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ON THE ELASTIC WAVE SCATTERING OF A HEMISPHERICAL CANYON IN TIME DOMAIN DUE TO NORMAL INCIDENT P WAVE

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ABSTRACT

In this paper the three-dimensional scattering of normal incident P waves by a hemispherical canyon in the homogeneous elastic half-space is analyzed in the frequency domain. Then the results in time domain are obtained by using the inverse Fourier transform. For this axisymmetric case, two sequences of point source potentials have been obtained in this study by extending the Gregory's method for the two-dimensional case (Gregory, 1967, 1970). They are singular at a specified point in the canyon and satisfy the free surface conditions at the extensive ground and radiation condition at infinity. The scattered wave that satisfies the free surface conditions and consists of outgoing wave at infinity is formed as a sum of these fundamental source potentials with coefficients which are determined from the traction free boundary conditions at the canyon surface in the least square sense. The error of boundary traction can be controlled in an acceptable level. The results in frequency domain can be compared with Mossessian and Dravinski's work (1989) that has used the indirect boundary integral equation method for analysis in the frequency domain. The ground motions generated by a normal incident P plane wave with the form of Ricker wavelet at nearby ground surface are shown, and the three major groups of incident P wave, surface P pulse wave and Rayleigh surface waves at the ground surface are shown in figures and the existence of the reflected Rayleigh surface wave is pointed out.

KEYWORDS

Three-dimensional scattering, hemispherical canyon, Ricker wavelet, source potentials, surface waves.

INTRODUCTION

Local topographic and geological irregularities may cause large spatial variations of seismic ground motion. The problems of two-dimensional irregularities for various incident waves have received many attentions. In reality the three-dimensional model is needed. There are much more difficulties for solving three-dimensional problem. The image method for solving incident *SH* wave can not be used anymore to analyze the three-dimensional case. The *P*, *S* and *Rayleigh* waves are converted mutually between surface irregularity medium and its flat ground surface. This phenomenon makes the three-dimensional scattering problem in semi-infinite medium more complicated than the two-dimensional case. Due to the existence of surface irregularity various kinds of surface waves will be induced. This phenomenon is very interesting and will be discussed in this article. Bard and Bouchon (1980a, 1980b) have used the method of Aki and Larner (1970) to solve the two-dimensional scattering problem in time domain. This method uses discrete wavenumber representation of wave field under the so-called Rayleigh assumption which causes the Rayleigh ansatz error. By applying the method of discrete wave number to the Green's function, Kawase (1988) has used the direct boundary element method to solve two-dimentional problem in time domain and Kim and Papageorgiou (1993) have solved the three-dimensional problem. Wong (1982) has introduced a indirect boundary element method by putting force points on a auxiliary surface to solve the two-dimensional surface topography scattering problem in the frequency domain. Mossessian and Dravinski (1989, 1990a) have extended this method to treat three-dimensional problems. They have also treated this problem in time domain (Mossessian and Dravinski, 1990b). Sanchez-Sesma *et al.* (1993) have used single-layer boundary sources formulation called indirect BEM to solve the three-dimensional problems. However, BEM needs many source points. Another method uses a series of functions to represent the scattered waves then through the least square method or the moment method to match the boundary condition required. Sanchez-Sesma (1983) has used the spherical wave functions of full field as the basis functions of scattered wave series to solve the diffraction problem of three-dimensional surface irregularity in frequency domain. Kawano *et al.* (1994) have used that method in time domain. The major disadvantage of that method is that their basis functions can not simulate the Rayleigh surface wave. Their basis functions are not complete . If the Rayleigh surface wave were not included in the scattered waves, the response in the high frequency range will cause large error.

Gregory (1967, 1970) has solved a problem of wave propagation in a two-dimensional half-space containing a circular cylindrical cavity. He has introduced two sequences of line source potentials that are singular along the axis of the cylinder, satisfy the free surface condition at the flat ground surface and represent the feature of outgoing wave at infinity (the radiation condition). Any solution of the governing equations (wave equations in frequency domain), which satisfies the free surface conditions and consists of outgoing waves at infinity, is expandable as a sum of these fundamental source potentials. The coefficients of source potentials can be determined from the boundary conditions at the cylindrical surface only. His approach provides a direct way to solve the wave problem in the semi-infinite medium. For three-dimensional problem we have adopted the same approach in frequency domain (Yeh and Yeh, 1994a). In this paper we use two sequences of point source potentials to solve the diffraction problem in time domain. This method provides a well behavior solution for numerical approach.

