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SEISMIC ANALYSIS OF STONE ARCH BRIDGES USING DISCONTINUOUS DEFORMATION ANALYSIS

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ABSTRACT

In the event of a strong earthquake, blocks of a stone structure may separate or slide apart. It is difficult for conventional analysis methods to capture the complete seismic response. Discontinuous Deformation Analysis (DDA) is proposed to simulate the response of a stone bridge during earthquake. DDA allows individual blocks to separate away from each other or slide along their contact area. At the same time, DDA fulfills the equilibrium and compatibility conditions within block and between blocks. Compared to other numerical methods with discontinuous feature, DDA provides more stable and accurate results.

The DDA method is demonstrated in this paper on the seismic analysis of the Mosca's bridge over the Doria Riparia in Turin which was constructed in 1827. The displacement of blocks, principle stress distribution, block sliding and separation of Mosca's bridge due to seismic loading are presented. A theoretical validation example of sliding blocks is also presented. Compatible DDA results with field observation of a slender tall masonry tower under seismic loading is discussed.

In conclusion, DDA is a promising new analysis tool with many potential applications in earthquake engineering. This paper demonstrates its seismic analysis application on stone bridge which can benefit programs for historical preservation and earthquake retrofit.

KEYWORDS

Stone bridge, discontinuous analysis, seismic analysis, discontinuous material, contact, sliding.

INTRODUCTION

Stone and masonry are essentially geomaterials, which are for the most part nonlinear and discontinuous in nature. Under extreme seismic loading, stone blocks may undergo large displacement and the joints between blocks may separate or slide apart. It would be very difficult for conventional analysis methods to capture the complete seismic response of stone bridge structures. Many of the fundamental assumptions implied by conventional finite element method (FEM) such as compatibility and continuity can not apply to such media. Although through significant modifications, FEM has been adapted for some discontinuous deformation problems in geomechanics, the essential incapability of these special elements have limited their practical applications. Another approach is the distinct element method (DEM) initiated by Cundall (1971) and based on Newton's second law. However, real time dynamic analysis of DEM can not be carried due to its inherent limitation (Ma et al., 1992). New concepts and methodologies, which can realistically take these natural or artificial discontinuous characteristics into consideration, need to be developed. The most attractive method is the discontinuous deformation analysis (DDA) method proposed by Shi in late 80's. In this paper, DDA background related to the seismic model of stone structure is give to some degree. The detail description of DDA is outside the scope of this paper and can be found in the book by Shi (1992).

BACKGROUND OF DDA AND EXAMPLES

In contrast to some discontinuous method like DEM, DDA is a displacement method formulated on the basis of the principle of minimum total potential energy. Thus the equilibrium conditions of moments, forces and stresses are satisfied. Using displacement as variables, it solves the equilibrium equation for a discontinuous system in dynamic formulation which avoids the ill-conditioned stiffness matrix in static formulation due to the separation of blocks (Chang, 1991). Kinetic damping is applied in its static solution which effectively filters out the motions with higher modes to bring the system to a static condition. Since it is a dynamic formulation, time step is used and the equilibrium equation is updated then solved in each time step. Small displacements and deformations accumulated in each time step will lead to larger displacements a nd deformations.

In DDA, the simultaneous equations of the system are derived according to the standard procedures of discrete techniques (including subdivision, element analysis, global assembling, considerations of boundary conditions, equation solving). The equilibrium equations for the block assemblage are physically derived through the minimization of the total potential energy of the system. In the numerical scheme, the simultaneous equations are solved iteratively in each time step until both constraints of no-tension and no-penetration between blocks are satisfied. Therefore the numerical model considers both statics and kinematics and the corresponding solution will be very close to the true solution of a discretized block system. The DDA method allows individual blocks to separate away from each other or slide along their contact area. When blocks are in contact, the Coulomb's friction law is applied on the interface to simulate frictional resistance. When this mobiliable strength of the interface between two blocks is not reached, there is no slip between these two blocks. Otherwise, when the externally-applied driving load exceeds the friction force, relative sliding will occur.

