



## THE USE OF SEMI-ACTIVELY CONTROLLED INTERACTIONS FOR EARTHQUAKE RESPONSE CONTROL OF STRUCTURES

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### ABSTRACT

This paper examines the performance of a structural response control approach which involves controlled interactions between structural systems or components to reduce resonance buildup during seismic excitation. The strategy of the control approach is to remove energy associated with vibration from one of the systems. Certain types of elements are employed to physically produce the interactions, which consist of reaction forces applied to the systems. The mechanical properties of the interaction elements are altered in real time by using switching components to effect changes in the reaction forces which are favorable to the control strategy. Using an ensemble of well-known earthquake time-histories as excitation input, numerical simulations are performed and some results are presented for several kinds of control cases.

### KEYWORDS

Active Interaction Control; Earthquake Response Controlled Structure; Semi-Active Control Approach.

### INTRODUCTION

A study has been initiated to examine the performance of a class of *semi-active* control approaches referred to as *Active Interaction Control*. The control approaches within this class utilize controlled interactions between two distinct structural systems — or different components of a single structural system — to reduce the resonance buildup in structural response that develops during an external excitation. Control devices (or *elements*) are employed to physically produce the interactions between the systems. The proposed approach differs from some other approaches in that the sensors, processors, and switching components involved all operate actively, whereas the interaction elements function passively. The major advantage of this semi-active technology is that relatively-large control forces can be generated with minimal power requirements, which is of prime importance for the control of relatively-massive systems. Early investigative efforts on this control approach are discussed in Hayen and Iwan (1994).

In the most simple form, the strategy of the control approach is to remove energy associated with vibration from only one system (the *primary* system). This process is accomplished through: the transfer of energy to another system (the *auxiliary* system) by means of the interaction elements; the dissipation of energy in the interaction elements; or a combination of both these methods. In a more complex form, the control strategy may be to minimize some composite response measure of the combined primary-auxiliary system. Only the most simple form is considered in the present study, and the auxiliary system is presumed capable, by nature or design, of absorbing any additional energy received as a result of the control effort.

Several scenarios for application of the proposed approach may be envisioned: one is that the systems represent two adjacent multi-story buildings; another is that the primary system represents a single multi-story building, whereas the auxiliary system could be an externally-situated resilient frame or possibly a relatively-small unrestrained mass — or even be completely absent (in this latter scenario, the interaction elements are internally-mounted control devices). The interactions consist of reaction forces developed within and transmitted through the elements attached between the two systems (or different points of a single system). The mechanical properties of these elements are altered in real time by control signals so that the reaction forces applied to the systems can be controlled in an optimal manner.

A preliminary study which used linear single-degree-of-freedom (SDOF) models for primary and auxiliary systems was initially conducted (Hayen, 1995). Numerical simulations were performed for several kinds of control cases using horizontal ground accelerations from an ensemble of earthquake time-histories as excitation input. In each of these cases, the system parameters were specified and a particular type of interaction element was considered. The simplicity of this study was very beneficial in formulating the structural control problem, guiding the selection of an appropriate set of parameters, and suggesting reasonable choices for the parameter values. Most importantly, this study facilitated the development of a basic methodology for implementing the control strategy, which could then be extended to higher-order systems.

A follow-on study which used linear multiple-degree-of-freedom (MDOF) models for primary and auxiliary systems, intended to better represent actual structural systems, was subsequently conducted (Hayen, 1995). Based upon insights obtained from results of the SDOF system study, a limited number of control cases were considered, which included those deemed most effective and implementable. Numerical simulations were again performed using the same excitation input as for the preliminary study. The control strategy used for these cases was aimed at reducing the response contributed by the dominant mode of vibration associated with the primary system. Uniformly-discretized models of a 6-story primary system were considered in most cases. Due to space limitations, only portions of the follow-on study are presented and discussed herein.

