to Liouville is the following (Oldham et al., 1974):

$${}_aD_t^q[f(t)] \equiv \frac{d^q f(t)}{[d(t-a)]^q} \equiv \frac{1}{\Gamma(n-q)}\frac{d^n}{dt^n}[\int_a^t \frac{f(\tau)}{(t-\tau)^{q-n+1}}d\tau] \tag{2}$$

In theory of viscoelasticity, we are concerned with the influence of the entire response history on the current response. The lower limit (a) should be thus be $-\infty$. However, if we assume $f(t) = 0,\ t < 0$, then ${}_{-\infty}D^q{}_t$ is identical to ${}_0D_t^q$. Hence a is usually taken as zero.

OUTLINE OF EXPERIMENTS

Viscoelastic Damper

Fig.(1) shows the construction of VE damper. This damper allows uniaxial shearing of VE material. The surface of the moveable shaft has been roughened by threading, this arrangement is enough to provide effective bond between the shaft and the VE material. This roughness is confined to middle third of the shaft's length, rest part is smooth thus allowing the shaft to move freely in and out of housing. The height of thread in 3mm, other important dimensions are shown in Fig.(1). Semisolid type VE material has been filled in the main housing. The shear deformation of the VE material traped between the roughened shaft and housing provides energy dissipation.

Fig.(2) shows a schematic representation of the testing arrangement. The mobile shaft of VE damper is attached to the actuator the other end of the damper is attached to load cell (mounted on rigid support). Dynamic tests on the viscous damper were conducted by imposing sinusoidal motion of specified amplitude (1 to 4 mm) and frequency (0.1 Hz. to 5 hz.) to the horizontally mobile shaft of the damper and measuring the force needed to maintain the motion.

Fig.(3) and Fig.(4) shows the variation of mechanical properties with excitation frequency. The hysteresis plots from Fig.(5) to Fig.(7) shows that area of loop decreases with increase in frequency. This indicates the reduction in energy-dissipation capacity, with the increase in frequency. Fig.(3) shows the stiffening of damper with increase in frequency.

Processing of Experimental Data

The recorded force-displacement loops had an almost precise elliptical shape see Fig.(5) to Fig.(7). These loops were used in obtaining the frequency dependent properties of the damper. The procedure is as follows: Under steady-state conditions, the force and displacement are $u = U_0 \sin(\omega t)$; $P = P_0 \sin(\omega t + \delta)$ where P_0 is the recorded amplitude of the force, U_0 is the recorded amplitude of displacement, ω is the frequency of motion and δ is the phase difference. The energy dissipated in a cycle of steady-state motion is $W_d = \oint P du = \pi \sin \delta P_0 U_0$. Physically W_d is the area of hysteresis in one cycle. Furthermore, P may be written as $P = K_1 U_0 \sin \omega t + K_2 U_0 \cos \omega t$; $K_1 = P_0 \cos \delta / U_0$; $K_2 = P_0 \sin \delta / U_0$; where K_1 and K_2 are the storage and loss stiffnesses of the damper. Quantity $P_0/U_0 = K_0$ represents the elastic stiffness. In the expression $P = K_1 u + K_2 \dot{u}/\omega$ it should be noted that the two parts of P represent respectively the in-phase and 90^o out of phase parts of the force(P). The quantity K_2/ω is the damping coefficient of the damper, $C = K_2/\omega$ and $K_2 = W_d/\pi U_0^2$. The above expressions are used to extract the frequency-dependent properties of the damper from the measured quantities P_0,U_0 and W_d. The experimental values of storage and loss stiffness are plotted in Fig.(3) and Fig.(4) . The strong dependency of mechanical properties of the damper on frequency is evident.