To obtain a complete solution, two conditions, equilibrium and compatibility, must be fulfilled. These conditions are automatically satisfied for every individual block by minimizing the total potential energy and using a continuous displacement function. At contact points or interfaces, the equilibrium condition is reached by minimizing the potential energy due to contact forces. DDA implementation of compatibility condition is imposing the no-tension and no-penetration constraint between contact blocks. It is always a simply and effective way to model the contact mechanism using a contact spring. A contact spring is added when blocks contact and is removed when blocks separate. However, whether contact or separate is unknown when governing equations are assembled. Adding a spring at a separate contact will impose a tension and removing a spring will result a penetration without contact force. Both cases violate the fundamental physical phenomena. It is necessary to impose the no-tension and no-penetration constraint to avoid such violation. Such constraint is modeled as a set of inequalities. Iteration named as open-close iteration has to be carried out by adding and/or removing springs in a trial-error manner until this constraint is met or the solution of the inequalities has been found. The block kinematics theory has been developed in DDA to insure the effectiveness and accuracy of setting the inequalities.

The current DDA takes the rigid body motion / displacements and deformation / strain components of blocks as the independent unknowns using the first order polynomial function as follows:

$$u = a_0 + a_1x + a_2y$$
$$v = b_0 + b_1x + b_2y$$

where (u,v) is the displacement of any point in a block. (a_0, b_0) are unknown variables representing rigid bodymotion and others (a_1, a_2, b_1, b_2) are deformation unknowns. By separating the rigid body motion and deformation components, DDA has the advantage to stabilize the numerical computation for an rigid body motion dominated system such as a masonry or hard rock structure. In dynamic solution of DDA, very small time intervals is specified, same as other dynamic methods (DEM, FEM etc.). The inertia matrices of DDA is two orders of magnitude higher than internal stress or stiffness matrices and mainly located at the diagonal positions in the global matrix to resist the rigid body motion. The inertia terms of the rigid body motion are located on the diagonal line, instead of around the diagonal line in FEM, since the rigid body motion unknowns are separated from others. Therefore, with same size of time interval, iteration in DDA should be easier to converge than that in FEM. In addition, the block in DDA can have any convex, or concave shape or a multi-connected polygon configuration with or without holes including complex shaped blocks used in some stone masonry structures, since the displacement function is not depending on the boundary of the block.