## PROBLEM FORMULATION

The following conditions are assumed to hold: 1) the primary system and the auxiliary system (when present) are subjected to the same base acceleration and respond linearly; 2) the systems are discrete models of multi-story structures, which account for only horizontal displacements of the nodal masses from their equilibrium positions; 3) the systems possess (classical) normal response modes, and the damping matrix resulting from the discretization process is obtained by specifying values for the modal damping ratios; 4) the interaction elements function passively, are considered to be massless, and are characterized by parameters representing mechanical properties that may be instantaneously changed in real time by control signals; 5) the system states are completely observable, and all system parameters have been identified in advance; and 6) only current values of the system state variables are available to determine the control input.

The models used in this study represent planar, uniformly-discretized structural systems, consisting of a series of identical stories formed by floor and roof slabs that are interconnected by elastic support columns which provide linear restoring forces. The mass of each slab is  $m_i$ , and the effective stiffness of all columns at each story is  $k_i$ , where  $i \in \{1, 2\}$  (1 denotes the primary system; 2 denotes the auxiliary system). The *relative* displacements of the slabs in each system are measured from a vertically-erected datum line fixed to the base of each system. A schematic illustration of a generic primary or auxiliary system is provided in Fig. 1.

Under these conditions, the equations of motion for the primary and auxiliary systems are expressible as

$$M_i \ddot{x}_i + C_i \dot{x}_i + K_i x_i = f_i - M_i \ddot{y}_i \quad (1)$$

where the vectors  $x_i \in R^{n_i}$ ,  $f_i \in R^{n_i}$ , and  $v_i \in R^{n_i}$  represent the relative displacements, control forces, and excitation input associated with the  $i$ th system, respectively. Each component of  $v_i$  is equal to  $\ddot{y}_i$ , where  $y_i$  is the absolute displacement of the base of each system. As usual, the matrices  $M_i$ ,  $C_i$ , and  $K_i$  (assumed to be symmetric and positive definite) are directly related to the kinetic, dissipative, and elastic properties of the uncontrolled  $i$ th system, respectively, and  $n_i$  is the number of structural stories of that system.

Let the components of the vector  $\mathbf{u} \in R^r$ , referred to as the control input, represent the reaction forces developed within each of the interaction elements. It is assumed that the relationship between the control forces acting on the  $i$ th system and the control input is of the form  $\mathbf{f}_i = -L_i \mathbf{u}$ . The precise definition of the matrix  $L_i$  depends upon the particular interaction configuration used but otherwise can be readily determined. Using this relation, (1) may be recast into a state space form

$$\dot{\bar{\mathbf{z}}}_i = \bar{A}_i \bar{\mathbf{z}}_i + \bar{B}_i^u \mathbf{u} + \bar{B}_i^v \mathbf{v}_i \quad (2)$$

where

$$\bar{\mathbf{z}}_i = \begin{Bmatrix} \bar{\mathbf{x}}_i \\ \dot{\bar{\mathbf{x}}}_i \end{Bmatrix}, \quad \bar{\mathbf{x}}_i = \Phi_i^T \mathbf{x}_i \Rightarrow \dot{\bar{\mathbf{x}}}_i = \Phi_i^T \dot{\mathbf{x}}_i; \quad \bar{A}_i = \begin{bmatrix} 0 & I \\ -\bar{D}_i^k & -\bar{D}_i^c \end{bmatrix}, \quad \bar{B}_i^u = \begin{bmatrix} 0 \\ -\bar{P}_i^u \end{bmatrix}, \quad \bar{B}_i^v = \begin{bmatrix} 0 \\ -\bar{P}_i^v \end{bmatrix} \quad (3)$$