FORMULATIONS OF THE POINT SOURCE POTENTIALS

In the absence of body force, the equation of motion (Pao and Mow, 1971) for a linear isotropic elastic body is

$$(\lambda + \mu)\nabla\nabla \cdot \mathbf{u} + \mu\nabla^2\mathbf{u} = \rho\ddot{\mathbf{u}}, \tag{1}$$

where λ and μ are known as the Lame's constants.

In this paper we consider the axisymmetric problem. In cylindrical coordinates (ρ,ϕ,z) all functions are independent of ϕ. The displacement vector $\mathbf{u}$ can be decomposed to two scalar potential function,

$$\mathbf{u} = \nabla\varphi + \nabla\times\nabla\times(\chi\mathbf{e}_z). \tag{2}$$

The equation of motion (1) can be replaced by two scalar wave equations,

$$c_p^2\nabla^2\varphi = \ddot{\varphi}, \qquad c_p^2 = (\lambda + 2\mu)/\rho, \tag{3}$$

and

$$c_s^2\nabla^2\chi = \ddot{\chi}, \qquad c_s^2 = \mu/\rho, \tag{4}$$

where c_p and c_s are the wave speed of P wave and S wave, respectively.
From the Fourier transform

$$\begin{cases} \hat{F}(\mathbf{x},\omega) = \dfrac{1}{2\pi}\int_{-\infty}^{+\infty} F(\mathbf{x},t)e^{i\omega t}dt, \\ F(\mathbf{x},t) = \int_{-\infty}^{+\infty} \hat{F}(\mathbf{x},\omega)e^{-i\omega t}d\omega,. \end{cases} \tag{5}$$

we can transfer the time domain problems to frequency domain. In the frequency domain (3) and (4) can be reduced to

$$\nabla^2\hat{\varphi} + \alpha^2\hat{\varphi} = 0, \qquad \alpha = \omega / c_p, \tag{6}$$

$$\nabla^2\hat{\chi} + \beta^2\hat{\chi} = 0, \qquad \beta = \omega / c_s, \tag{7}$$

where α and β are the wave number of P wave and S wave, respectively.
We now construct two sequences of point source potentials for $\hat{\varphi}$ *and* $\hat{\chi}$ sources. Each point source potential forms a displacement field that has one singular point in $z > 0$ and satisfies the traction free boundary condition at $z = 0$ surface and radiation condition at $z \to +\infty$.
For $\hat{\varphi}$ point source potential, consider a point source at $z = z_0 > 0, \rho = 0$,

$$\hat{\varphi}_n^0 = \int_0^\infty S^p(\xi)e^{-\nu_\alpha|z-z_0|}J_0(\xi\rho)d\xi, \tag{8}$$

$$S^p(\xi) = \begin{cases} \dfrac{\xi^{n+1}\cdot a^{n+3}}{\nu_\alpha}, & \text{for } n = 0,2,4,6,\ldots \\ \operatorname{sgn}(z_0 - z)\cdot\xi^n a^{n+3}, & \text{for } n = 1,3,5,7,\ldots \end{cases} \tag{9}$$

where J_0 is the zero order first kind Bessel function and $\nu_\alpha = \sqrt{\xi^2 - \alpha^2}$. The condition $\operatorname{Re}\nu_\alpha > 0$ and $\operatorname{Im}\nu_\alpha \le 0$ will guarantee the radiation condition satisfied. For each source potential, a pair of potentials $\hat{\varphi}_n^p$ and $\hat{\chi}_n^p$ are required to satisfy the following conditions:

(a) $\hat{\varphi}_n^p - \hat{\varphi}_n^0$ and $\hat{\chi}_n^p$ are regular functions in $z > 0$, and satisfy the reduced wave equations (6),

(b) the scattered wave formed by $\hat{\varphi}_n^p$ and $\hat{\chi}_n^p$ satisfies the traction free conditions at $z = 0$ plane,

and

(c) the pair $\hat{\varphi}_n^p$ and $\hat{\chi}_n^p$ represent outgoing waves as $z \to +\infty$.

