



## ON THE DEFINITION OF SEISMIC SOURCE ZONES FOR HAZARD ESTIMATES

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### ABSTRACT

Defining seismic source zones was - and still is - a very difficult task. Especially when it comes to seismic hazard assessments of critical industrial facilities, such as nuclear power plants, experts are often puzzled by the number of diverse interpretations based on the same seismic information. In this case, we try to demonstrate, that a counting-method, which incorporates Mandelbrot's fractal dimension, can assist in pin-pointing, whether a defined polygon-border of a certain seismic zone needs to be re-defined or further investigated in order to arrive at a satisfactory spatial definition of the desired seismic region. The degree, to which each polygon describes the spatial distribution of epicentres at a given scale, can also be used for comparing different regions. The presented method is applied to earthquake-data from Austria. The application of the proposed method not only highlights the advantages, but also some pitfalls which need to be considered. Further generalisations of the method could include the incorporation of other than earthquake-data. The method itself is not limited to seismological applications alone.

### KEYWORDS

Seismic zones, spatial extent, fractals, quality factor, verification

### INTRODUCTION

Probabilistic seismic hazard analysis is based on earthquake statistics and was introduced by Allin Cornell in 1968. The necessary earthquake-data are normally selected from a certain region. The subjective definition of the spatial extent of these regions gave often rise to discussions (see Krinitsky, 1993a-c, and Hanks & Cornell, 1994), for changes of the region-boundaries are automatically reflected in subsequent results regarding the local seismic hazard at a defined point.

In this context - and for clarification purposes -, we propose to distinguish among

- seismic source zones - an area, in which earthquakes of a particular kind (clusters, mechanism, stress drop, etc.) tend to occur
- seismic regions - larger areas with a certain seismotectonic background
- seismic zones - zones of similar ground motions: used in building codes
- data zones - serve statistical purposes for completing data-sets.

The distinction between *seismic source zones* and *seismic regions* is somewhat cumbersome, but worthwhile to consider for enhancing seismic hazard estimates. In addition, the kind and shape of source-zones was treated very individually during the past years. *Rectangular* source zones, which bordered each other, were used by e.g. Schenk & Karnik (1978) or Peruzza & Slejko (1993). *Arbitrary* shaped source zones, which again bordered on each other, were utilised by e.g. Schenk et al. (1981) or Lenhardt (1995). Isolated source regions of irregular shape were used - besides many others - by Schenk et al. (1989), Grünthal (1995) or Stavrakis & Drakopoulos (1995) for calculating seismic hazard maps.

In the following paragraphs we just deal with the first category: seismic source zones, which can have an *arbitrary* shape. We will try to quantify, how good the defined borders of the source zone describe the spatial distribution of epicentres.

## METHOD

The general formulation of the annual probability of exceeding the ground motion ' $u_0$ ' at a certain point is given by (see also Frankel, 1995):

$$P(u > u_0) = \sum_k \sum_l 10^{\log(N_k/T) - b*(M_l - M_{ref})} * P(u > u_0 | D_k, M_l) \quad (1)$$

with ' $N_k$ ' being the number of occurrences in of ' $u > u_0$ ' during the time window of ' $T$ '-years in a distance range ' $D_k$ ', ' $M_{ref}$ ' is the lowest completely detectable magnitude, and ' $M_l$ ' is the magnitude of the seismic event. Hence, the probability ' $P$ ' represents a sum over hypocentral distances and magnitudes from a particular region - a *single* seismic source zone. Considering a number of source regions, forces us to introduce source-zone specific ' $b$ -values' and threshold magnitudes ' $M_{ref}$ '. Even if ' $M_{ref}$ ' can be considered as constant, the crucial factor ' $b$ ', which describes the relation between the number of large and small seismic events from a particular region - or area - needs to be established for each seismic source zone. This value can be very sensitive to the applied statistic - and to the spatial extent of the zone itself. At this point, the basic question arises:

### *To which extent is the 'b-value' representative for the seismic source region?*

In order to answer this question, we can utilise the dot-counting method, which can be understood as a by-product of the fractal theory (Mandelbrot, 1982). The method can be split into six steps:

1. Defining a source region (= polygon).
2. Select a scale of a regular grid (e.g. 20 km).
3. Determine the number of grid-cells, having their centre inside of the polygon.
4. Count the number of grid-cells, which are occupied by at least one epicentre.
5. Divide the scale by  $\sqrt{2}$ , and start at point '3' until a considerable small scale has been reached (e.g. 5 km).
6. Plot the logarithm of the chosen scale versus the logarithm of the number of qualified cells and determine the slope.

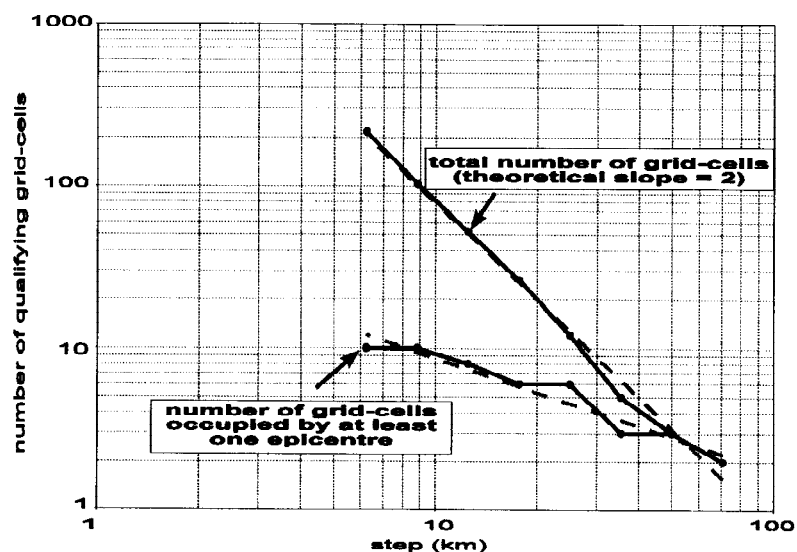


Fig.1. Principle (see text)

The slope of the number of occupied grid-cells versus the step-size represents the fractal dimension of the epicentre distribution. The higher the fractal dimension (derived from the 2-dimensional Euclidean space), the better the definition of the polygon describing the seismic source zone. If all epicentres are equally distributed, than they should always occupy *all* grid-cells which are qualifying within in the polygon-borders. Theoretically, this should lead to a fractal dimension of  $D = 2$ . In Fig.1 one example is shown to illustrate the problem. The top slope represents the total number of grid cells at each step of resolution. This slope actually amounts to 2,00, as it should be. This is an exemption, however. Region-dependent - or better 'polygon-dependent' - deviations from the theoretical value of  $D = 2,00$  can be seen in Table 1. The slope was calculated in each case with a least-square-fit procedure. The problem of fractal rabbits (see Kaye, 1993), which are essentially inaccuracies due to too large scales, is circumvented by neglecting scales exceeding the minimum lateral extent of the polygon under consideration. The dashed bottom line in Fig.1 represents the desired fractal dimension of the epicentre-distribution. The smaller the 'gap' between the top- and the bottom slope, the better the 'quality' of the polygon-approximation at a given scale. Figure 1 shows a bad example on purpose (region 1 in Tab.1), to be able to distinguish the bottom- and the top slope. Both slopes are represented by regressions of the kind

$$\log(N_{\text{poly}}) = a_1 + b_1 * s, \quad (2)$$

$$\log(N_{\text{epi}}) = a_2 + b_2 * s \quad (3)$$

with 's' being the step-size (or scale of resolution) in km's, and  $N_{\text{poly}}$  and  $N_{\text{epi}}$  are the number of qualifying (either total number, or the number of epicentre-occupied) grid-cells. The absolute value of the negative slope 'b2' represents the desired fractal dimension 'D<sub>epi</sub>'. The step-size dependent 'quality' of the 'coverage' of the epicentres by the grid-cells can then be calculated by taking the ratio of (3) and (2), leading to