0 and  $I$  represent null and identity matrices, respectively, of appropriate dimensions. In addition,

$$\bar{D}_i^k = \begin{bmatrix} \omega_{i,1}^2 & & 0 \\ & \ddots & \\ 0 & & \omega_{i,n_i}^2 \end{bmatrix}, \quad \bar{D}_i^c = \begin{bmatrix} 2\zeta_{i,1}\omega_{i,1} & & 0 \\ & \ddots & \\ 0 & & 2\zeta_{i,n_i}\omega_{i,n_i} \end{bmatrix}; \quad \bar{P}_i^u = \Phi_i^T M_i^{-1} L_i, \quad \bar{P}_i^v = \Phi_i^T \quad (4)$$

The  $x_{i,j}$  are *modal coordinates* for the  $i$ th system, with  $j \in \{1, \dots, n_i\}$ , reflecting the well-known result that the total response of a linear dynamical system may be decomposed into specific modes of vibration.  $\Phi_i$  is an orthogonal matrix whose columns consist of the eigenvectors of  $M_i^{-1}K_i$ , denoted as the  $\eta_i^j$ . Based upon the assumptions stated, the natural frequencies and mode shapes of the  $i$ th system are given by

$$\omega_{i,j} = 2\omega_{i,o} \sin \left[ \frac{(2j-1)\pi}{(2n_i+1)2} \right], \quad \omega_{i,o} = \sqrt{\frac{k_i}{m_i}}, \quad \eta_i^j = \begin{Bmatrix} \varphi_{i,1j} \\ \vdots \\ \varphi_{i,n_i j} \end{Bmatrix}, \quad \varphi_{i,kj} = c_{i,j} \sin \left[ \frac{(2j-1)k\pi}{(2n_i+1)} \right] \quad (5)$$

where the  $\varphi_{i,kj}$  are the entries of  $\Phi_i$ , and the  $c_{i,j}$  are constants selected so that the  $\eta_i^j$  are orthonormal with respect to one another. For simplicity, it is assumed that a single value for the fraction of critical damping applies equally to all response modes (*i.e.*,  $\zeta_{i,j} = \zeta_{i,o}$ ). Hereafter, attention will be focused on the primary system. When the leading subscript on the notation is absent, the primary system is implied.

## CONTROL STRATEGY

The objective of the proposed approach is to reduce the maximum absolute values of the *story drifts* (*i.e.*, the relative displacements between adjacent nodal masses) of the controlled system from those which occur for the uncontrolled system. In Hayen (1995), it is shown that the primary system *relative vibrational energy*,  $E$ , provides an upper bound for the absolute values of the story drifts, where

$$E \equiv \frac{1}{2} \dot{\mathbf{x}}^T M \dot{\mathbf{x}} + \frac{1}{2} \mathbf{x}^T K \mathbf{x} \Rightarrow E = \frac{1}{2} \bar{\mathbf{z}}^T \bar{S} \bar{\mathbf{z}} \quad (6)$$

and

$$\bar{S} = \begin{bmatrix} D^k & 0 \\ 0 & D^m \end{bmatrix} = \begin{bmatrix} D^m & 0 \\ 0 & D^m \end{bmatrix} \begin{bmatrix} \bar{D}^k & 0 \\ 0 & I \end{bmatrix}; \quad D^k = \Phi^T K \Phi, \quad D^m = \Phi^T M \Phi \quad (7)$$

As may be verified,  $D^k$  and  $D^m$  are also diagonal matrices, in which case it is observed that

$$E = \sum_{j=1}^n E_j; \quad E_j \equiv \frac{1}{2} d_j^m (\dot{\bar{x}}_j^2 + \omega_j^2 \bar{x}_j^2) \quad (8)$$

where  $E_j$  is that portion of  $E$  contributed by the  $j$ th response mode, and  $d_j^M$  is the  $j$ th diagonal entry of  $D^M$ . In addition, by differentiating (6) and using (2) and (8), it may be shown that

$$\dot{E}_j = d_j^M (\ddot{\bar{x}}_j + \omega_j^2 \bar{x}_j) \dot{\bar{x}}_j = -d_j^M \left[ \sum_{k=1}^r \bar{p}_{jk}^u u_k + \sum_{l=1}^n \bar{p}_{jl}^v v_l + 2\zeta_o \omega_j \dot{\bar{x}}_j \right] \dot{\bar{x}}_j \quad (9)$$

The control strategy is to remove energy from the dominant response mode — say, the  $s$ th response mode — to the extent allowed by the constraints on the control input. This strategy emerges because the dominant response mode is expected to provide the largest contribution to the total energy. Hence, the control effort is aimed at regulating  $E_s$  in order to maintain  $E$  as small as possible.

