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RELIABILITY ANALYSIS OF LIFELINE NETWORKS WITH VERTEX AND CORRELATION FAILURES

WANG DONGWEI and HUO DA
Department of Civil Engineering, Zhengzhou Institute of Technology, 450002, P. R. China

LI GUIQING
Department of Civil Engineering, Wuhan University of Technology, 430070, P. R. China

CHEN YIANYIAN
Institute of Engineering Theory, Harbin University of Architecture, 150001, P. R. China

ABSTRACT

Because of differences between lifeline networks and classic networks on the aspects of the vertex failure and the correlation failure, the computing method of reliability for the classic network can not be applied to the reliability analysis of lifeline systems directly.

The present paper provides a reliability computing method of the general weighted network with the failure probability of vertices. The correlation failures of lifeline systems under the seismic action are classified as the environmental correlation failure and the response correlation failure, and the response correlation failure is divided into strong response correlation failure and weak response correlation failure. A specific method for the aseismic reliability analysis of lifeline network considering the correlation failure is suggested in this study.

KEYWORDS

General weighted network; Environmental correlation failure; Response correlation failure.

INTRODUCTION

When analyzing the reliability of classic network or arc weighted network, a basic hypothesis is taken as follows:

Hypothesis 0
(1) For arc and network, there are only two states, reliable and failure;
(2) Vertex does not failure;
(3) Failure is independent among arcs; The failure of a arc can not result in the failure of other arcs.

It was pointed out that the algorithm of classic network reliability could not be the reliability analysis of lifeline system directly because of the diversity between lifeline network and classic network on vertex failure, correlation failure and other aspects (Wang *et al.*, 1992).

In this paper, It is deduced an algorithm for reliability of general weighted network with the possibility of vertex failure and classified the correlation failure types of lifeline under earthquake environment; proposed some basic conceptions of environmental correlation failure and response correlation failure, strong response correlation failure and weak response correlation failure; and suggested a concrete method for aseismic reliability analysis of lifeline network considering correlation failure.

ALGORITHM FOR THE RELIABILITY OF GENERAL WEIGHTED NETWORK

In a lifeline network, vertices may be some stress-focus places (such as tee joint on pipeline, voltage-divided valve), uncommon stress suffered places (such as underground pipeline going through different soil fields) or some structures (such as power transmission tower, overline bridge). They also have the possibilities in fault which even greater then those of arcs (various kinds of pipeline, circuit, *etc.*). Then it needed to dissolved the second hypothesis of hypothesis 0 for classic network where analyzing the reliability of lifeline network and to change the hypothesis 0 as follows:

Hypothesis 1

(1) There are only two states, reliable and failure for network units (vertices and arcs) and network system.

(2) Failure is independent for each unit, which means the failure of one unit can not result in the failure of other units.

The network comparing with this basic hypothesis is called general weighted network or generalized weighted network. In existing network analysis theory, the disposing method for the vertex weight is to equivalence it to arc weight. Suppose there is a weighted vertex *i*, split it into starting point *i*' and end point *i*", all arcs shooting into *i* are joined with *i*', arcs shooting from *i* are joined with *i*", Then add the weight of vertex *i* to the nominal arc between *i*' and *i*" (see Fig. 1).

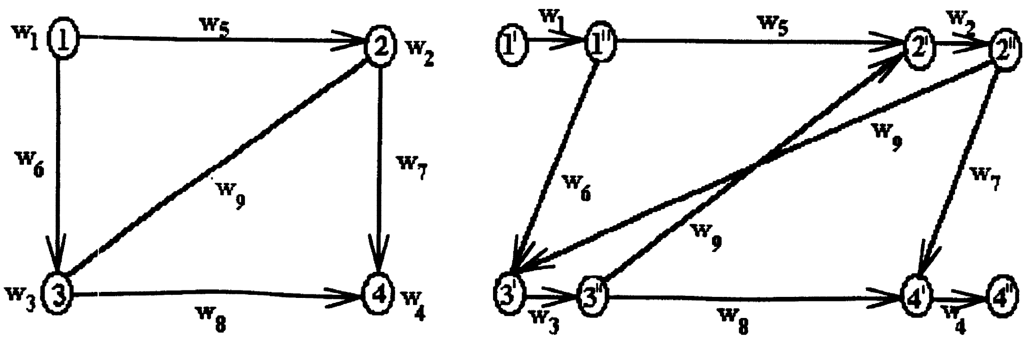

Fig. 1 Former method changing a general weighted network to a classic network

The advantage of this method is: after changing, the theory of classic network can apply to the analysis of general weighted network. But, it is can be gotten from figure 1, this method will multiple the number of vertices and arcs of the original network. In the meantime it is inconvenience to realized it on computer plotting and program.