To achieve these three conditions, we observe the source potential near the $z = 0$ plane,

$$\hat{\varphi}_n^0 = \int_0^\infty F(\xi;\alpha)a^{n+3}e^{-\nu_\alpha z_0}e^{\nu_\alpha z}J_0(\xi\rho)d\xi, \tag{10}$$

where

$$F(\xi;\alpha) = \begin{cases} \dfrac{\xi^{n+1}}{\nu_\alpha}, & n = 0,2,4,\ldots \\ \xi^n, & n = 1,3,5,\ldots \end{cases} \tag{11}$$

Let

$$\hat{\varphi}_n^p - \hat{\varphi}_n^0 = \int_0^\infty A(\xi) e^{-\nu_\alpha z_0} e^{-\nu_\alpha z} J_0(\xi\rho) d\xi, \tag{12}$$

$$\hat{\psi}_n^p = \int_0^\infty B(\xi) e^{-\nu_\alpha z_0} e^{-\nu_\beta z} J_0(\xi\rho) d\xi, \tag{13}$$

where $\nu_\beta = \sqrt{\xi^2 - \beta^2}$, $\mathrm{Re}\,\nu_\beta > 0$, $\hat{\varphi}_n^p$ and $\hat{\chi}_n^p$ satisfy the conditions (a) and (c). Solving for the condition (b), we get (Lamb, 1904)

$$A(\xi) = \frac{(2\xi^2 - \beta^2)^2 + 4\xi^2 \nu_\alpha \nu_\beta}{R(\xi)} \cdot F(\xi;\alpha) \cdot a^{n+3}, \tag{14}$$

$$B(\xi) = \frac{4\nu_\alpha (2\xi^2 - \beta^2)}{R(\xi)} \cdot F(\xi;\alpha) \cdot a^{n+3}, \tag{15}$$

where $R(\xi) = 4\xi^2 \nu_\alpha \nu_\beta - (2\xi^2 - \beta^2)^2$, and the root of $R(\xi) = 0$ is the Rayleigh wave number ξ_R.
For $\hat{\chi}$ point source potential, we consider a sequence of point source potentials at the point $z = z_0, \rho = 0$,

$$\hat{\chi}_n^0 = \int_0^\infty S^{sv}(\xi) e^{-\nu_\beta |z - z_0|} J_0(\xi\rho) d\xi, \tag{16}$$

$$S^{sv}(\xi) = \begin{cases} \dfrac{\xi^{n+1} \cdot a^{n+4}}{\nu_\beta}, & \text{for } n = 0,2,4,6,\ldots \\ \mathrm{sgn}(z_0 - z) \cdot \xi^n a^{n+4}. & \text{for } n = 1,3,5,7,\ldots \end{cases} \tag{17}$$

Follow the same procedure for $\hat{\varphi}$ source potentials we have a pair of potentials $\hat{\varphi}_n^s$ and $\hat{\chi}_n^s$,

$$\hat{\varphi}_n^s = \int_0^\infty C(\xi) e^{-\nu_\beta z_0} e^{-\nu_\alpha z} J_0(\xi\rho) d\xi, \tag{18}$$

$$\hat{\psi}_n^s - \hat{\psi}_n^0 = \int_0^\infty D(\xi) e^{-\nu_\beta z_0} e^{-\nu_\beta z} J_0(\xi\rho) d\xi. \tag{19}$$

where

$$C(\xi) = \frac{4\nu_\beta \xi^2 (2\xi^2 - \beta^2)}{R(\xi)} \cdot F(\xi;\beta) \cdot a^{n+4}, \tag{20}$$

$$D(\xi) = \frac{(2\xi^2 - \beta^2)^2 + 4\nu_\alpha \nu_\beta \xi^2}{R(\xi)} \cdot F(\xi;\beta) \cdot a^{n+4}. \tag{21}$$

FREQUENCY-DOMAIN RESPONSE OF A HEMISPHERICAL CANYON

The geometry of the problem is illustrated in Fig.1. The flat ground surface $z = 0$ and canyon surface are traction free. The shape of the canyon is hemispherical surface with radius a. The tremor of the ground is due to a normal incident P plane wave.