For the purposes of both formulating a control algorithm and implementing the control strategy, efforts are directed toward minimizing the change in  $E_s$  (or, preferably, causing this change to be as negative as possible) over some time interval  $[t_c, t_d]$ . This change, denoted by  $\Delta E_s$ , is evaluated from

$$\Delta E_s = -d_s^M \int_{t_c}^{t_d} \sum_{k=1}^r \bar{p}_{sk}^u u_k \dot{\bar{x}}_s dt - d_s^M \int_{t_c}^{t_d} \sum_{l=1}^n \bar{p}_{sl}^v v_l \dot{\bar{x}}_s dt - d_s^M \int_{t_c}^{t_d} 2\zeta_o \omega_s \dot{\bar{x}}_s^2 dt \quad (10)$$

The approach adopted is to focus only on the first term in (10) and its effect upon  $\Delta E_s$ , since only this term can be affected by the control input; the second and last terms are ignored since they are externally imposed and nonpositive definite, respectively. Ideally, the  $k$ th interaction element should be operated such that the quantity  $\bar{p}_{sk}^u u_k \dot{\bar{x}}_s$  is nonnegative definite at every instant in time during the interval  $[t_c, t_d]$ , ensuring that the first term in (10) is nonpositive definite (recall  $d_s^M > 0$ , by hypothesis). Practically, the enforcement of this condition is accomplished only at isolated points in time, at which information is sampled and an appropriate operating state is determined for each element: *activated* or *deactivated*. In the activated state, interactions are enabled; in the deactivated state, interactions are disabled. When an element operating state other than the current one is selected, the element mechanical properties are altered by switching components.

## INTERACTION ELEMENTS

Originally, several types of interaction elements were considered. Due to space limitations, only the most effective element type is discussed here. This element consists of a linear elastic member, of fixed stiffness  $k_{int}$ , assembled in series with a connecting member which remains rigid when locked but internally yields when unlocked. In the activated state, the connecting member is locked and the element behaves simply as a linear elastic device. In the deactivated state, the connecting member yields in an extremely-fast manner, rapidly dissipating the strain energy accumulated in the elastic member and quickly reducing the reaction force level to zero. During activation, the element serves as both an energy storage and energy transfer device for the purpose of extracting vibrational energy from the primary system. During deactivation, the element internally yields, commencing dissipation of stored energy and termination of the interaction.

For the sake of definiteness, a linear viscous damper is considered for the connecting member, whose damping coefficient  $c_{int}$  may be switched to either one of two fixed values:  $c_h$ , the *high* value of  $c_{int}$ ; or  $c_l$ , the *low* value of  $c_{int}$ . Using  $u$  to denote the element reactive force and  $\varepsilon$  to denote the element deformation (see Fig. 2), the governing differential equation for the element is given by

$$\dot{u} = k_{int} \dot{\varepsilon} - \frac{u}{\tau}; \quad \tau \equiv \frac{c_{int}}{k_{int}} \quad (11)$$

Thus, the device may be modelled as a *Maxwell viscoelastic element* with a variable relaxation time  $\tau$ . In the activated (*i.e.*, locked) state,  $c_{int} = c_h$ , with  $c_h \rightarrow \infty$ . In the deactivated (*i.e.*, unlocked) state,  $c_{int} = c_l$ , with  $c_l \rightarrow 0$ . The limiting values indicated for  $c_h$  and  $c_l$  ensure the desired operating behavior for the element. This conclusion can be confirmed by first recognizing that (11) may be integrated to obtain

$$u(t) = u(t_0) e^{-(t-t_0)/\tau} + k_{int} \int_{t_0}^t e^{-(t-\bar{t})/\tau} \dot{\bar{e}}(\bar{t}) d\bar{t} \quad (12)$$

and then inspecting the behavior of  $u(t)$  as  $c_h$  or  $c_l$  (hence,  $\tau$ ) approaches its limiting value.

