



POSSIBILITY OF ESTIMATION FOR AMPLIFICATION CHARACTERISTICS OF SOIL DEPOSITS BASED ON RATIO OF HORIZONTAL TO VERTICAL SPECTRA OF MICROTREMORS

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ABSTRACT

This paper investigates the fundamental characteristics of ratio between horizontal and vertical spectra (H/V-R) of short-period microtremors and elucidates its relationship with ground amplification. In addition, the paper investigates the possibility to predict earthquake ground motion using spectral ratio. The results are summarized as follows:

- 1) Clearly observed peaks in H/V-R fairly correspond to the 1st mode of ground amplification due to S-wave propagating above bedrock which is relatively strong reflection boundary.
- 2) H/V-R can be simulated in 2-dimensions by FEM considering surface waves generated by ground noise. From the numerical simulation of single layer model, it is considered that the abovementioned relationship is valid in the range of impedance ratio exceeding 2-3.
- 3) H/V-R can be efficiently used to estimate the relative strength of earthquake motion in case of minor changes in ground conditions.

KEYWORDS

short-period microtremors, spectral ratio, surface wave, predominant period, ground amplification, seismic motion characteristics

INTRODUCTION

Many studies on engineering use of short-period microtremors have been carried out since the leading studies by Kanai in the 1950's (for example, KANAI et al., 1954). The study, however, which directly estimated the ground amplification characteristics from microtremors observation was comparatively few. Under such circumstances, Nakamura proposed a new method which estimates predominant period and ground amplification by means of ratio between horizontal and vertical spectra (abbreviated as H/V-R), (Nakamura et al., 1986). However the fundamental characteristics and the range of application on engineering use of H/V-R is not necessarily elucidated. The authors carried out the observation of short-period microtremors at the sites where the underground structure was investigated in detail and the earthquake observation was conducted. This paper examines the fundamental characteristics of H/V-R based on the observation result and its simulation analysis and investigates the possibility to predict earthquake ground motion using H/V-R.

1. OBSERVATION SITE AND DATA PROCESSING

1.1. Observation site and measurement condition

4 sites in Tokyo and 11 sites in Sendai of Japan were selected as the microtremors observation sites. Fig.1 and Fig.2 show the observation sites of each area. These sites have a various type of surface geology. The predominant period due to surface layer is between about 0.1-0.2 second and 1.2 second. Velocity component of microtremors was measured for two horizontal and vertical directions using the survo type seismometer.

1.2. Data processing

Data processing for microtremors records is as follows. a) 10 sets of data for 20.48 second each are selected

from the record in order of small amount of noise. b) Each data is transformed in frequency domain by FFT and the two horizontal spectra are composed as considering the phase information. c) Parzen window with 0.3Hz of band-width is took for each spectra. d) A geometric mean of 10 horizontal and vertical spectra is calculated. e) H/V-R is defined as a ratio of these spectra.