Fractional Derivative Modeling

The mathematical model of the VE damper is written in a form analogous to that of the stress-strain relationship of the Viscoelastic material. This is based on the assumption that the VE material is primarily subjected to shearing action while the shaft moves in the horizontal direction. The force-displacement relationship in the horizontal motion is expressed as

$$P(t) + bD^{\beta}P(t) = ku(t) + cD^{\alpha}u(t) \tag{3}$$

In the above equation P and u are force and displacement respectively. Taking Fourier Transform of Eq.(3) and assuming zero initial condition. The force-displacement relationship becomes:

$$P(\omega) + b(i\omega)^{\beta}P(\omega) = ku(\omega) + c(i\omega)^{\alpha}u(\omega) \tag{4}$$

From Eq.(4) complex stiffness can be deduced as

$$K^{\star} = \frac{k + c(i\omega)^{\alpha}}{1 + b(i\omega)^{\beta}} \tag{5}$$

By using $i^{\alpha} = \cos(0.5\pi\alpha) + i\sin(0.5\pi\alpha)$ and simplyfing Eq.(5); then separating real and imaginary parts of complex stiffness we get

Storage Stiffness:

$$K_1 = \frac{k(1 + c\omega^{\alpha}\cos(0.5\pi\alpha) + b\omega^{\beta}\cos(0.5\pi\beta) + cb\omega^{\alpha+\beta}\cos 0.5\pi(\alpha - \beta))}{(1 + b\omega^{\beta}\cos(0.5\pi\beta))^2 + (b\omega^{\beta}\sin(0.5\pi\beta))^2} \tag{6}$$

Loss Stiffness:

$$K_2 = \frac{k(1 + c\omega^{\alpha}\sin(0.5\pi\alpha) - b\omega^{\beta}\sin(0.5\pi\beta) + cb\omega^{\alpha+\beta}\sin 0.5\pi(\alpha - \beta))}{(1 + b\omega^{\beta}\cos(0.5\pi\beta))^2 + (b\omega^{\beta}\sin(0.5\pi\beta))^2} \tag{7}$$

Parameter Estimation

To estimate the parameters involved in the expression of storage stiffness, a nonlinear least square fit has been used on the experimental values of storage and loss stiffnesses. A powerful alogrithm for least square fit, known as "Leverberg Marquart" alogrithm has been used herein, which takes into account the partial derivative of the involved parameter. These fitted plots of storage and loss modulus is shown in Fig.(3) and Fig.(4).

The values of the estimated parameters are: $k = 0.01$, $c = 0.0193$, $\alpha = 1.0$, $b = 0.48$ and $\beta = 0.6$ Hence the force-displacement relationship in terms of time derivative becomes

$$P(t) + 0.48D^{0.6}P(t) = 0.01u(t) + 0.0193Du(t) \tag{8}$$

ANALYTICAL SCHEMES

In this section, the analytical schemes, to solve the force-displacement relationship developed in the above has been detailed, for the analysis in the frequency domain as well as in the time domain .

Laplace or Frequency Domain

Generally, viscoelastic stress analysis problems becomes more complicated due to involvement time derivative of variables. The time variable can be removed by employing the Laplace Transform. When a solution for the desired variable has been found in terms of the Laplace Transform variable s the inverse Laplace transformation yields the desired solution in the time variable t for the time dependent behavior in the viscoelastic problem. This systematic method is known as Elastic-Viscoelastic Analogy or the 'Correspondence Principle' (Findley et al., 1975). A similiar approach can be used in frequency domain.

Due to above mentioned principle, the numerical schemes in the frequency domain are much more convenient to use. $K^* = K_1(\omega) + iK_2(\omega)$ which represents the amplitude and phase angle of the steady-state force in the damper for a harmonic displacement input of unit amplitude. Accordingly, the time history of force is expressed as:

$$P(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} [K_1(\omega) + iK_2(\omega)]\,\bar{u}(\omega)e^{i\omega t}d\omega \tag{9}$$

where $\bar{u}(\omega)$ represents the Fourier Transform of the imposed motion. The computation of the force is thus obtained by the Discrete Fourier Transform (DFT) approach in combination with Fast Fourier Transform (FFT) algorithms. (Velestos et al., 1985).

Time Domain

Rewritting the force-displacement relationship of Eq.(9)

$$0.48D^{0.6}P(t) = 0.01u(t) + 0.0193Du(t) - P(t) \tag{10}$$

In evaluating $D^{\beta}P(t)$, $P(t)$ is needed, which is unknown. Therefore a iteration process is needed (Markis et al., 1990). Moreover to evaluate the term $D^{\beta}P(t)$ numerically, (Oldham et al., 1974) has given quadrature formulas. The typical term in Eq.(10) is $D^{\alpha}u$ which is evaluated by quadrature formula's by (Oldham et al., 1974). The $D^{\alpha}u$ can be expressed in quadrature form as; $D^{\alpha}u_n = 1/h^{\alpha}\sum_{j=0}^{n} w_j u_j, \quad 0 \leq \alpha < 1$ where w_0, w_{n-j} and w_n are weights (Oldham et al., 1974). Where n = Total no. of time steps ; h = time step size. Acceleration is approximated by central difference method. This renders a multistep numerical scheme:$\bar{w}_{n+1}u_{n+1} = f(nh) - \sum_{j=0}^{n}\bar{w}_j u_j$ (Koh et al., 1985).