$$q(s) = 10^{(a_2 - a_1 + (b_2 - b_1) * \log(s))} \quad (4)$$

ranging from 0 to 1 (truncated). A value of  $q(s) = 1$  simply means, that all grid-cells are occupied by epicentres above the scale 's'. We can determine this scale-threshold with

$$s_{\text{thresh}} = (a_1 - a_2) / (b_2 - b_1). \quad (5)$$

The level, at which the 'quality' falls below 50-percent ( $q(s) = 0,5$ ), is given by

$$s_{50\%} = (\log(0,5) + a_1 - a_2) / (b_2 - b_1), \quad (6)$$

and the final quality-factor of a particular region was defined as

$$Q = \log(s_{\text{thresh}} / s_{50\%}). \quad (7)$$

As depicted in Fig.2, the less steep the slope of 'q(s)', the better the quality of the spatial choice of the polygon-border. Equation (7) takes account of this fact, for it represents nothing else then the reciprocal slope in Fig.2. Figure 2 shows three examples as a demonstration. The top curve was calculated from region 10 (see next paragraph), representing a 'good quality' example of a polygon-choice for describing the spatial epicentre-distribution. Also larger changes in the applied do not reflect dramatically in the coverage of the epicentres. The bottom curve, however, illustrates a 'poor quality'-region. All numbers referring to regions are related to Fig.3 and Tab.1.

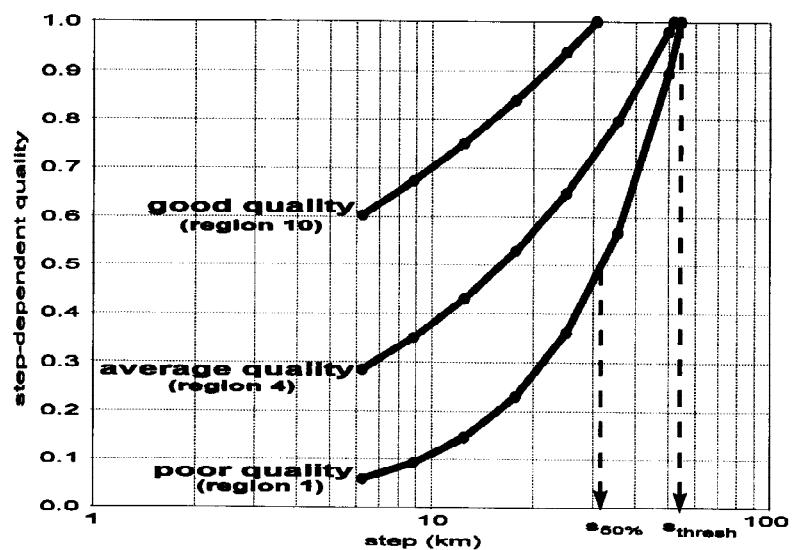


Fig. 2. Step-dependent quality of three different regions (for further explanation see text)

## APPLICATION

For this purpose, we sub-divided Austria into fifteen regions, which are actually not deemed as seismic source regions, but rather as of similar seismotectonic background and focal depth's-distribution, are investigated in the presented manner.