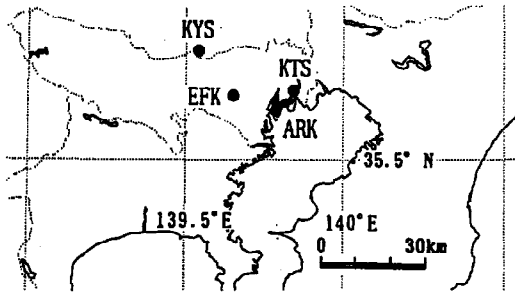


Fig.1 Measurement sites of microtremors in Tokyo

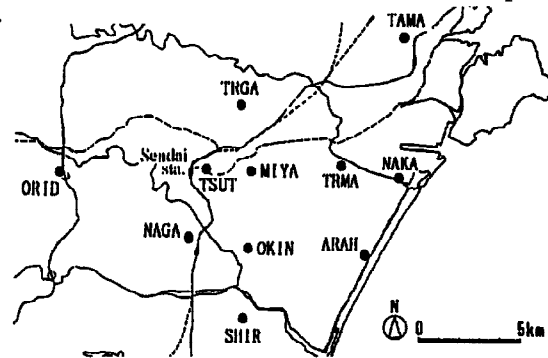


Fig.2 Measurement sites of microtremors in Sendai

2.COMPREHENSION FOR FUNDAMENTAL CHARACTERISTICS OF H/V-R BASED ON OBSERVATION

2.1.Peak period

2.1.1.Ground that clear boundary exist in the surface layer

1)Ground regarded as a single layer structure The representative type of such ground is KTS(in Fig.1). The velocity structure based on PS logging at KTS is shown in Table 1. In this table, KTS has the soft soil with about 100m/sec of shear velocity(V_s) to a depth of about 30m. In deeper ground of the soft soil, the 1st period obtained from 1-dimensional analysis due to S-wave does not almost change irrespective of a setting level of basement, so KTS is the ground regarded as a single layer structure which the soft soil has a predominant influence. H/V-R observed and amplification characteristics(material damping of soil: $h=1\%$ & 5%) due to S-wave propagating above the lowest layer(GL-62.15m) obtained from PS logging were shown in Fig.3. From this figure, about 1.2 second of peak period recognized in H/V-R is good agreement with the 1st period due to S-wave. As to the 2nd period, however, the peak seen in H/V-R is small in comparison with the one due to S-wave. Horizontal average Fourier spectrum for the main part of some earthquake ground motions recorded at the surface of KTS is shown in Fig.4. In this figure, it is recognized that the Fourier spectrum of earthquake is in harmony with H/V-R except for the higher modes. From these circumstances, in case of ground regarded as a single layer, it seems that the peak period of the 1st mode is able to find in H/V-R but the one corresponding to higher modes is hard to appear in comparison with the one of the 1st mode. This phenomenon is recognized in the numerical analysis shown later(See Fig.14 and Fig.17).

2)Ground regarded as double layer structure The representative type of such ground is NAGA(in Fig.2). The velocity structure based on PS logging at NAGA is shown in Table 2. From this table, it is understood that NAGA is the ground regarded as a double layer structure which has the clear two boundaries at GL-4.85m and GL-28.45m. H/V-R and amplification characteristics due to S-wave($h=1\%$ & 5%) above the lowest layer(GL-81m) were shown in Fig.5. From this figure, about 0.55 and 0.2 second of peak period seen in H/V-R is good agreement with the 1st and 2nd period due to S-wave. The 0.2 second of peak, however, is also corresponding with the 1st period due to S-wave above GL-4.85m and the meaning of peak is different from the case of KTS. This phenomenon is explained by the numerical simulation analysis shown later(See Fig.12). It is considered that about 2 second of peak in the same Fig.5 is the one which reflect the influence of deep underground structure. Horizontal average Fourier spectrum for the main part of some earthquake ground motions recorded at the surface of NAGA is shown in above-mentioned Fig.4. In this figure, it is recognized that the shape of Fourier spectrum is in harmony with the one of H/V-R. From these circumstances, in case of ground regarded as multi-layer structure, it seems that the peaks in H/V-R are corresponding to the 1st mode of ground amplification due to S-wave propagating above bedrock which is relatively strong reflection boundary.

2.1.2.Ground that clear boundary does not exist in the surface layer

S-wave velocity of MIYA(in Fig.2) is increasing gradually with a depth as shown in Table 3. In such ground, a clear boundary does not exist in the surface layer and predominant period is not appeared clearly. H/V-R and amplification characteristics above the lowest layer(GL-54m) of PS logging are shown in Fig.6. From this figure, in a short period range less than about 1 second, the tendency which clear peaks are not appeared is common but the shape of the two does not necessarily show good correspondence. In case of such ground, it seems that it is difficult to grasp the ground amplification characteristics from H/V-R. On the other hand, about 2 second of large peak, which is considering due to the influence of deep underground structure, is seen in Fig.6. The deep underground structure was proposed as shown in Table 4 (Kashima *et al.*, 1990). The amplification characteristics above the layer having $V_s=3000$ m/s in this table are shown in above-mentioned Fig.6. The 1st period of this model shows a good correspondance to the peak period of H/V-R and it is

considered that the peak of H/V-R reflects the influence of deep underground structure. About 1-2 second of peak in H/V-R was seen at other sites in Sendai. This phenomenon suggests that it is important to investigate the meaning of the peaks in H/V-R by ground data and so on. Horizontal average Fourier spectrum for the main part of some earthquake ground motions recorded at MITA is shown in Fig.7. In this figure, it is recognized that the 1st period of Fourier spectrum is in harmony with the one of H/V-R. The higher modes, however, does not estimated sufficiently.

Table.1 Velocity structure based on PS logging at KTS

Depth(m)	Vs(m/sec)	Vp(m/sec)
0~ 4.50	100	1320
~ 6.50	140	
~ 10.50		1170
~ 20.50	110	1220
~ 28.50		1400
~ 32.50	150	
~ 36.75		1270
~ 46.50	240	1400
~ 57.00	290	1580
~ 60.60	370	1870
~ 62.15	300	1720
~ 64.50	400	1870

Table.2 Velocity structure based on PS logging at NAGA

Depth(m)	Vs(m/sec)	Vp(m/sec)
0~ 0.80		240
~ 4.85	105	920
~ 9.30	290	1600
~ 20.25	170	1050
~ 28.45	290	1640
~ 56.50	600	1990
~ 81.00	530	1700
81.00~	700	

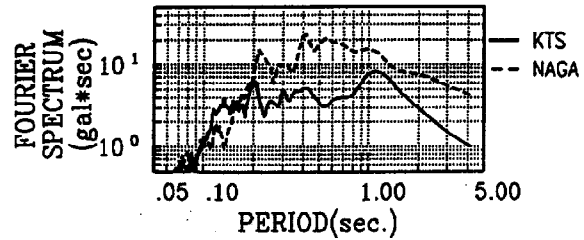


Fig.4 Horizontal average Fourier spectrum(2-dimension) of earthquake ground motion at KTS and NAGA