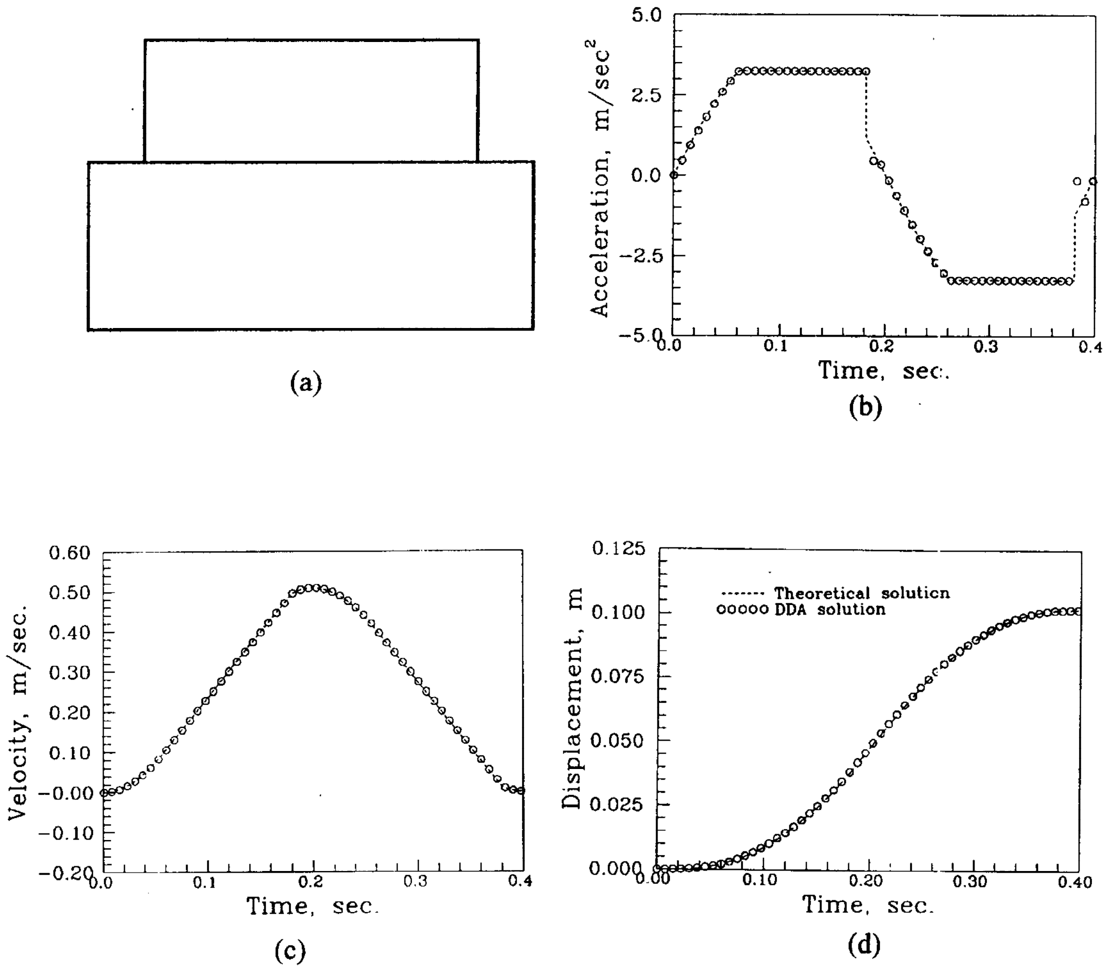
Input parameters for DDA computation can be diverted into five categories as listed in the first column in Table 1. For stone and masonry structure, masonry block is usually represented by DDA's element (block), which is linear elastic and require input values for Young's Modulus and Possion's Ratio as well as initial stresses. The contact at the interface is represented by contact spring in normal and shear direction. The failure criteria is constituted by Morh-Coloumb theory. Tension strength also can be included. A block consists of individual lines (segments) and the input parameters for geometry of a block are the coordination of these lines. The boundary condition in DDA can be displacement and/or force controlled which impose on the selected points in a block. The coordination of such points, and magnitude and direction of displacement and force are needed. In addition, there are two curtail input parameters: assumed maximum displacement ratio and time interval. Assumed maximum displacement ratio is a dimensionless quantity and used for finding the possible contact at the current step. The detailed discussion of these two parameters is presented by Ma et al. (1995).

Table 1. Input Parameters in DDA.

Block (Soil Mass)	Young's Modulus, Possion's Ratio, Density, Initial Stress
Interface	Normal/Shear Contact Stiffness, Friction Angle, Cohesion and Tension
Geometry	Coordinates of Blocks
Computation	Assumed Maximum Displacement Ratio, Time Step and Interval
Boundary Conditio	Coordinates of Points for Displacement or Force Controlled

Two Sliding Blocks

Failure or loss of structural integrity of a stone bridge is often the result from the progressing sliding between blocks at one or a number of locations under seismic loading. The dynamic analysis of a two block system was used to verify the solution given by DDA (Figure 1.a). A sinusoidal acceleration with maximum acceleration of 0.5g and frequency of 0.25 is applied on bottom block. The friction angle between the blocks is 18 degrees. The analytical solution is based on assumption of rigid body block. Therefore, Young's modulus of 10 GPa and Possion's ratio of 0.25 were used. The excellent agreement between the theoretical and the DDA solution on acceleration, velocity, and displacement of the top block can be seen in Figure 1.



(a) (b)

(c) (d)

Figure 1. A Sliding two-block system.

Seismic Response of a Slender Tall Tower

Figure 2. shows the DDA model of a stone masonry tower. This type of tower can be found in the near East and the Mediterranean area. The seismic survivability of these towers appears to be higher than expected. No major damage has been reported during strong earthquake, which has attracted some researchers to study its seismic response. DDA was used to simulate the tower under earthquake loading by applying a sinusoidal acceleration