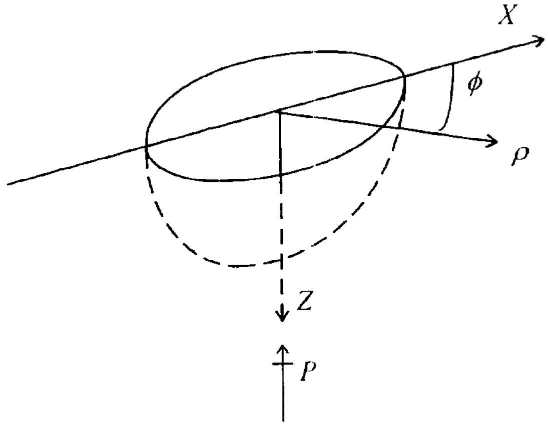

Fig. 1. Hemispherical canyon

Conveniently the total wave field can be separated into three parts that are incident wave $\mathbf{u}^i$, reflected wave $\mathbf{u}^r$ and scattered wave $\mathbf{u}^s$. In the absence of the canyon, the reflected wave is caused by the incident wave imparting on the flat ground surface. The free field response,

$$\mathbf{u}^{i+r} = \mathbf{u}^i + \mathbf{u}^r, \tag{22}$$

satisfies the traction free condition at surface $z = 0$. The free field response of normal incident P plane wave is

$$\begin{cases} u_z^{i+r} = a \cdot \left(e^{-i\alpha z} + e^{i\alpha z}\right), \\ u_\rho^{i+r} = 0. \end{cases} \tag{23}$$

Because of the existence of canyon, the scattered waves are induced. However, the total wave field must satisfy the traction free conditions at canyon surface and the flat ground as well. Thus the scattered wave $\mathbf{u}^s$ must satisfy the traction free condition at $z = 0$, and the radiation condition as $z \to +\infty$.

Considering the nature of the scattered wave, its corresponding potentials $\hat{\varphi}$ and $\hat{\chi}$ can be formed by two source potential sequences $(\hat{\varphi}_n^p, \hat{\chi}_n^p)$ and $(\hat{\varphi}_n^s, \hat{\chi}_n^s)$,

$$\hat{\varphi} = \sum_{n=0}^{\infty}\left(A_n \hat{\varphi}_n^p + B_n \hat{\varphi}_n^s\right), \tag{24}$$

and

$$\hat{\chi} = \sum_{n=0}^{\infty}\left(A_n \hat{\chi}_n^p + B_n \hat{\chi}_n^s\right). \tag{25}$$

In numerical approach the infinite series in (24) and (25) must be truncated at $n = N - 1$. The $2N$ unknown coefficients (A_n, B_n) are determined by applying the traction free condition at canyon surface. At the canyon surface we have

$$\begin{cases} \sigma_{nn}^s + \sigma_{nn}^{i+r} = 0, \\ \sigma_{nt}^s + \sigma_{nt}^{i+r} = 0. \end{cases} \tag{26}$$

According Gaussian quadrature rule we choose $2N$ points at canyon surface, then (26) gives $4N$ equations for determining $2N$ unknowns (A_n, B_n) in least square sense. The number of N depends on the nondimensional frequency $\eta = \omega a/(\pi C_s)$. For the class of examples considered in this paper, N is between 10 to 20. Our results are vary close to the results of Mossessian and Dravinski (1989) and Yeh and Yeh (1994b).

TIME-DOMAIN RESPONSE OF A HEMISPHERICAL CANYON

After we have the results of the frequency domain solutions, we can get the time domain solutions from the inverse Fourier transform. We consider the shape of a normal incident P plane wave is a Ricker wavelet defined as (Ricker, 1977)

$$u_z(z,t) = (2b^2 - 1)e^{-b^2}, \tag{27}$$

where $b = \pi(t - t_0)/t_c$ and $t_0 = (z_r - z)/C_p$, z_r is a reference position, t_c is the characteristic period. The nondimensional time τ is defined as $t \cdot C_s/a$. The dominate frequency η_c of Ricker wavelet is at $2/\tau_c$. The calculated frequency η ranges from 0 to $7/\tau_c$. In the example, τ_c is set to be 1.55 and Poisson ratio is set to be $1/3$. The calculated frequencies η are 30 in total, ranging from 0.15 to 4.5. Figures 2 and 3 show the vertical and radial displacements of a canyon, respectively. The incident P wave and refracted surface P pulse which is the sPs (Lapwood, 1949) wave along the surface, Rayleigh surface wave with different phase velocity are easy identified in these Figures. In Fig. 4 the vertical and radial displacements relative to time at position $x = 8a$ are shown. From this figure we can see the phase difference between vertical and radial displacements for sPs wave and Rayleigh wave.