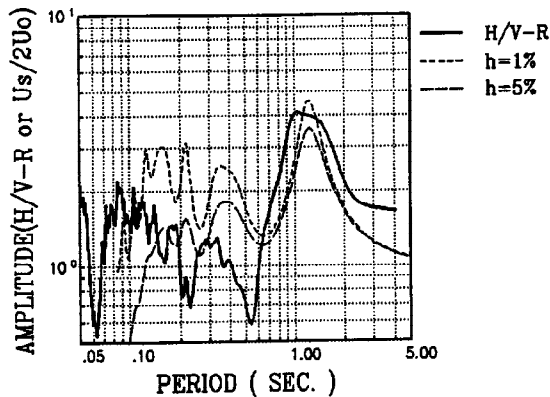


Fig.3 Comparison between H/V-R(bold line) and amplification due to S-wave(fine lines:bedrock;GL-62.15m) at KTS

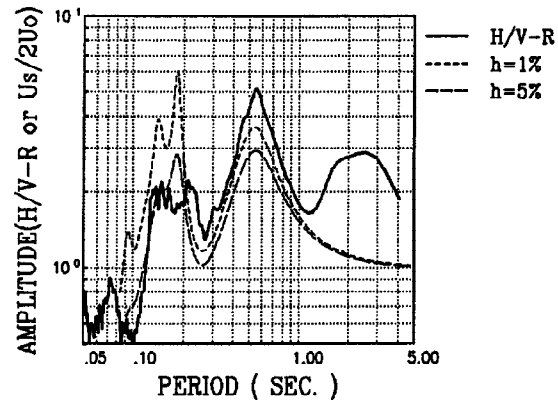


Fig.5 Comparison between H/V-R(bold line) and amplification due to S-wave(fine lines:bedrock;GL-81.0m) at NAGA

Table.3 Velocity structure based on PS logging at MIYA

Depth(m)	Vs(m/sec)	Vp(m/sec)
0~ 1.50	210	380
~ 3.00	350	580
~ 7.00		1000
~ 11.00	430	1200
~ 22.00	480	1600
~ 34.00	640	
~ 46.00	570	1800
~ 54.00	480	1900
54.00~	680	1900

Table.4 Assumed deep velocity structure at MIYA¹³⁾

Depth(m)	Vs(m/sec)	Vp(m/sec)
PS logging to the depth of 54m (Table.3)		
~ 154.0	680	1900
~ 654.0	1400	—
~ ∞	3000	—

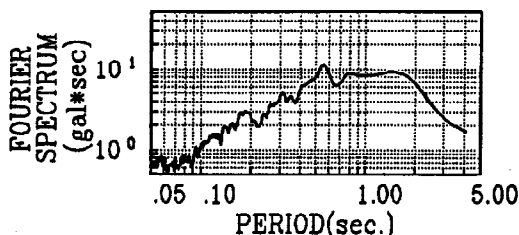


Fig.7 Horizontal average Fourier spectrum(2-dimension) of earthquake ground motion at MIYA

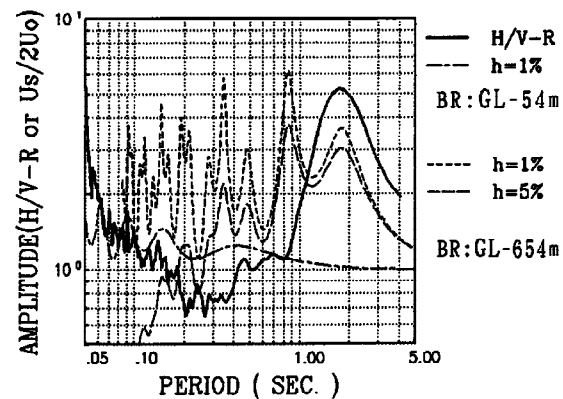


Fig.6 Comparison between H/V-R(bold line) and amplification due to S-wave(fine dot and dashed line:bedrock; GL-54.0m, fine dotted and broken lines;GL-654m) at MIYA

2.1.3. Relationship between the peak period in H/V-R and the one due to S-wave Fig.8 shows the relationship between peak period in H/V-R and predominant period due to S-wave in case of considering the lowest layer obtained from PS logging as a bedrock. ● indicates the 1st period and ■ shows the 2nd period respectively.

From this figure, the relationship of the two is in comparatively harmony except for some sites where the peak period in H/V-R is longer than predominant period due to S-wave. As a representative site where does not show good correspondence, H/V-R and amplification characteristics due to S-wave at TSUT and ARAH are shown in Fig.9. It is impossible to investigate about this difference in detail because of the lack of information for deep underground structure. Judging from the fact, however, that the peak seen in power spectrum of earthquake records shown later(See Fig.20) is in comparatively harmony with the one in H/V-R, it is considered that H/V-R reflects the influence of deeper underground structure. This phenomenon suggests the importance to investigate the meaning of peaks seen in H/V-R.