## NUMERICAL STUDY

Numerical simulations are performed for several control cases using horizontal ground accelerations from an ensemble of earthquake time-histories as excitation input. This ensemble is comprised of the 1940 Imperial Valley (El Centro) S00E, 1952 Kern County (Taft Lincoln School Tunnel) S69E, and 1971 San Fernando (Holiday Inn) N00W earthquake records. However, the accelerograms from these events were normalized to have an effective peak ground acceleration of  $0.40g$  according to a procedure discussed in Hayen (1995). Subsequently, these earthquake records will be denoted by the symbols ELC, TAF, and HOL, respectively.

For prescribed  $u$  and  $v_1$ , the dynamical behavior of the primary system is fully characterized upon specification of the undamped fundamental frequency of vibration  $\omega_{1,1}/2\pi$ , the modal damping ratio  $\zeta_{1,0}$  (which is the same for all response modes), and the number of structural stories  $n_1$ . In all of the control cases examined,  $\zeta_{1,0} = 0.01$ . The control cases considered herein involve a 6-story primary system for which  $\omega_{1,1}/2\pi$  is set to 1.00 Hz. For prescribed  $u$  and  $v_2$ , the dynamical behavior of the auxiliary system (when present) is fully characterized upon specification of the parameters

$$\alpha \equiv \frac{k_2}{k_1}, \quad \beta \equiv \frac{m_2}{m_1}, \quad \zeta_{2,0}, \quad n_2 \quad (13)$$

Although each interaction element is operated independently, all of the elements are characterized by the same set of possible parameter values (activated or deactivated). The parameters used for this characterization are

$$\mu \equiv \frac{k_{int}}{k_1}; \quad \delta \equiv \frac{c_{int}}{c_1}, \quad c_1 \equiv 2m_1\zeta_{1,0}\omega_{1,1} \Rightarrow \tau = \frac{2\delta\zeta_{1,0}\omega_{1,1}}{\mu\omega_{1,0}^2} \quad (14)$$

Results from three categories of control cases are presented below. In the first category, the auxiliary system is absent, and the interaction elements are attached between adjacent floor and roof slabs, as illustrated in Fig. 3. In the second category, the auxiliary system is a multi-story elastic frame, with both mass and stiffness properties, as shown in Fig. 4. In the third category, the auxiliary system is a relatively-small unrestrained mass located at the top of the primary system, as illustrated in Fig. 5.

## SELECTED RESULTS

The effectiveness of the proposed approach is judged by comparing the response of the controlled primary system to that of an uncontrolled primary system. As mentioned above, only a small subset of the control cases examined are presented. The simulation output for each case is summarized in response time-history diagrams, on which modal coordinates of the primary system are plotted versus time for the first 30 seconds of each excitation record. It was previously observed that the first response mode of the uncontrolled primary system exhibits the most activity for the ensemble of excitation records considered. Therefore, only the first response mode is specifically targeted for reduction. In each of the time-history diagrams, solid and dashed lines are used to indicate the values of  $\bar{x}_1$  (*i.e.*, the first modal coordinate) over time for a controlled and uncontrolled primary system, respectively. Note that  $(\varphi_{j1} - \varphi_{(j-1)1}) \bar{x}_1$  represents the contribution (in centimeters) to the  $j$ th story drift by  $\bar{x}_1$  (where  $\varphi_{01} \equiv 0$ ).