to the base block as shown in Figure 2. The maximum acceleration is 0.5g and frequency of the acceleration is 2.5. The dimension of tower is presented(see figure 2). The results of DDA simulation is fairly consistent with field observation. The tower structure with the height over width ratio of 10 and about 100 layers of masonry block apparently performs well under the applied acceleration. The bottom portion of the tower has higher normal stresses due to the height of tower and the heavy unit weight of masonry blocks. Therefore, this portion of tower has much greater resistant force to the seismic loading. Blocks in increasing higher position tend to slide due to the lower normal stress and friction force. However, the sliding between blocks consume the seismic energy and less seismic force will travel to the upper portion of the tower. In the present case, only 30 degrees friction angler and no damping was assumed. It is also believed that the sliding between blocks in reality will occur in all the directions, which will consume more seismic energy than those in the two dimensional case. Detailed results pertaining to this case will be found in authors' upcoming paper.

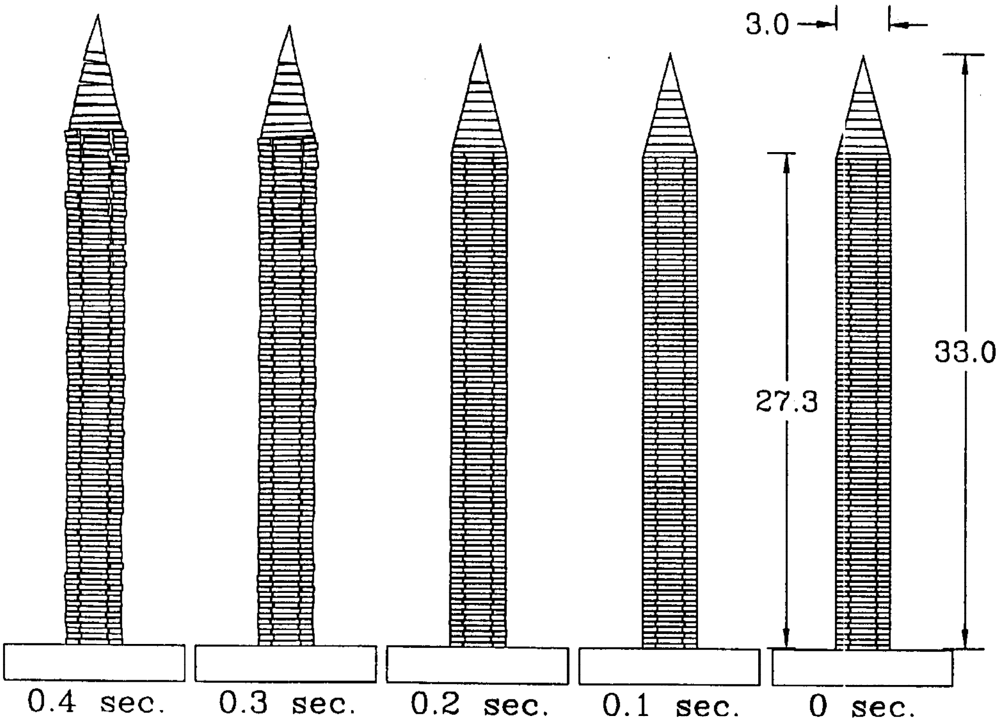


Figure 2. Graphic outputs of slender tall tower under seismic loading.

MOSCA'S BRIDGE UNDER EARTHQUAKE LOADING

Mosca's bridge, constructed in 1827, spans the Doria Riparia in Turin, Italy (Figure 3). The bridge is made of Malanaggio granite, 93 voussoirs make up the 45 m span and the intrados rise 1.5 m. The thickness varies from 2 m at the springing to 1.5 m at the crown. In DDA simulation, a single block element defines each voussoir and stone block in the bridge. Each abutment is modeled as a large polygonal block element. In DDA simulations, Young's modulus and normal contact stiffness are assigned to be 4.5 GPa. The unit weight is 2.4 T/m^3. 0.002 sec. is the time interval. Earthquake loading was applied as a sinusoidal acceleration of 0.5g and frequency of 2.5 same as above two cases at both abutments. It was assumed that the lateral resistance of abutments from the foundation are not accounted during earthquake. Therefore,abutments can freely move lateral and are fixed in the vertical direction. Prior to applying the dynamic loading, the bridge was subjected to the gravity for 0.02 sec in order to obtain the initial stress of blocks and initial contact forces. Then a sinusoidal acceleration was applied for 0.4 sec. and bridge was also under the gravity during this period time.