2.2. Peak value

Relationship between the peak value of H/V-R and the amplification due to S-wave corresponding to the period of Fig.8 is shown in Fig.10. In this figure, ● and ■ have similar meaning as in Fig.8. From this figure, the relationship of the two shows the comparative large unevenness, so the relation between the underground structure and the amplification was investigated for EFK and NAGA. PS logging at EFK is shown in Fig.11. H/V-R and amplification due to S-wave in case of considering the lowest layer(GL-70.5m) and middle layer(GL-8.7m) as a bedrock are shown in above-mentioned Fig.11. As shown in this figure, the amplification due to middle layer is better agreement than the one due to the lowest layer. In case of hard layer exist in the middle, peak value of H/V-R is in harmony with the amplification due to S-wave propagating above the hard layer.

NAGA has the double layer structure having GL-4.85m and GL-28.45m of the boundary as shown in Table 2. Fig.12 shows the H/V-R and amplification due to S-wave in case of setting the bedrock at GL-4.85m, GL-28.45m and GL-681m(adding assumed deep velocity structure in Table 4). From this figure, it is considered that the peak values of H/V-R is in harmony with the amplification due to S-wave propagating above the each corresponding bedrock. That is, the peak value of H/V-R does not express the absolute amplification of S-wave, it is considered that the one correspond to the 1st mode of ground amplification due to S-wave propagating above bedrock which is a relatively strong refraction boundary.

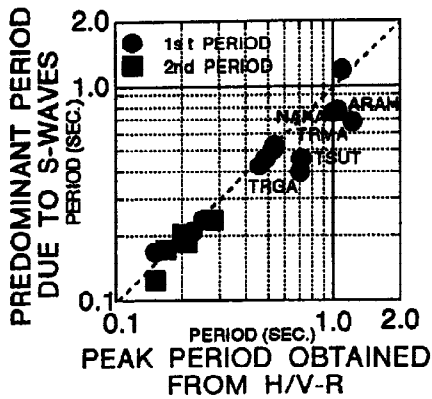


Fig.8 Relationship between peak period of H/V-R and predominant period due to S-wave propagating above lowest layer obtained from PS logging.

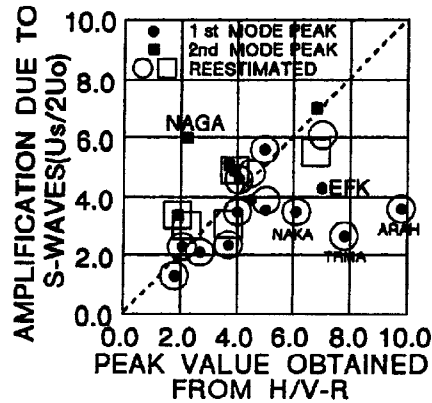
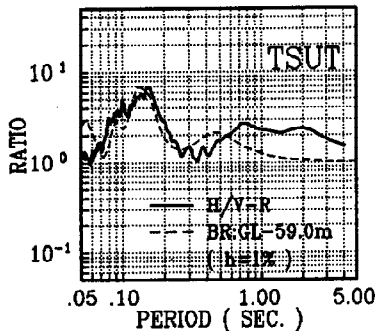
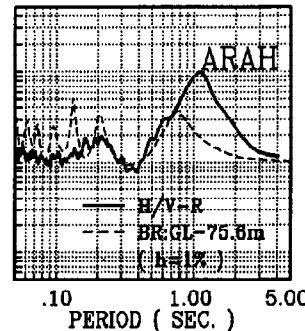


Fig.10 Relationship between peak value of H/V-R and amplification due to S-wave based on PS logging. (●, ■ : amplification above lowest layer, ○, □ : amplification for 1st mode above bedrock corresponding in peak period)



PS logging at TSUT

Depth(m)	Vs(m/sec)	Vp(m/sec)
0~ 2.80	95	680
~ 4.80	180	820
~ 15.70	500	1730
~ 25.30	480	
~ 34.80	380	
~ 48.40	500	1120
~ 51.30	500	
~ 59.00	650	
69.00~	960	1730



PS logging at ARAH

Depth(m)	Vs(m/sec)	Vp(m/sec)
0~ 2.00	130	260
~ 6.00	170	
~ 14.00	270	1570
~ 20.90	180	
~ 25.80	100	
~ 31.10	150	1150
~ 35.00	400	
~ 49.70	570	1680
~ 78.80	810	1880
75.60~	750	

Fig.9 Comparison between H/V-R(bold line) and amplification due to S-wave(fine line) at TSUT and ARAH. It is considered that deep underground structures greatly influence H/V-R.

Relation shown in Fig.10 was reexamined considering above-mentioned characteristics. The result was shown in symbol \circ and \square in the same Fig.10. This result is improved from the previous one and the relation of the two shows comparatively good correspondence except for NAKA, ARAH, TRMA reflecting the influence of deep underground structure. In these circumstances, it is recognized that the peak of H/V-R has the possibility to be able to estimate the amplification for the 1st mode. It is necessary, however, to be careful that these relationship is concluded on the data processing shown in chapter 2.