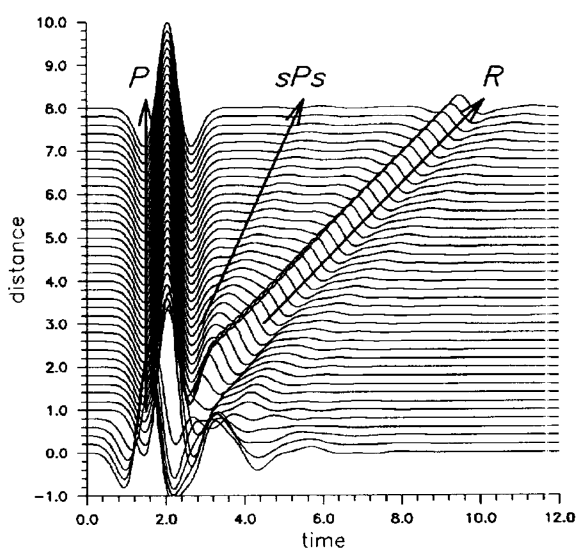

Fig. 2. The vertical displacements of canyon

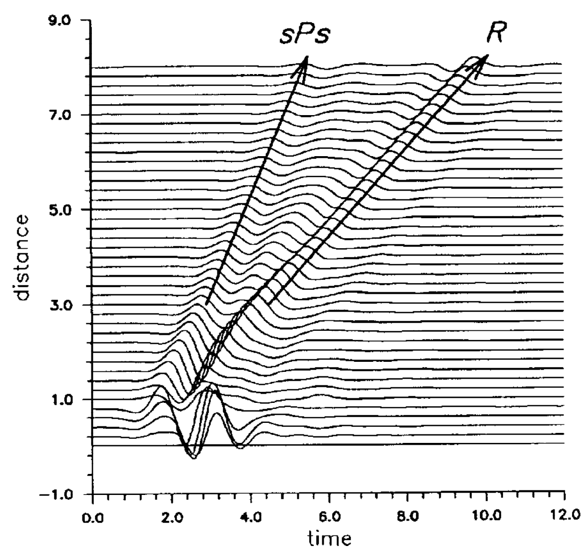

Fig. 3. The radial displacements of canyon

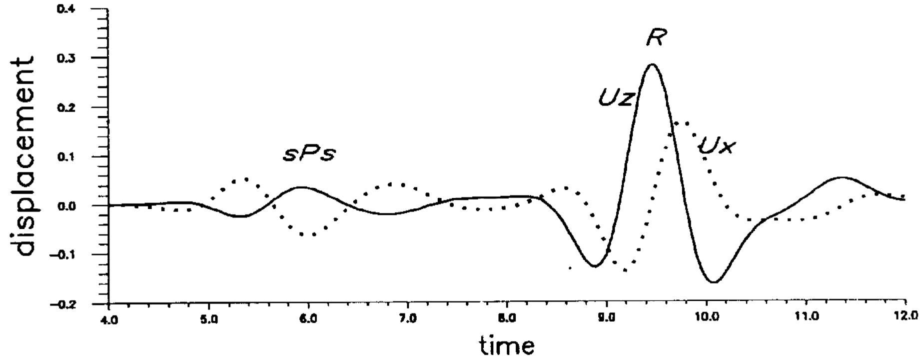

Fig. 4. The phase difference between vertical and radial displacements

CONCLUDING REMARKS

Time-domain response of a hemispherical canyon subject to normal incident P wave with a Ricker wavelet shape is studied. To calculate the response in wave field, a new set of basis functions of series solution is developed. The basis functions based on point source potentials will not lose the contribution of Rayleigh surface wave which is proved in the time-domain response. The accuracy of the proposed method was successfully tested against published results in the low frequency range. From the results we also believe the method can be used in a wide range of frequency.