Figure 6 shows the first mode response for control cases from the first category, with  $\mu = 0.50$  and  $\delta_l = 5.00$ . It is apparent that significant response reduction is achieved in the first mode for each of the excitation records. Figure 7 shows the first mode response for control cases from the second category, with  $\alpha = 1.00$ ,  $\beta = 0.15$ ,  $\mu = 1.00$ , and  $\delta_l = 5.00$  ( $\zeta_{2,0} = 0.01$  and  $n_2 = 6$ ). Again, significant response reduction is achieved in the first mode for each of the excitation records. A larger value of  $\mu$  is used in this instance to match the

value selected for  $\alpha$ . Finally, Fig. 8 shows the first mode response for control cases from the third category, with  $\alpha = 0.00$ ,  $\beta = 0.30$ ,  $\mu = 0.025$ , and  $\delta_l = 0.25$ . In this instance, moderate-to-significant response reduction is achieved in the first mode for each of the excitation records. The value selected for  $\beta$  represents five percent of the total primary system mass. In all of these cases,  $\delta_h \rightarrow \infty$ .

Although not indicated herein, it is found that under the influence of the control effort, the second and higher response modes are not adversely affected when compared with those in the uncontrolled case. Also, extensive data on the peak values of the nodal mass accelerations and story drifts was collected for each of the control cases examined. For the first and third categories, these data indicate that both the acceleration levels and the drift levels are generally reduced from that which results in the uncontrolled case. For the second category, the drift levels are generally reduced, whereas the acceleration levels are typically elevated.

## CONCLUSION

The study presented herein considered MDOF models of structural systems and investigated several possible interaction arrangements for the proposed approach. Results from the first and second categories of control cases show that very significant response reduction is achieved (a 50 to 75 percent decrease in the peak values of the first modal coordinate, which provided the largest contribution to the story drift levels), indicating great promise for the proposed approach. Results from the third category of control cases show that a remarkable response reduction capability exists in view of the facts that only a single interaction element is utilized and a relatively-small auxiliary system is employed. Although the control effectiveness of these cases is not as great as that observed in the other categories, such an interaction arrangement might prove useful for applications involving wind gust excitation (such as those considered for tuned mass dampers).

It should also be mentioned that a comparison of the results of some special reference cases, in which the interaction elements remain either activated or deactivated for the entire duration of the excitation, with the results of the control cases that use the proposed method of element operation indicates the efficacy of the switching process. In addition, results of some cases involving a primary system configured as described for the first category, which is not externally forced but is given nonequilibrium initial conditions, reveal that the proposed method is quite capable of reducing the response of higher frequency modes during the ensuing free vibration, as well as suppressing a response arising from several dominant modes. These results may suggest that such capabilities are also possible for a primary system which is externally forced.

It is anticipated that such issues as system state estimation and parameter identification, thermal loading of control devices, and time delays in switching processes need to be addressed in subsequent research efforts. The next steps likely to be taken in further investigation include: development of enhanced computational models for the structural systems and interaction elements utilized in simulations, models which would more accurately capture the dynamics (both mechanical and thermal) of the actual systems and devices involved; and incorporation of the capability to identify and target the most dominant response modes in real time.

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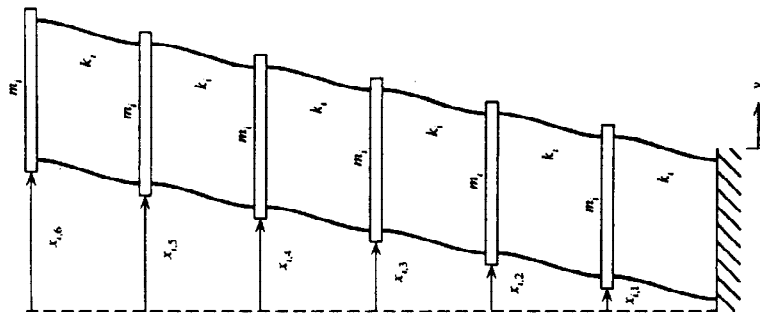


Fig. 1. Schematic Representation of Generic Primary or Auxiliary System.