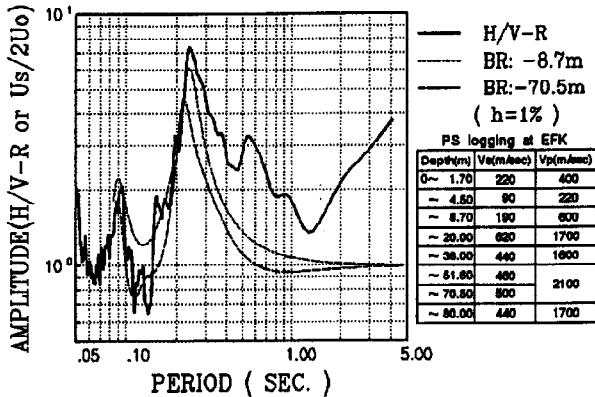


Fig.11 Comparison between H/V-R (bold line) and amplification due to S-wave in case of varying depth of bedrock (fine dotted line:bedrock;GL-8.7m, fine broken line;GL-70.5m) at EFK

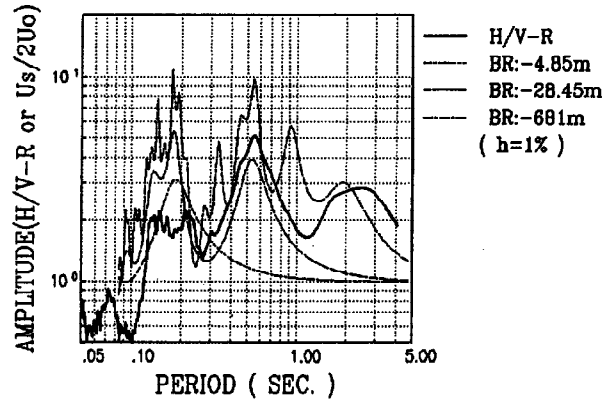
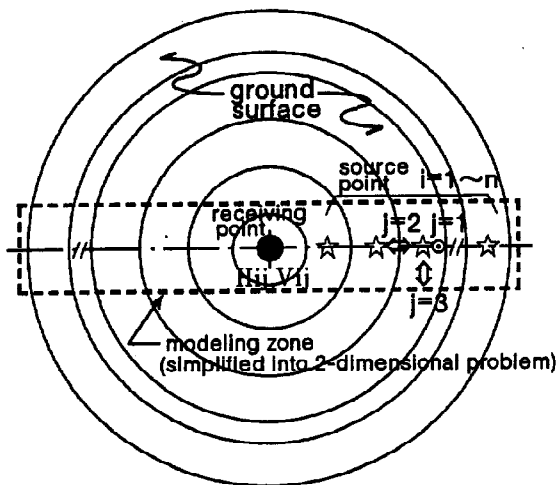


Fig.12 Comparison between H/V-R (bold line) and amplification due to S-wave in case of varying depth of bedrock (fine dotted line:bedrock;GL-4.85m, fine brokenline;GL-28.45m, fine dot and dashed line:GL-681m[assumed]) at NAGA

3.SIMULATION ANALYSIS OF H/V-R

Simulation analysis of H/V-R which suppose surface noise were carried out using 2 dimensional FEM (Waas,G., 1972). As for the modering of source and receiving point, the one by LACHT was referred (LACHT,C. *et al.* 1994). The simulation model and estimation formula of H/V-R are shown in Fig.13. The process of simulation analysis is as follows. (a)Modeling zone of 2-D FEM including receiving point are cut out from the ground. (b)Microtremors sources distributed uniformly on the ground surface is represented by some source points. (c)At the source points, unit amplitude vibration is made in vertical and horizontal(in-plane and out-of-plane) component. (d)Amplitude at receiving point is estimated as root-mean-squear of maximum amplitude of each vibrational pattern. (e)At the end, H/V-R is calculated by the formula in Fig.13. In this model, as the ground surface is vibrated and the far field is treated, it is considered that the Reiley wave in case of in-plane vibration and the Love wave at out-of-plane vibration are prominent respectively. So the influence of body wave is not mentioned here.

Simulation analysis for KTS and NAGA was carried out. Analytical model was composed based on Table 1 and 2. The results are shown in Fig.14 and Fig.15. From these figure, observed H/V-R is good simulated by calculated one including short period range. It is considered that short-period microtremors is able to almost interpret as a assemblage of Rayleigh and Love waves. As to a vertical line of Fig.14 and 15, there is a problem for validity of vibration amplitude ratio of each directions at source point and the material damping of soil, so the absolute value of the two is not able to be compared strictly. However It is considered that the amplitude of H/V-R is able to be roughly estimated by this simulation model.