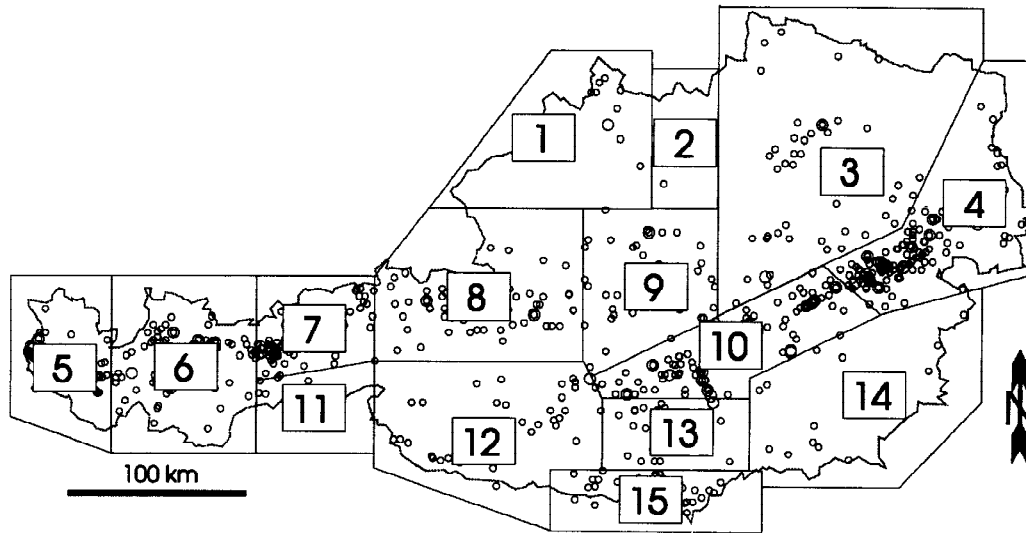


Fig. 3. Earthquakes exceeding M1,5 in Austria since 1900, and superimposed seismic regions (see also Lenhardt, 1995)

As it can be seen from Figure 3, Austria is unevenly clustered with epicentres - according to seismotectonic features. Some regions are more, some are less seismotectonic active. The degree of seismicity does not concern us here, but the simple shape of each polygon, - and the degree to which the polygon satisfies our demands in terms of deriving representative focal depth's, b-value's et cetera.

The presented method allows us now, not only to judge the epicentral coverage of each polygon, but also to investigate the validity of the presented approach and its pit-falls. Firstly, we note, that the mean fractal dimension of all polygons amounts to  $2,09 \pm 0,11$  (Tab.1). This is rather unsatisfactory, for the theoretical value should be 2,00 instead of 2,09. Hence, we have to acknowledge a bias towards a 5 % higher fractal dimension, depending of the shape of the polygon, starting point of the grid, grid size et cetera. This effect can also be observed in the box-counting method, which also tends to overestimate the fractal dimension (Lenhardt, 1996).

The mean fractal dimension of the spatial epicentre-distribution amounts to  $1,25 \pm 0,33$ , a value which is typical for natural processes (Turcotte, 1992). The mean step-size, from which onwards all epicentres occupy all grid-cells within the polygon ( $s_{\text{threshold}}$ ) varies around 42 km's, and the size of the grid-cell above which the quality exceeds '0,5' ( $s_{50\%}$ ), lies close to 17 km's. Hence, with a grid of a cell-size below 17 km we can already encounter problems in terms of matching the epicentral distribution. Only in few exemptions (regions 7 and 10) we find an ' $s_{50\%}$ ' with less than 10 km. In this context, one should note, that the accuracy of epicentres in the Austrian earthquake-catalogue is approx. 5 km for earthquake-data of this century. Finally, the average 'quality' of all seismic regions of Austria was calculated to be 0,38.

Obviously, region Nr.10 received the highest 'score', followed by the regions 7, 6 and 4. On the other hand, the polygon-borders of regions 1, 2, 11, 13 and 14 should either be improved, if one would attempt to use these seismic regions as source regions, or other than direct seismological evidence is needed to verify the spatial definition of these seismic regions. In order to improve the average 'quality' of all seismic regions in Austria, we could also simply discard those regions, which are deemed of low seismotectonic activity, thus increasing the average 'quality'-factor.