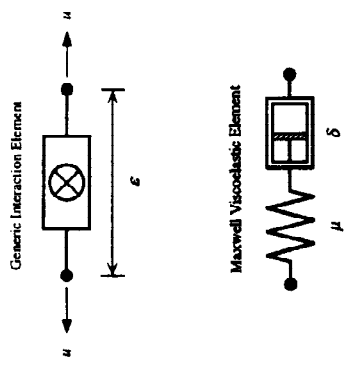


Fig. 2. Schematic Representations of Interaction Element.

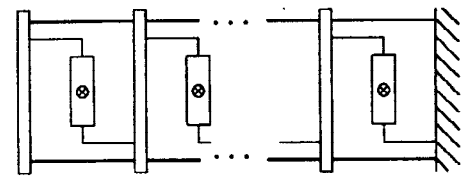


Fig. 3. Configuration of Primary System for First Category Control Cases. Generic interaction elements are shown.

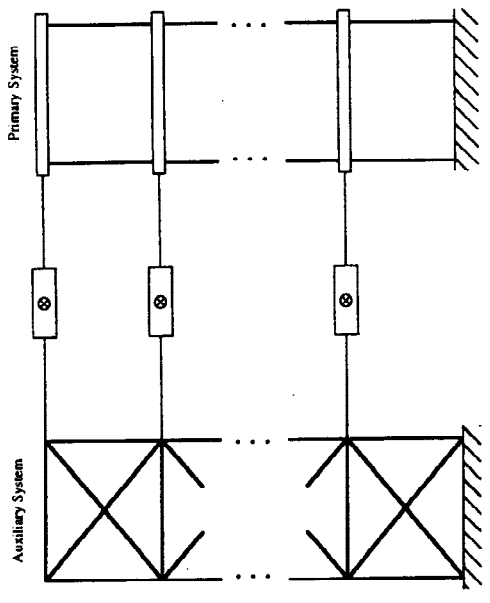


Fig. 4. Primary-Auxiliary System Configuration for Second Category Control Cases. Generic interaction elements are shown.

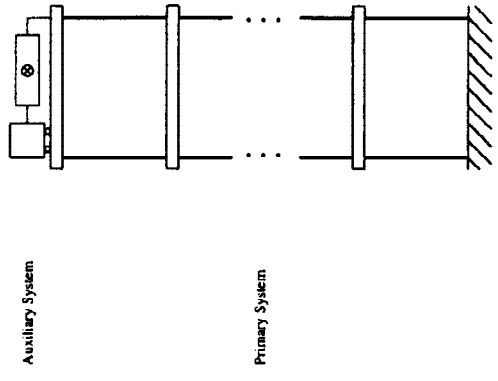


Fig. 5. Primary-Auxiliary System Configuration for Third Category Control Cases. A generic interaction element is shown.