Fig.(5) to Fig.(7) demonstrate good agreement between the analytical prediction and recorded force-displacement loops. The force-displacement relationship corresponding to 1 Hz, 2 Hz and 3 Hz has been analyzed to predict experimental results. This analysis has been conducted in the frequency domain.

Looking at the hyteresis loop from Fig.(5) to Fig.(7), it is to be noted, that the major axis of these ellipitical loops rotates towards the force axis as the frequency of excitation increases from 1 Hz to 3 Hz. This indicates the stiffening effect of VE material with the increase in frequency. It is clear from Fig.(5) to Fig.(7) that the area of hystersis loop decrease with the increase in frequency, this indicates that the energy dissipation capacity of the VE material is decreased, with the increase in frequency of excitation.

Therefore it can be concluded that the proposed 5-parameter fractional derivative model accurately predict the hyteresis of the damper used herein.

OPTIMAL DESIGN OF VE DAMPER

In broader sense the passive control device should be attached between those two points which have a relative displacement or in other words there must be a phase lag between the motion of those points. The location of such devices should be decided keeping this point in view. To have a better understanding of relative displacement let us take an example of three span continuous girder bridge as shown in Fig.(8).

Under strong ground motion the maximum relative displacement will take place at the displacement discontinuity at two roller support. This relative displacement are induced, primarly due to vibration of piers as they are more flexible than girder. Therefore the passive control device should be attached between the girder and the pier cap this arrangement will facilate maximum energy dissipation.

MATHEMATICAL FORMULATION

Transformation of Governing Equation of Motion

Firstly, the equations of motion of linear multi degree of freedom structure with viscoelastic dampers has been formulated. The total number of degree of freedom is n, and the structure is subjected to external forces. The governing equation of motion can be expressed as :

$$\mathbf{M}\ddot{\mathbf{z}}(t) + \mathbf{D}\dot{\mathbf{z}}(t) + \mathbf{K}\mathbf{z}(t) = \mathbf{B_0}w(t) \tag{11}$$

In order to apply optimal control theory, the governing equations of motion in physical coordinates $\mathbf{z}$(t) are transformed into modal coordinate $\mathbf{q}$(t) as;

$$\mathbf{I}\ddot{\mathbf{q}}(t) + \mathbf{D_0}\dot{\mathbf{q}} + \mathbf{\Lambda}\mathbf{q} = \mathbf{\Phi_0^T}\mathbf{B_0}w(t) \tag{12}$$

Where $\mathbf{z}(t) = \mathbf{\Phi}\mathbf{q}(t)$, $\mathbf{\Phi}$ is a modal matrix where each column is a modal vector, $\mathbf{\Lambda} = \mathbf{\Phi}^T\mathbf{K}\mathbf{\Phi}$ is a diagonal matrix where its diagonal elements are the square of modal frequencies, $\mathbf{D} = \mathbf{\Phi^T}\mathbf{D}\mathbf{\Phi}$ is modal damping matrix which is usually not a diagonal matrix, i.e non-proportional damping system. Therefore, these equations in the modal coordinates are still coupled. Thus, these equations can not be treated as independent SDOF equations and cannot be solved by classical method. By this reason, the transformation into first order state equations is helpful.