Figures 4 and 5 illustrate the principle stresses distribution at the end of loading (at 0.42 sec.) with or without damping. Since implementation of the damping of masonry block into DDA model is still not clear at this point of time, the damping is incorporated by reduction of the velocity at every time step. It was found that most blocks experienced high compression stress up to 5 MPa and tension of 10 KPa in the case without

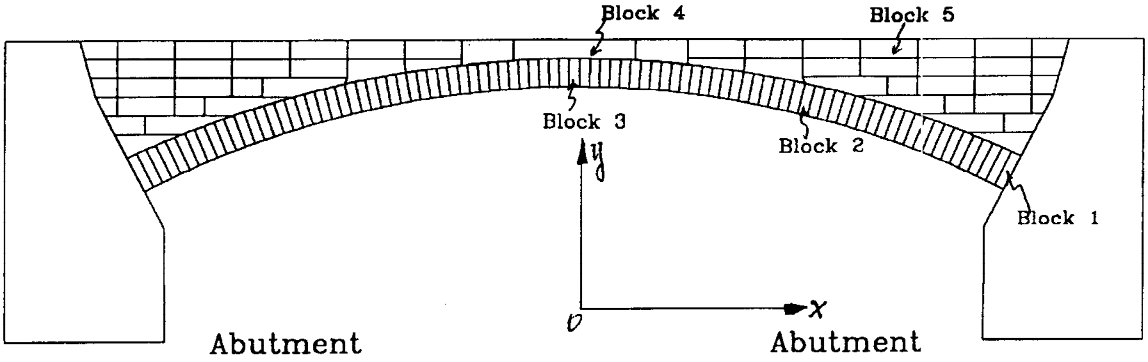


Figure 3. DDA model of Mosca's bridge.

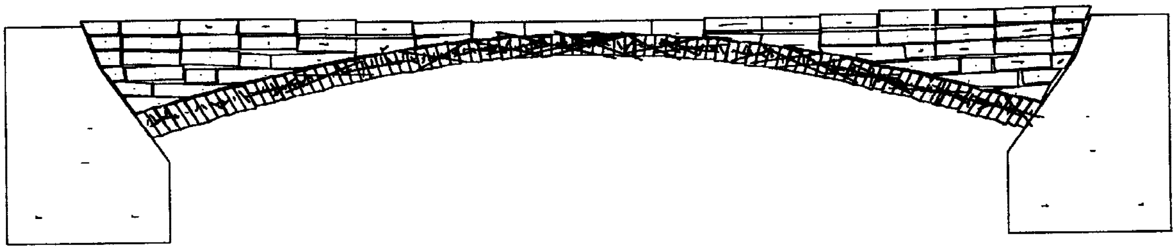


Figure 4. Principle stress distribution without damping.

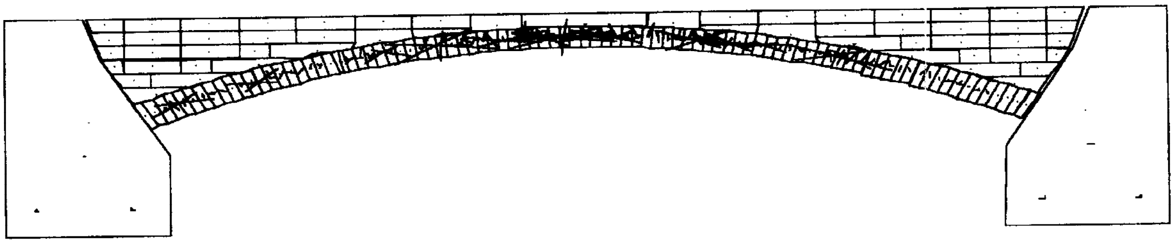


Figure 5. Principle stress distribution with damping.

damping. The tension apparently is resulted from the deformation rebounded from the compression deformation at very short time period. High stresses were found in a relatively large area around the center of span. In damping case with 6% reduction however, the higher stresses are more concentrated at the center span due to the narrow cross section at this area. Displacement of right abutment was monitored and presented in Figure 6. It seems that the velocity reduction can be effectively used as a damping measure. Figure 7a and 7b demonstrate the time history of seismic-induced horizontal stress of marked blocks numbered in Figure 3. The stress propagation within the bridge block system is clearly simulated. It is interesting to observe that the horizontal stresses of center blocks No.3 and No. 4 increasing in phase but different magnitude, almost 2 times difference.