Table 1. Fractal dimensions and quality-factors of seismic regions in Austria

| region   | $D_{\text{Polygon}} = -b_1$ | $D_{\text{Epicentre}} = -b_2$ | $S_{\text{threshold}}$ | $S_{50\%}$ | Q    |
|----------|-----------------------------|-------------------------------|------------------------|------------|------|
| 1        | 2,00                        | 0,69                          | 54                     | 32         | 0,23 |
| 2        | 2,19                        | 0,78                          | 38                     | 23         | 0,22 |
| 3        | 2,04                        | 1,17                          | 65                     | 29         | 0,35 |
| 4        | 1,88                        | 1,29                          | 51                     | 16         | 0,50 |
| 5        | 2,05                        | 1,21                          | 42                     | 18         | 0,37 |
| 6        | 2,17                        | 1,68                          | 45                     | 11         | 0,61 |
| 7        | 2,23                        | 1,78                          | 32                     | 7          | 0,66 |
| 8        | 2,04                        | 1,40                          | 42                     | 14         | 0,48 |
| 9        | 2,04                        | 1,34                          | 33                     | 12         | 0,44 |
| 10       | 2,10                        | 1,73                          | 31                     | 5          | 0,79 |
| 11       | 2,13                        | 0,9                           | 36                     | 20         | 0,26 |
| 12       | 1,98                        | 1,27                          | 46                     | 17         | 0,43 |
| 13       | 2,36                        | 1,37                          | 27                     | 14         | 0,29 |
| 14       | 2,05                        | 0,82                          | 54                     | 31         | 0,24 |
| 15       | 2,08                        | 1,38                          | 32                     | 12         | 0,43 |
| mean     | 2,09                        | 1,25                          | 42                     | 17         | 0,38 |
| std.dev. | 0,11                        | 0,33                          | 10,3                   | 8,0        | 0,11 |

## DISCUSSION

Highest 'qualities' (>0,5) were found in the regions of highest seismic activity, whereas regions of low 'quality' (< 0,3) also exhibit the lowest seismic activity. In the latter case, the evidence of earthquakes is therefore poor, and the definition of the polygon is based on weak arguments. One could also argue, one should not use those regions of 'low quality' as 'source zones', because they should be considered as *aseismic* or *limited seismically active* rather than *seismic active*, unless additional evidence proves the opposite.

The fact, that the 'quality' correlates with the seismic activity is surprising. A bias due to the proposed algorithm can be excluded, however, for multiple epicentre-occupations of one and the same grid-cell - which would be reflected in the seismic activity (= number of earthquakes exceeding a certain size from a specified time-span and region) - are discarded. Hence, the weighting of each cell is always 'one', - nevertheless how many epicentres qualified within that specific grid-cell - or how active this area is.

As improvements to the above-mentioned method, one could consider including other than earthquake-catalogue data, such as paleoseismological evidence of surface ruptures. An optimisation-algorithm for automatic adjustment of polygon-borders could finally assist in the cumbersome re-adjustment of pre-defined polygon-borders.

## SUMMARY

Defining seismic source zones was - and still is - a very difficult task. Therefore a method was presented, which permits quantifying the 'quality' of the subjective spatial extent of seismic source regions. The dot-counting-method, which incorporates Mandelbrot's fractal dimension, was used to assist in pin-pointing, whether a defined polygon-border of a certain seismic zone needs to be re-defined or further investigated in order to arrive at a satisfactory spatial definition of the desired seismic region. The degree, to which each polygon describes the spatial distribution of epicentres at a given scale - referred to as *quality* - , can also be used for comparing different regions.

The presented method is applied to earthquake-data from Austria. The application of the proposed method not only highlights the advantages, but also some pitfalls which need to be considered.