Eq.(2.5) can be representated by a canonical form of first order state equations as:

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}w(t) \tag{13}$$

$$\mathbf{x} = \left\{ \begin{array}{c} \mathbf{q} \\ \dot{\mathbf{q}} \end{array} \right\}; \mathbf{A} = \left[\begin{array}{cc} \mathbf{0} & \mathbf{I} \\ -\mathbf{\Lambda} & -\mathbf{D}^0 \end{array} \right]; \mathbf{B} = \left\{ \begin{array}{c} \mathbf{0} \\ \mathbf{\Phi}_0^T\mathbf{B}_0 \end{array} \right\} \tag{14}$$

In the above equations, $\mathbf{0}$ and $\mathbf{I}$ denote the null matrix and the identity matrix of appropriate dimension respectively. The advantage of this transformation is that most of the control theory have been developed based on this general form of first order state equation. So it is easy to adopt the general control theory in the following procedure. When the values of passive control parameters are changed, some elements in $\mathbf{A}$ and $\mathbf{B}$, and then the response $x(t)$ is modified. For this reason, if we can define a function to represent average response such that it can be efficently evaluated for a given $\mathbf{A}$ and $\mathbf{B}$, we will get desired optimal parameters in a new effective way. The definition of a function which represents response and optimization technique is as follows.

PERFORMANCE INDEX

To conduct optimal design, the first important thing is to set up a criteria or an index to indicate the performance of structure and effectiveness of vibration control system. Performance index is usually defined as a function of response $x(t)$. In order to formulate the performance index for free vibration, and as a matter of fact the response varies between positive and negative sign, performance index should be a integral quadratic form of response. A linear MDOF structure with linear MDOF passive control system. Its performance index is defined for respresenting a time integration of global vibration energy of system, E(t).

$$\mathbf{J} = \int_0^\infty E(t)dt \tag{15}$$

$$\text{E(t)} = \text{Potential Energy} + \text{Kinetics Energy}$$

$$E(t) = \frac{1}{2}\mathbf{z}^T(t)\mathbf{K}\mathbf{z}(t) + \frac{1}{2}\dot{\mathbf{z}}^T(t)\mathbf{M}\dot{\mathbf{z}}(t) \tag{16}$$

Since $\mathbf{z}(t) = \mathbf{\Phi}\mathbf{q}(t)$, then

$$E(t) = \frac{1}{2}\left[\mathbf{q}^T(t)\mathbf{\Phi}^T\mathbf{K}\mathbf{\Phi}\mathbf{q}(t) + \dot{\mathbf{q}}^T(t)\mathbf{\Phi}^T\mathbf{M}\mathbf{\Phi}\dot{\mathbf{q}}(t)\right] \tag{17}$$

$$= \frac{1}{2}\left[\mathbf{q}^T(t)\mathbf{\Lambda}\mathbf{q}(t) + \dot{\mathbf{q}}^T(t)\mathbf{I}\dot{\mathbf{q}}(t)\right] \tag{18}$$

$$= \frac{1}{2}\left\{ \begin{array}{cc} \mathbf{q}^T(t) & \dot{\mathbf{q}}^T(t) \end{array} \right\} \left[\begin{array}{cc} \mathbf{\Lambda} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{array} \right] \left\{ \begin{array}{c} \mathbf{q}(t) \\ \dot{\mathbf{q}}(t) \end{array} \right\} \tag{19}$$

$$= \mathbf{x^T}(t)\mathbf{Q}\mathbf{x}(t) \tag{20}$$

Thus matrix $\mathbf{Q}$ becomes:

$$\mathbf{Q} = \frac{1}{2} \begin{bmatrix} \mathbf{A} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \tag{21}$$

For free vibration oscillation, $w(t) = 0$, the state equation is reduced to the form as

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) \tag{22}$$

The solution is

$$\mathbf{x}(t) = e^{\mathbf{A}(t)}\mathbf{x}(0) \tag{23}$$

where $\mathbf{x}(0)$ is a initial vector of $\mathbf{x}$(t) Substitute Eq.(2.21) into Eq.(2.12), leads to;

$$\mathbf{J} = \int_0^\infty \left[\mathbf{x}^T(0)e^{\mathbf{A}^T(t)}\mathbf{Q}e^{\mathbf{A}(t)}\mathbf{x}(0)\right] dt \tag{24}$$

$$= \mathbf{x}^T(0)\left\{\int_0^\infty e^{\mathbf{A}^T(t)}\mathbf{Q}e^{\mathbf{A}(t)}\right\}\mathbf{x}(0)dt \tag{25}$$

Eq.(23) show the dependence of $\mathbf{J}$ on the initial state $\mathbf{x}(0)$. In order to use $\mathbf{J}$ to evaluate optimal parameters, it is usually necessary to eliminate this dependence on $\mathbf{x}(0)$. Mathematically, a simple way to eliminate the dependence on the initial state is to average $\mathbf{J}$ for a linearly independent set of initial states. This is equivalent to assuming the initial state $\mathbf{x}(0)$ to be a random variable uniformly distributed on the surface of n-dimensional unit sphere (Levine et al., 1974).