To solve classic network reliability, firstly it is needed to search minimal path set between source and sink, and the disjoint minimal path sets, then replace the unit reliability with reliable unit in disjoint minimal path, and replace the failure unit with failure probability. In the end, the connectivity reliability of the network system can be solved out as follows:

$$R_{st} = \mathrm{P}\left\{\bigcup_{i=1}^{m} \mathrm{A}_i\right\} = \mathrm{P}\left\{\sum_{i=1}^{m_{\mathrm{dis}}} \mathrm{A}_{\mathrm{dis},\,i}\right\} \tag{1}$$

Here, A_i is the ith minimal path between source (s) and sink or terminal (t); m is the number of the minimal paths between source and sink; $\mathrm{A}_{\mathrm{dis},\, i}$ is the ith disjoint minimal paths between s and t; $m_{\mathrm{dis},i}$ is the number of the disjoint minimal paths between s and t.

It is easily to know that if extended the definition of the minimal path in classic network into that in general weighted network, the connectivity reliability of the general weighted network can be easily solved out with formula (1).

In classic network, if
(1)There is a path from vertex s to vertex t, which means it is reachable from s to t though a continuous arc sequence;
(2) The path is content with the minimum character of the path, which means original arc sequence will not be a path from s to t if any arc is deleted from the path;
the arc sequence is then called a minimal path from s to t. The arc number in the path is called the length of the minimal path.

Depending on the above-mentioned definition, it is set up a basic conception of minimal path in a general weighted network as follows:

Definition 1 In a general weighted network, if
(1)There is a path from vertex s to vertex t, which means it is reachable from s to t though a continuous vertex - arc sequence;
(2) The path is content with the minimum character of path, which means original vertex - arc sequence will not be a path from s to t if any vertex or arc is deleted from the path.
the vertex-arc sequence is called a minimal path from s to t, the number of the vertices and arcs in the path is called the length of the minimal path.

Suppose that there is a general weighted network $\mathrm{N_g}$, A_i is a minimal path of classic network $\mathrm{N_a}$ which has the same structure with $\mathrm{N_g}$. The length of the path A_i is l. Because two continues arcs are joined with only one vertex, after considering the l+1 vertices of A_i (including source s and sink t on it), a minimal path of general weighted network Ng (noted as A'_i) will be formed. Apparently, A'_i is satisfied with the minimum character of arc and does not include the isolated or duplicated vertices (otherwise there will be a circle in A_i). So, A'_i is also satisfied with the minimum character of vertex. Then it is gotten a following theorem:

Theorem 1 For a minimal path with length l of an arc weighted network, after string with the $l+1$ vertices in the path, it will form a minimal path of a general weighted network with same structure.

Then, the following theorem can also be proved:

Theorem 2 The number of minimal paths in a general weighted network is the same with that in a same-structured arc weighted network.

Prove: Assume there is a general weighted network N_g with the minimal paths numbered m_g, and there is an arc weighted network N_a which has the same structure with N_g, with the minimal paths numbered m_a. If $m_g < m_a$, it expresses there is at least one minimal path of N_a can not form the minimal path of N_g after stringing the vertices the path passing through, which is contradict with theorem 1. If $m_g > m_a$, it expresses that there is at least one minimal path of N_g not meeting with the condition of arc minimum character, which is contradict with definition 1.

□

Depend on the above theorem, it can be solved out the minimal path set of general weighted network directly from that of classic networks, then the method to solve the connectivity reliability of general weighted network is as follows:

(1) Use the universal program to set up the minimal path set $\{A_a\}$ of a arc weighted network which has the same structure with the general weighted network calculated;

(2) For each minimal path, string all vertices the path passing through and form a minimal path set of the general weighted network $\{A_g\}$;

(3) Solve out disjoint minimal path set $\{A_{dis,\ g}\}$ with the universal program.