Hij : horizontal component , Vij : vertical component

l : source point (100~1000m,50m pitch)

j : source direction (1:vertical, 2:horizontal(in-plane), 3:horizontal(out-of-plane))

$$(H/V-R) = \frac{\sqrt{\sum_{l=1}^n \left(\sum_{j=1}^3 (\alpha_l * H_{lj})^2 \right)}}{\sqrt{\sum_{l=1}^n \left(\sum_{j=1}^3 (\alpha_l * V_{lj})^2 \right)}}$$

α_l : area ratio dominated by each source point

Fig.13 Simulation model of short-period microtremors and estimation formula for H/V-R

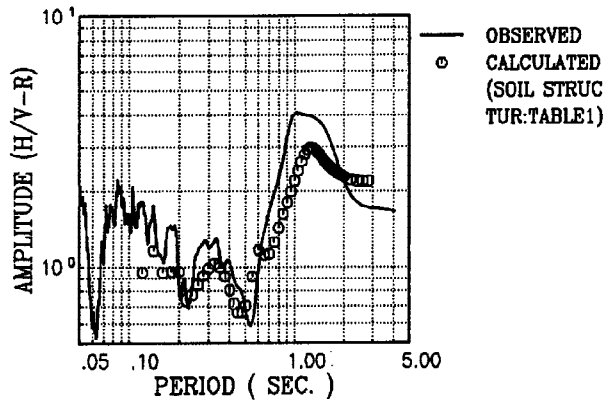


Fig.14 Simulation results for H/V-R at KTS

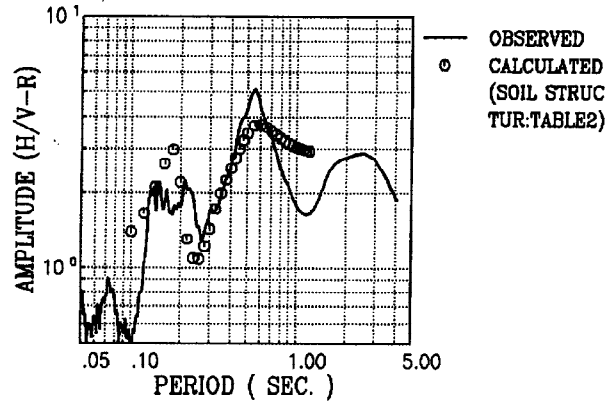


Fig.15 Simulation results for H/V-R at NAGA

4.COMPARISON BETWEEN AMPLIFICATION CHARACTERISTICS DUE TO S-WAVE AND H/V-R FOR MODEL GROUND

The characteristics of H/V-R is compared to the amplification characteristics due to S-wave using the similar method as previous simulation analysis. Groun model and calculation case are shown in Fig.16. Single layer model having fixed V_s and V_p of surface layer is assumed. V_s and V_p of bedrock are changed in 4 cases as Fig.16. Material damping of soil are changed in $h=0\%$ and 2% . Fig.17(a)-(c) show the comparison between H/V-R and amplification due to S-wave in case of varing S-wave velocities of bedrock only(corresponding to CASE 1, 2, 3). H/V-R show the result of 3-directional vibration source($j=1-3$ [ref.Fig.14]) and 2-directional vibration source($j=1,2$) without Love wave component. From these figure, first of all, it is seen that the whole shape of H/V-R is almost determined by Rayleigh wave and raised by Love wave. At the 1st period, H/V-R due to Rayleigh wave($j=1,2$) is not almost influenced by the material damping of soil, and it seems that the amplitude which is almost equivalent to amplification due to S-wave is obtained by adding some extend of Love wave to its value. This relationship is valid in wide range of inpedance ratio when some extent material damping exist in the surface layer. In case of 0% damping, however, raising by love wave influence strongly the peak value.

In the short period(higher mode) domain, as contrast between surface and bedrock become small, peaks are showing a tendency not to be clear. In the case of ground having large contrast, as becoming higher mode, peaks are showing a tendency not to be well-defined. It is considered that this tendency is due to influence which many higer mode drown out raising and falling by H/V-R of each mode.

Fig.18 shows the comparison between H/V-R and amplification due to S-wave in case of varing P-wave velocity of bed rock(CASE 4). From comparison this figure and Fig.17(b)($V_p=1600\text{m/s}$), whole shape of the two does not almost change, so it is considered that the influence of P-wave velocity is small in this inpedance range.

Fig.19 shows the relationship between ground impedance ratio and peak period ratio, respectively impedance ratio and amplification ratio. Peak period and amplification ratios express a relationship between H/V-R(2-directional vibration source) and S-wave($h=1\%$). From this figure, in the range of over from 2 to 3 of inpedance ratio, it is recognized that the peak period ratio of the two is equivalent and the peak value ratio shows the almost constant value. This result shows a application range for this method.