Although the results shown here are only for normal incident P plane wave, the other cases for various incident waves can be studied by this approach without difficulty. To understand the diffracted waves generated from canyon, the case of normal incident P plane wave is enough and clear.

For the general three-dimensional problem three sequences of point source potentials and one plane source potential are needed. The more general cases can be analyzed by this method and will be present in the sequential papers.

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REFERENCE

Aki, K. and Larner, K. L. (1970). Surface motion of a layered medium having an irregular interface due to incident plane SH waves. *J. Geophys. Res.* 75: 1921-1941.

Bard, P. Y. and Bouchon, M. (1980a). Seismic response of sediment-filled valleys, Part 1, The case of incident SH waves. *Bull. Seism. Soc. Am.* 70(4): 1263-1286.

Bard, P. Y. and Bouchon, M. (1980b). Seismic response of sediment-filled valleys, Part 2, The case of incident P and SV waves. *Bull. Seism. Soc. Am.* 70(5): 1921-1941.

Gregory, R.D. (1967). An expansion theorem applicable to problems of wave propagation in an elastic half-space containing a cavity. *Proc. Comb. Phil. Soc.* 63: 1341-1367.

Gregory, R.D. (1970). The propagation of waves in an elastic half-space containing a cavity. *Proc. Comb. Phil. Soc.* 67: 689-709.

Kawano, M., Matsuda, S., Toyoda K. and Yamada, J. (1994). Seismic Response of three-dimensional alluvial deposit with irregularities for incident wave motion from a point source. *Bull. Seism. Soc. Am.* 84(6): 1801-1814.

Kawase, H. (1988). Time-domain response of a semi-circular canyon for incident SV, P, and Rayleigh waves calculated by the discrete wavenumber boundary element method. *Bull. Seism. Soc. Am.* 78(6): 1415-1437.

Kim, J. and Papageorgiou, A. S. (1993). Discrete wave-number boundary-element method for 3-D scattering problems. *J. Eng. Mechanics* 119(3):603-624.

Lamb, H. (1904). On the propagation of tremors over the surface of an elastic solid. *Phil. Trans.* A203: 1-42.

Lapwood, E.R. (1949). The disturbance due to a line source in a semi-infinite elastic medium. *Philos. Trans. Roy. Soc. (London).* A242: 63-100.

Mossessian, T. K. and Dravinski, M. (1989). Scattering of elastic waves by three-dimensional surface topographies. *Wave motion* 11: 579-592.

Mossessian, T. K. and Dravinski, M. (1990a). Amplification of elastic waves by a three dimensional valley. Part 1: steady state response. *Earthquake Engineering and Structural Dynamics* 19: 667-680.

Mossessian, T. K. and Dravinski, M. (1990b). Amplification of elastic waves by a three dimensional valley. Part 2: transient response. *Earthquake Engineering and Structural Dynamics* 19: 681-691.

Pao, Y.H. and Mow C.C. (1971). *Diffraction of elastic waves and dynamic stress concentrations.* A Rand Corporation Research Study.

Ricker, N. H..(1977). *Transient Waves in Visco-elastic Media.* Elsevier. Amsterdam.

Sanchez-Sesma, F.J. (1983). Diffraction of elastic waves by three-dimensional surface irregularities. *Bull. Seism. Soc. Am.* 73(6): 1621-1636.

Sanchez-Sesma, F.J., Ramos-Martinez, J. and Campillo, M. (1993). An indirect boundary element method applied to simulate the seismic response of alluvial valleys for incident P, S and Rayleigh waves. *Earthquake Engineering and Structural Dynamics* 22: 279-295.

Wong, H.L. (1982). Effect of surface topography on the diffraction of P, SV, and Rayleigh waves. *Bull. Seism. Soc. Am.* 72(4): 1167-1183.

Yeh, Y.-K. and Yeh, C.-S. (1994a). On the predominant period of a hemispherical canyon. *Proc. 10th European Conf. on Earthquake Engineering,* Vienna, Austria.

Yeh, Y.-K. and Yeh, C.-S. (1994b). Earthquake basin topography effect 3D model research. *Report NSC84-2621-P-319-003-B,* National Science Council, Taipei, Taiwan, R.O.C..