CLOSURE REMARKS

The discontinuous feature of stone bridge poses a great challenge for engineers to understand its mechanism, especially during earthquake. A novel numerical method, DDA, has demonstrated its unique capability to simulate the mechanical response under dynamic loading. Better understanding under different boundary conditions can be generally gained based on DDA simulations in order to protect, maintain and retrofit stone structures, which often have historical significance.

DDA simulations conducted in this study suggest that although discontinuities or joints between blocks inherently reduce the integrate and strength of stone structure, sliding on these joints consumes the seismic energy during earthquake. Therefore, while local failures occur due to sliding or separating, the probability of overall structural collapse is low.

At the present time, applications of DDA in real-world problems among engineering professionals are still limited, in part due to the lack of a user-friendly DDA program. The authors are currently developing a Microsoft Windows version of the DDA code. This version will take advantage of the Windows feature called Object Linking and Embedding (OLE) which provides a tool for easy preparation of input data files and analysis of computed results through popular spreadsheet, database and CAD programs.

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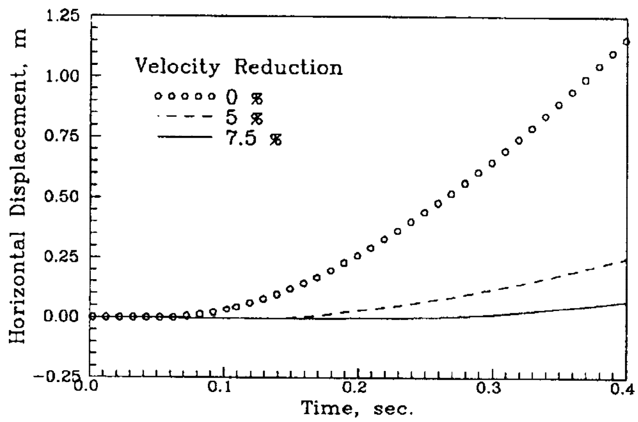
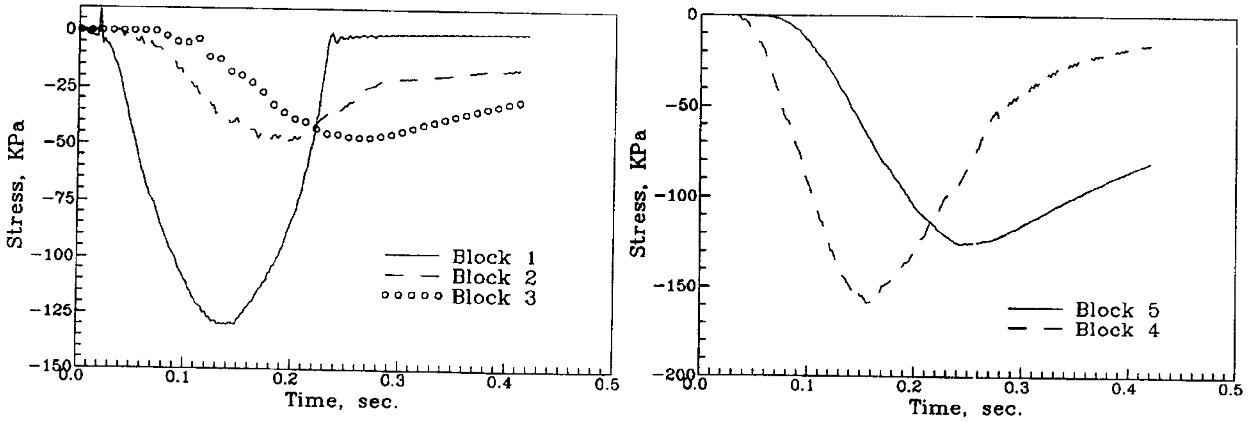


Figure 6. Time history of displacement of the right abutment.



(a)

(b)

Figure 7. Time history of horizontal stress of selected blocks.