In order to permit comparisons between different pre-defined polygons, we had to make use of a 'quality-factor', which was subjectively defined as  $Q = \log(s_{\text{thresh}}/s_{50\%})$ . 's<sub>thresh</sub>' and 's<sub>50%</sub>' are representing the scale at which all grid-cells are occupied by at least a single epicentre, and the scale at which 50 % of all grid-cells are occupied by at least one epicentre. Highest 'qualities' (> 0,5) were found in Austria to be related with regions of highest seismic activity, whereas regions of low 'quality' (< 0,3) also exhibit the lowest seismic activity. In the latter case, the evidence of earthquakes is therefore poor, and the definition of the polygon is based on weak arguments. One could also argue, one should not use those regions of 'low quality' as 'source zones' at all, because they should be considered as *aseismic* or *limited seismically active* rather than *seismic active*, unless additional evidence proves the opposite.

Other than earthquake-catalogue data, such as paleoseismological evidence of surface ruptures, can also be included in the proposed method, by adding the appropriate coordinates to the data-set. Besides, the method is not limited to the presented application alone, but can be applied whenever similar problems are experienced in other data-sets.

## REFERENCES

- Cornell, A.C. (1968). Engineering seismic risk analysis. *Bull.Seis.Soc.Am.*, Vol.58, 1583-1606.
- Frankel, A. (1995). Ground shaking hazard maps for the Eastern United States.Proc. of the 2nd USA-France workshop on assessing earthquake hazards in the Central and Eastern United States and Western Europe, Nice, France.
- Grünthal G., Bosse Ch., Mayer-Rosa D., Rüttener E., Lenhardt W. A. & Melichar P. (1995). Joint seismic hazard assessment project for Austria, Germany and Switzerland. Proc. '10th European Conference on Earthquake Engineering', Balkema, 57-62.
- Hanks, T.C. and Cornell, C.A. (1994). Probabilistic seismic hazard analysis: A beginner's guide. Proc. of 5th Symposium on *Current Issues Related to Nuclear Power Plant Structures, Equipment and Piping*. North Carolina State University, Raleigh.
- Kaye, B. (1993). *Chaos & complexity: discovering the surprising patterns of science and technology*. VCH Verlagsgesellschaft mbH, Weinheim, Germany.
- Krinitzsky, E.L. (1993a). Earthquake probability in engineering-part 1: the use and misuse of expert opinion. *Engineering Geology*, Vol.33, 257-288.
- Krinitzsky, E.L. (1993b). The hazard in using probabilistic seismic hazard analysis. *Civil Engineering*, Nov., 60-61.
- Krinitzsky, E.L. (1993c). Earthquake probability in engineering-part 2: earthquake recurrence and the limitations of Gutenberg-Richter b-values for the engineering of critical structures. *Engineering Geology*, Vol.36, 1-52.
- Lenhardt, W.A. (1995). Regional earthquake hazard in Austria. Proc. of *10th European Conference on Earthquake Engineering*, Balkema, 63-68.

- Lenhardt, W.A. (1996). On fractal theory and its application to earthquakes in Austria. Proc. of '*Aspects of Tectonic Faulting*', Graz/Austria, in preparation.
- Mandelbrot, B.B. 1982. The fractal geometry of nature. Freeman, San Francisco.
- Schenk, Z. & Karnik, V. (1978). The third asymptotic distribution of largest magnitudes in the Balkan earthquake provinces. *Pure and Appl. Geophys.*, Vol.116, 1314-1325.
- Schenk, V., Schenkova, Z. & Karnik, V. (1981). Seismic hazard assessment in the Bohemian Massif. Proc. of 2nd International Symposium on the *Analysis of Seismicity and Seismic Hazard*, Liblice, Czechoslovakia.
- Schenk, V., Schenkova, Z. & Grünthal, G. (1989). Seismic hazard assessment for Central Europe, Version 1989. Proc. of 4th International Symposium on the *Analysis of Seismicity and Seismic Risk*, Bechyne Castle, Czechoslovakia.
- Stavrakakis, G.N. & Drakopoulos, J. (1995). Bayesian probabilities of earthquake occurrences in Greece and surrounding areas. *Pure and Appl. Geophys.*, Vol.144, 307-319.
- Turcotte, D.L. (1992). Fractals and chaos in geology and geophysics. University Press, Cambridge.