(4) Bring $\{A_{dis,\ g}\}$ into formula (1), and solve out the connectivity reliability of the network.

This method does not change the universal program solving the network reliability and not increase the number of vertices and arcs of the network itself. So, it is one of the recommendable effective method for connectivity reliability of lifeline network.

ASEISMIC RELIABILITY ANALYZES OF LIFELINE NETWORK CONSIDERING CORRELATION FAILURE.

While an earthquake occurred, there exists correlation failure between vertex and vertex. arc and arc, vertex and arc. So, the third note of hypothesis 0 should also be dissolved. But, it is very difficult or even not feasible to decide the correlated degree of network units exactly. So it is necessary to use simplified method to research the correlation failure of lifelines.

Correlation Failure Analysis under Earthquake Environment

There are two main kinds of structural damage during an earthquake:

(1) Dependent Damage The damage of a structure depends on the environment factors such as the break of fault zone, the liquefaction, the landslide or land sink. The character of this kind damage is that the

structural failure probability mainly depends on the damage probability of the surrounding that is much higher than the failure probability of the structure itself. The area forms the serious failure zone.

(2) Response Damage The damage of a structure is due to that the maximum response value of the structure is higher than the response limit under the action of the earthquake wave. The character of which is that the structural failure probability mainly depends on the joint probability density function of the maximum response value and the response limit value.

So, there exists strong correlation failure among those structures that is on a same fault zone, sand saturation area or adverse zone. But when the response damage is primary, weather the structure is failure or not mainly depends on the structural aseismic defensive. Besides, the aseismic ability of a structure has close relation with its dynamic character. The Michoacan earthquake resulted in many damages of high-soft aseismic designed buildings in Mexican. The main reason is that the geography condition of that place is volcano ash clay. The structures with similar self-vibration frequency under the same aseismic standard can be thought having higher correlation failure. On the contrary, the correlation failure may be relatively weaker when the structures have different aseismic standard and much different self-vibration freqeuncy. So, it is classified and defined as follows:

Definition 2 It is called the environment correlation failure among lifeline units where they are dependent damage. It is called the response correlation failure when response damage happens, in which, it is called the strong response correlation failure when the units have the same design standard and similar dynamic parameter, or it is called the weak response correlation failure.

Suppose that there are unit i and j in a lifeline network. It is known that the design intensity and the self-vibration frequency are I_{di}, I_{dj} and ω_i, ω_j respectively, then when

$$I_{di} = I_{dj} \tag{2}$$

and satisfy

$$0.8 < \beta_{ij} = \frac{\omega_i}{\omega_j} < 1.25 \tag{3}$$

The two units are regarded as the strong response correlation failure, otherwise they are weak response correlation failure.

Algorithm for Network Reliability of Correlated Failure Lifeline

In accordance with the above analysis, it is built a basic hypothesis of the algorithm for the network reliability of correlated failure lifeline in this paper:

Hypothesis 2

(1) There are only two states, reliable and failure for network units and network system.

(2) Dispose environment correlation failure units and strong response correlation failure units with the hypothesis of complete correlation failure. Dispose weak response correlation failure units with the hypothesis of failure independence.

Depended on the above-mentioned, a program is made to analyze the aseismic reliability of failure correlation lifeline network. The basic steps are as follows:

(1) Analyze correlation failure for all vertices and arcs in the network system and decide the complete correlation failure units subset.

a). All the units on the same fault zone are in one subset. n_1 of faults form n_1 unit subsets. The unit failure probability in each subset is replaced by the break probability of the fault.

b). All the units on the same sand saturation zone are in one subset. n_2 of saturation zones form n_2 unit subsets. The unit failure probability in each subset is replaced by the liquefaction probability.

c). All the units with same design intensity and on the same self-vibration field are in one subset. Various combinations form n_3 unit subsets, the unit failure probability in each subset is replaced by the highest one in the subset.

d). Take one representative unit from each failure complete correlated subset, the numbers of the representative units are $n_1+n_2+n_3$.