5.ESTIMATION OF SEISMIC MOTION CHARACTERISTICS BASED ON H/V-R

Power spectrum of earthquake records observed in Sendai during 1993 Kushiro-oki earthquake($M=7.8$) were compared to the one estimated by H/V-R. Power spectrum based on H/V-R is calculated from the product of the square of H/V-R regarded as a transfer function of ground and power spectrum at bedrock. Main part for about 20 second of Record obtained at GL-33m($V_s=1400\text{m/s}$) in TAMA was used as bedrock wave. Fig.20 shows the comparison between power spectrum obtained from earthquake records and the one estimated by H/V-R. It is difficult to compare the amplitude of spectra of the two in detail because the bedrock wave is not able to be specified at each site. The whole shape of spectrum, however, is able to be compared. From this figure, the calculated spectrum is in harmony with the observed one at every sites and the validity that H/V-R is approximately regarded as a transfer function of ground is recognized.

Considering the earthquake ground motion as a random vibration, strength of seismic motion estimated from H/V-R were compared with observed maximum acceleration. Relative value of acceleration was compared because the bedrock wave is not able to be specified at each site. Used earthquakes are the above-mentioned

Kushiro-oki EQ, 1989 Sanriku Haruka-oki EQ(M=6.5) and 1989 Iwate-oki EQ(M=7.3). As a average value of these three earthquake, comparison between maximum acceleration ratio based on earthquake records and the ratio estimated by H/V-R(both are normalized by NAKA) is shown in Fig.21. From this figure, it is recognized that H/V-R is valuable to estimate the relative strength of earthquake motion.

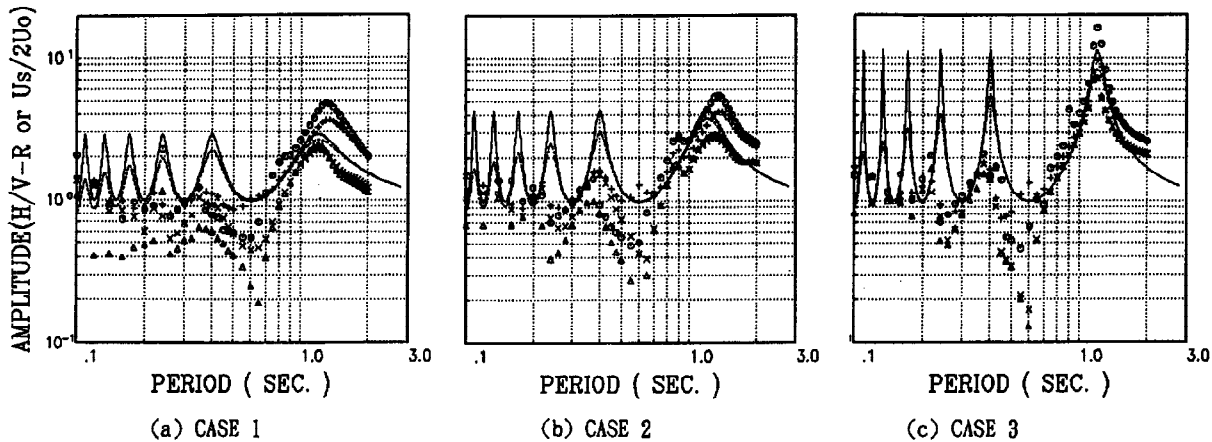


Fig.17 Comparison between H/V-R (○ :h(damping factor of soil)=0%, + :h=2% for 3-directional vibration source (j=1-3[ref.Fig.14]) & △ :h=0%, × :h=2% for 2-directional vibration source (j=1,2)) and amplification due to S-wave(solid line;h=0%,dotted line;h=2%) in case of varying S-wave velocities of bedrock.

V_s	V_p	ρ	H
(m/s)	(m/s)	(t/m ³)	(m)
100	500	1.4	30

(200) (1600) 2.0 - ∞
 (300) (800) (h=0%, 2%)
 (800)

	BEDROCK	
	V_s (m/sec)	V_p (m/sec)
CASE 1	200	1000
CASE 2	300	*
CASE 3	800	*
CASE 4	300	800

Fig.16 Ground model and numerical simulation conditions

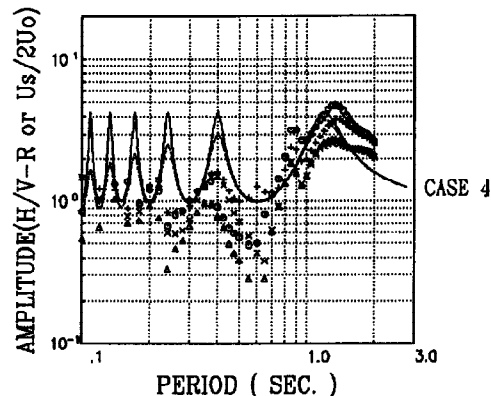


Fig.18 Comparison between H/V-R and amplification due to S-wave in case of varying P-wave velocities of bedrock. (Symbols have similar meaning as in Fig.18)

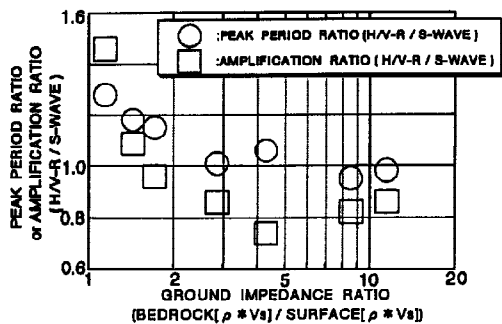


Fig.19 Relationship between ground impedance ratio and peak period ratio, respectively impedance ratio and amplification ratio. Peak period and amplification ratios express a relationship between H/V-R (2-directional vibration source) and S-wave(h=1%).