$$\mathbf{J}_a = \frac{1}{n} tr \int_0^\infty \left[e^{\mathbf{A}^T(t)}\mathbf{Q}e^{\mathbf{A}(t)}\right] dt \tag{26}$$

Where tr is a trace operator which is a summation of all the diagonal elements in the square matrix. Note that $\mathbf{J}_a$ is identical to average value of $\mathbf{J}$ in Eq.(26) for all possible unit initial state vector $\mathbf{x}(0)$.
In addition, let's define

$$\mathbf{V} \equiv \int_0^\infty \left[e^{\mathbf{A}^T(t)}\mathbf{Q}e^{\mathbf{A}(t)}\right] dt \tag{27}$$

The evaluation of $\mathbf{J}_a$ is reduced into performing infinite integration which is, however, very time consuming. So the numerical optimization, which requires an evaluation of $\mathbf{J}_a$ many times is inefficient. However, there is more computational efficient way to evaluate $\mathbf{V}$ in Eq.(27) is the positive definite solution of the following Lyapunov equation:

$$\mathbf{V}\mathbf{A} + \mathbf{A}^T\mathbf{V} = -\mathbf{Q} \tag{28}$$

Hence, $\mathbf{J}_a$ can be evaluated in effective manner. Without losing generality, the constant $\frac{1}{n}$ is dropped for simplicity, as

$$\mathbf{J}_a = tr[\mathbf{V}] \tag{29}$$

OPTIMIZATION

To obtain the optimal frequency dependent parameter of VE damper it is assumed that the performance index J is only a function of dampers parameters represented by vector (P_a). Now the problem is to find optimal vector P_a which minimizes J. This optimal search of large dimensional vector (P_a) is feasible only by means of computer based numerical optimizer provided that the response index function $J(P_a)$ and the gradient $\partial J/\partial P_a$ are evaluated in efficent manner Rao (Rao et al., 1984). A computer program to calculate $J(P_a)$ is then based on the analytical approach described in the previous section as shown in Fig.(9).
Once these calculation programs are established, various types of numerical optimizer can be incorporated. Since J is nonlinear function of P_a and no explicit constraints in the parameters is involved, Davidon-Fletcher -Powell(DFP) nonlinear programming strategy is employed for the main iterative searching loop. In each loop, the problem is simplified to one dimensional optmization and the optimal solution is searched by cubic interpolation technique.

CONCLUSION

It has been concluded that the five parameter fractional derivative model accurately models the mechanical properties of the viscoelastic damper. The new viscoelastic damper configuration works very well to exploit the energy dissipation capacity of VE material. It is found that the frequency domain analysis is more convenient then time domain for prediction of force-displacement relationship of the damper. A new approach has been developed for rigorous design of VE damper which takes into account not only the frequency dependent behaviour of VE damper but also incorporates the desired performance index for the structure. The mathematical framework developed herein can be applied to any passive control system without many simplifcations and assumptions.

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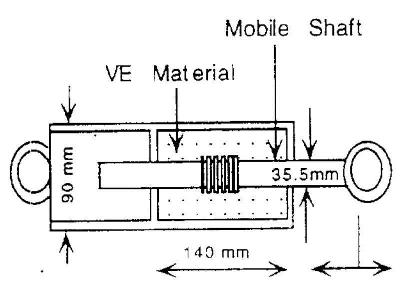