(2) Build the minimal path set and disjoint minimal path set between source and sink.

a). Solve out minimal path set of the network assuming the units are failure independent.

b) .Replace failure complete correlated units in minimal paths with representative units.

c). Make the absorbing compute for minimal path set and search out minimal path set considering correlation failure.

d). Disjoint the minimal paths having been absorbing computed, search out disjoint minimal path set considering correlation failure.

(3) Bring the unit reliability and failure probability into matching reliable unit and failure unit in disjoint minimal path set, solve out the network connectivity reliability considering correlation failure.

EXAMPLE

For convenient with the output of computer, we set up following basic rules
(i) expresses the vertex numbered *i*, [i]expresses the arc numbered *i*; in the meantime, (i) expresses the vertex *i* is reliable, and [i] expresses the arc *i* is reliable, while (-i) expresses the vertex *i* is failure, and [-i] expresses the arc *i* is failure.

Suppose there is a lifeline shown in Fig. 2. There is a fault going through the arc [3] and [4], its break probability is 0.06, and there is a sand saturation zone across the arc [7], [10] and [14], the liquefaction

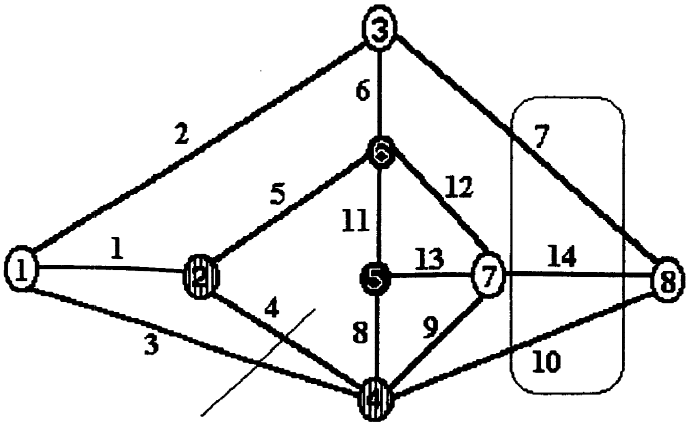

Fig. 2. Lifeline network of the example

Table 1. Parameters of the lifeline network shown in Fig. 2

unit	reliability	subset	unit	reliability	subset	unit	reliability	subset
(1)	0.9999	—	(2)	0.9800	1	(3)	0.9800	—
(4)	0.9800	1	(5)	0.9500	2	(6)	0.9500	2
(7)	0.9000	—	(8)	0.8700	—			
[1]	0.9700	—	[2]	0.9700	—	[3]	0.9400	3
[4]	0.9400	3	[5]	0.9400	—	[6]	0.9400	—
[7]	0.800	4	[8]	0.8800	—	[9]	0.8500	—
[10]	0.800	4	[11]	0.9000	—	[12]	0.8500	—
[13]	0.8500	—	[14]	0.8000	4			

probability equals to 0.2, vertices (2) and (4), (5) and (6) have the same design intensities and on the same self-vibration fields separately. Parameters of vertices and arcs are shown is Table 1.

Where all the units are considered as failure independent, from source (1) to sink (8), there are 38 of minimal paths. Tring all vertices the minimal paths passing through, it forms the minimal path set of the lifeline network:

A1: (8)[10](4)[3](1)
A2: (8)[7](3)[2](1)
A3: (8)[14](7)[9](4)[3](1)
A4: (8)[10](4)[4](2)[1](1)
A5: (8)[14](7)[12](6)[6](3)[2](1)
A6: (8)[14](7)[12](6)[5](2)[1](1)
A7: (8)[7](3)[6](6)[5](2)[1](1)
A8: (8)[14](7)[13](5)[8](4)[3](1)
A9: (8)[14](7)[9](4)[4](2)[1](1)
A10: (8)[14](7)[12](6)[11](5)[8](4)[3](1)
A11: (8)[10](4)[9](7)[12](6)[6](3)[2](1)
A12: (8)[14](7)[13](5)[11](6)[6](3)[2](1)
A13: (8)[10](4)[8](5)[11](6)[6](3)[2](1)
A14: (8)[10](4)[4](2)[5](6)[6](3)[2](1)
A15: (8)[7](3)[6](6)[11](5)[8](4)[3](1)
A16: (8)[10](4)[9](7)[12](6)[5](2)[1](1)
A17: (8)[14](7)[13](5)[11](6)[5](2)[1](1)
A18: (8)[10](4)[8](5)[11](6)[5](2)[1](1)
A19: (8)[14](7)[12](6)[5](2)[4](4)[3](1)
A20: (8)[7](3)[6](6)[5](2)[4](4)[3](1)
A21: (8)[7](3)[6](6)[12](7)[9](4)[3](1)
A22: (8)[14](7)[13](5)[8](4)[4](2)[1](1)
A23: (8)[10](4)[8](5)[13](7)[12](6)[5](2)[1](1)
A24: (8)[7](3)[6](6)[11](5)[13](7)[9](4)[3](1)
A25: (8)[10](4)[9](7)[13](5)[11](6)[6](3)[2](1)
A26: (8)[10](4)[9](7)[13](5)[11](6)[5](2)[1](1)
A27: (8)[14](7)[13](5)[11](6)[5](2)[4](4)[3](1)
A28: (8)[14](7)[9](4)[8](5)[11](6)[5](2)[1](1)
A29: (8)[14](7)[9](4)[8](5)[11](6)[6](3)[2](1)
A30: (8)[10](4)[8](5)[13](7)[12](6)[6](3)[2](1)
A31: (8)[14](7)[9](4)[4](2)[5](6)[6](3)[2](1)
A32: (8)[7](3)[6](6)[12](7)[9](4)[4](2)[1](1)
A33: (8)[7](3)[6](6)[12](7)[13](5)[8](4)[3](1)
A34: (8)[14](7)[12](6)[11](5)[8](4)[4](2)[1](1)
A35: (8)[7](3)[6](6)[11](5)[8](4)[4](2)[1](1)
A36: (8)[7](3)[6](6)[12](7)[13](5)[8](4)[4](2)[1](1)
A37: (8)[14](7)[13](5)[8](4)[4](2)[5](6)[6](3)[2](1)
A38: (8)[7](3)[6](6)[11](5)[13](7)[9](4)[4](2)[1](1)

where considing the correlation falures, there are 4 failure complete correlated subsets:

{1}: { (2), (4) }   {2}: { (5), (6) }   {3}: { [3], [4] }   {4}: { [7], [10], [14].}

〈1〉

And only remains 6 minimal paths:

A1: (8) {4}{1}{ 3}(1)
A2: (8){4}(3)[2](1)
A3: (8) {4}(7) [12]{2}[5]{1}[1](1)
A4: (8){4}(3)[6]{2}[5]{1}[1](1)
A5: (8) {4}{1}[8]{2}[11][5][1](1)
A6: (8){4}(7)[13]{2}[11][5]{1}[1](1)

The disjoint minimal paths:

Adis1: A1
Adis2: A2 ※{ -3}
Adis3: A2 ※{ -1}{ 3}
Adis4: A3 ※[-2]{-3}
Adis5: A3 ※[2](-3){-3 }
Adis6: A4 ※[-12]{-3}[-2]
Adis7: A4 ※[12](-7){-3 }[-2]
Adis8: A5 ※(-3){-3}[-12]
Adis9: A5 ※(-3){-3}(-7)[12]
Adis10: A5 ※[-6](3)[-2]{-3}[-12]
Adis11: A5 ※[-6](3)[-2]{-3}(-7)[12]
Adis12: A6 ※[-8][-12]{-3}(-3)
Adis13: A6 ※[-2][-8][-12]{-3}[-6](3)

Bring the unit reliability and failure probability into matching reliable unit and failure unit in disjoint minimal path set, the connectivity reliability of the lifeline network equal to 0.694944. But if not considering the correation failure not, the connectivity reliability of the lifeline network will be 0.857625.

CONCLUSION

(1) The algorithm for network reliability considering vertex failure proposed in this paper is recommendable because it does not change the subprogram searching for minimal paths and disjoint minimal paths.

(2) The influence of correlation failure to network connectivity reliability is notable. It is rational and feasible to classify the correlation failure response under the earthquake action into the environmental correlation failure and response correlation failure, and divide the response correlation failure into strong one and weak one.

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