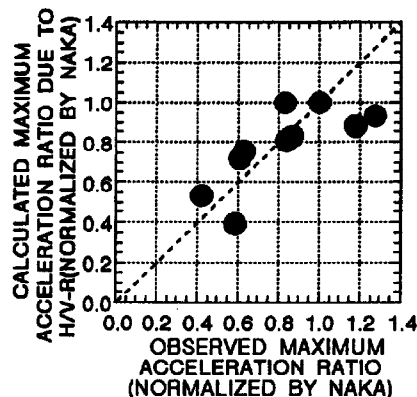


Fig.21 Comparison between maximum acceleration ratio based on earthquake records and the ratio estimated by H/V-R. (Both are normalized by NAKA)

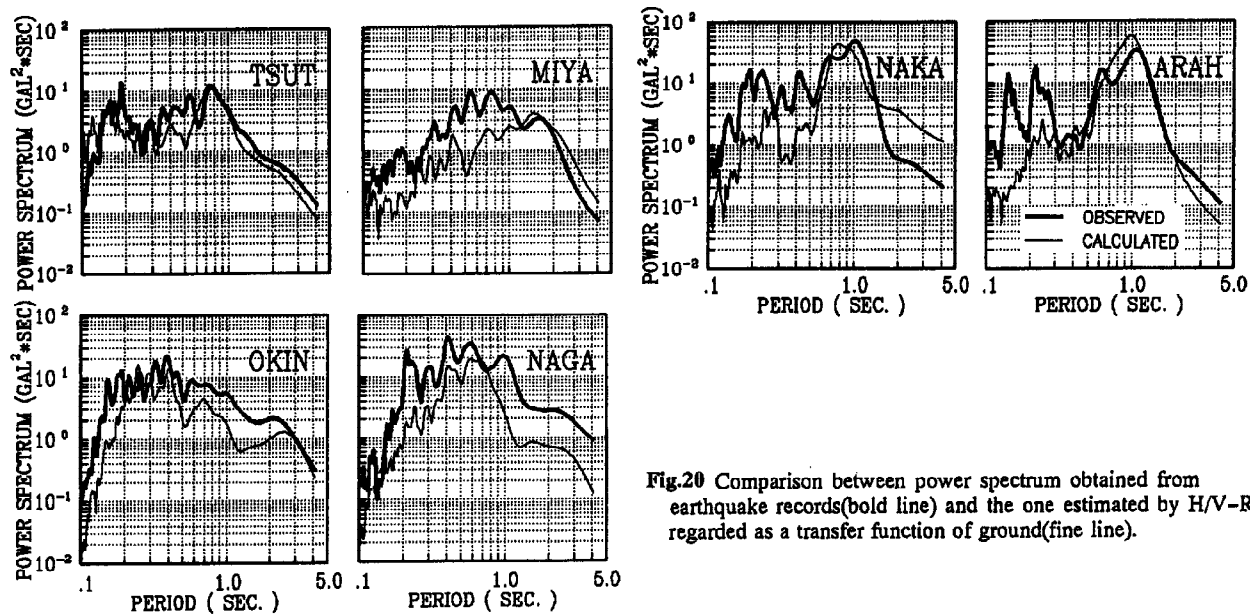


Fig.20 Comparison between power spectrum obtained from earthquake records (bold line) and the one estimated by H/V-R regarded as a transfer function of ground (fine line).

CONCLUSIONS

The fundamental characteristics of ratio between horizontal and vertical spectra (H/V-R) of short-period microtremors and its application range for engineering use are investigated. The results are summarized as follows:

- 1) Clearly observed peaks in H/V-R fairly correspond to the 1st mode of ground amplification (i.e. predominant period and peak value) due to S-wave propagating above bedrock which is relatively strong reflection boundary. On the other hand, peaks corresponding to the higher modes are hard to appear as compared with 1st mode.
- 2) H/V-R can be simulated in 2-dimensions by FEM considering Rayleigh and Love waves which include the higher modes generated by ground noise.
- 3) From the numerical simulation by FEM of single layer model, it is considered that the relationship in 1st mode mentioned item 1) is valid in the range of impedance ratio exceeding 2-3.
- 4) Contrast of P-wave velocity does not largely influence the peaks of H/V-R in the range of this investigation.
- 5) H/V-R does not express the absolute ground amplification above a bedrock. H/V-R, however, can be efficiently used to estimate the relative strength of earthquake motion in case of minor changes in ground conditions.
- 6) In case of estimating the ground amplification characteristics of the wide area by means of H/V-R, it is most important to make an investigation based on ground data and a confirmation at some reference points with regard to the meaning of the peaks of H/V-R.

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