Fig. 1 Viscoelastic Damper

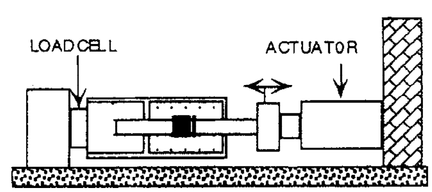

Fig. 2 Experimental Setup

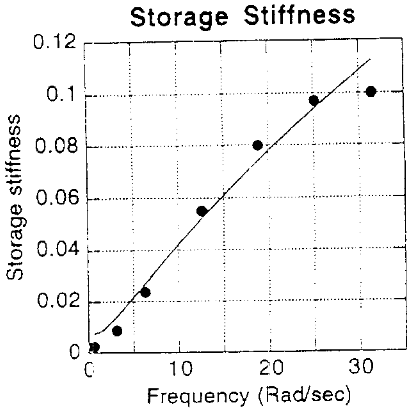

Fig. 3 Fitted Storage Stiffness of VE Damper

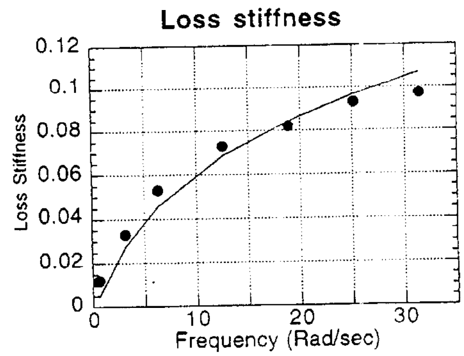

Fig. 4 Fitted Loss Stiffness of VE Damper

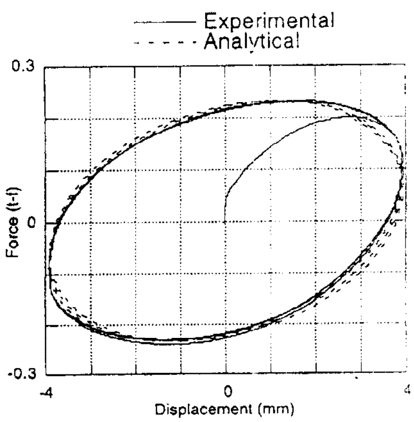

Fig. 5 Hyteresis loop 1 Hz

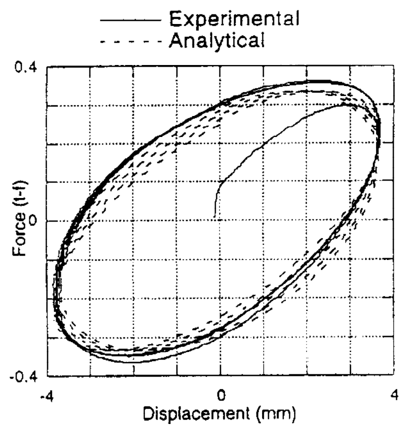

Fig. 6 Hyteresis loop 2 Hz

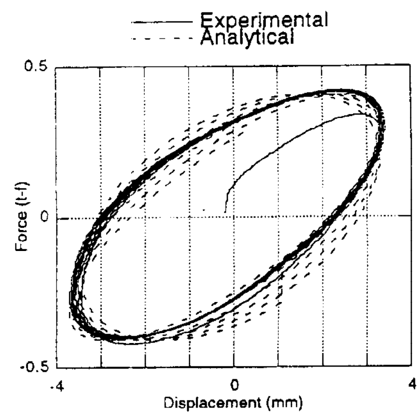

Fig. 7 Hyteresis loop 3 Hz

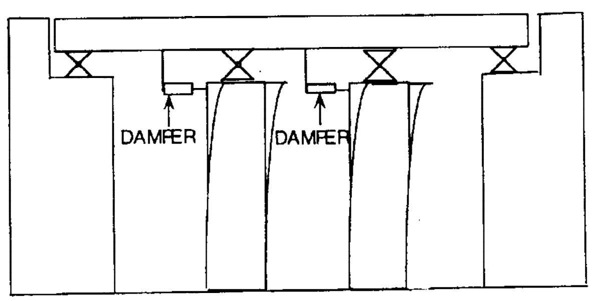

Fig.8 A Typical Bridge With VE Dampers

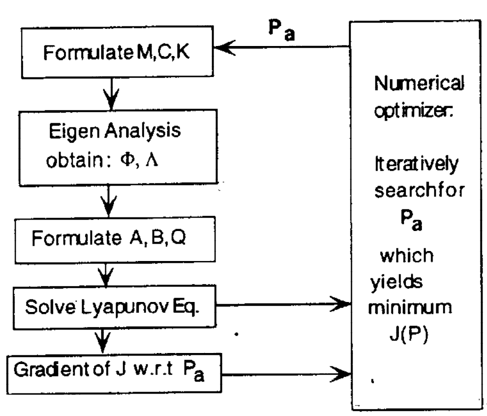

Fig.9 Flow Chart For Optimization