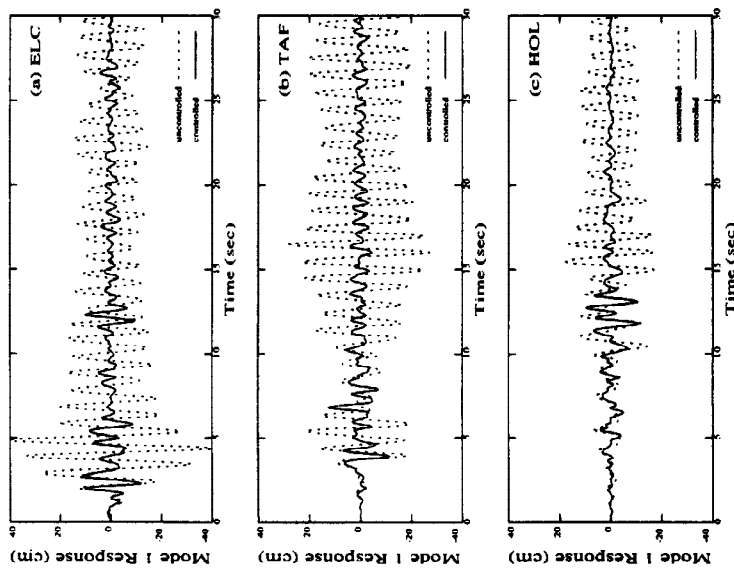


Fig. 6. Response Time-History of Mode 1 for a 6-Story First Category Controlled Primary System, Subjected to (a) EJ Centro, (b) Taft Lincoln School Tunnel, and (c) Holiday Inn Excitation Records.

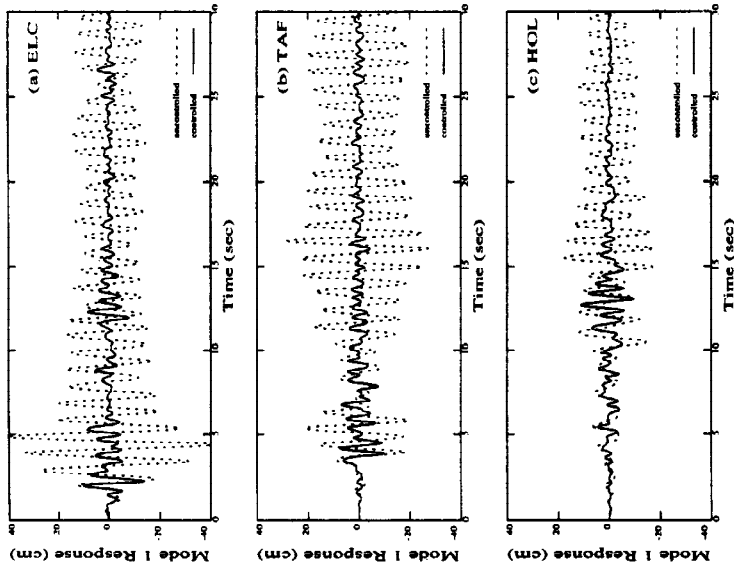


Fig. 7. Response Time-History of Mode 1 for a 6-Story Second Category Controlled Primary System, Subjected to (a) EJ Centro, (b) Taft Lincoln School Tunnel, and (c) Holiday Inn Excitation Records.

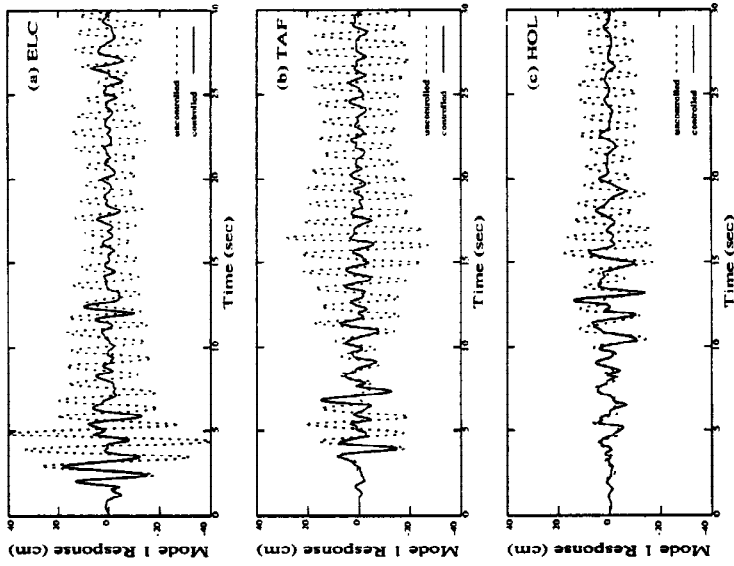


Fig. 8. Response Time-History of Mode 1 for a 6-Story Third Category Controlled Primary System, Subjected to (a) EJ Centro, (b) Taft Lincoln School Tunnel, and (c) Holiday Inn Excitation Records.