



SWV RESOLUTION OF SASW TESTS

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ABSTRACT

The paper introduces the concept of phase and shear wave velocity (SWV) reference profiles to characterize the variation of the SWV and corresponding dispersion of rayleigh waves in soil profiles. This concept is used to conduct a parametric study of the resolution and accuracy of the SASW tests. The analysis show that for practical purposes there are no unique interpretation of the SASW tests. Also, the resolution decreases with depth and it is not possible to define distinct layers within the profile. The tests can be used to obtain the general variation of SWV as described by means of the reference profiles.

KEYWORDS

SASW; Rayleigh Waves; Dispersion of Rayleigh waves; Dispersion Inversion; Reference Profiles; Shear Wave Velocity; SASW resolution; Surface waves.

INTRODUCTION

The purpose of the paper is to evaluate the resolution of shear wave velocity (SWV) profiles obtained from interpretation of dispersion of surface waves. In this type of methods, the phase velocity (V_f) of the fundamental mode of Rayleigh waves is measured by some means in the field. The SWV is then obtained by an inversion procedure, often based on the forward computation of the dispersion from a given SWV profile and a non-linear optimization. The most common variation of this technique is the so called Spectral Analysis of Surface Waves (SASW) method. To be more general and consistent, the technique is referred here as the V_f - V_s method.

The V_f - V_s method is useful for non-intrusive fast and efficient measurement of SWV for engineering applications. Potential uses of data obtained by this SWV measurement method are: site response evaluation, liquefaction potential evaluation, static and dynamic foundation parameters evaluation, soil layering, and pavement systems evaluation. There has been however, a lack of understanding of the resolution of the V_f - V_s method and the SWV layer data that can be obtained from this type measurements. This is fundamental to properly asses the applicability of this data in engineering. The paper will show quantitative results that can be

used to define the resolution of this type measurements. The concept of phase velocity (Vf) and SWV (Vs) reference profiles is introduced first, based on experimental and empirical evidence from SASW measurements. This concept is then used to evaluate the behavior of dispersion curves for the actual range of SWV values in soils. The resolution of single layers within the reference profile is then examined.

PHASE AND SHEAR WAVE VELOCITY REFERENCE PROFILES

The shear wave velocity of soils depend on several parameters. According to Mitchell (1993), the most important are:

- The type of soil, defined by compositional and environmental factors such as mineralogy, internal structure, cementation, overconsolidation, and geologic age.
- The cyclic strain, strain rate and number of loading cycles.
- The effective confining stress.

The basic form of the relationship between these parameters was determined by Seed et al. (1984) and others from laboratory tests. For non cohesive materials the form is given by:

$$G = 1000 K_2 \sqrt{\sigma_m} \quad (1)$$

The equation units are pounds per square foot (psf), where G is the shear modulus, σ_m is the mean effective stress and K_2 is a coefficient that depends primarily on grain size, relative density, and shear strain.

For cohesive soils Dobry and Vucetic (1987), reported the following equation derived from laboratory data to compute the shear modulus at low strains G_{max} :

$$G_{max} = 625 \frac{OCR^k}{0.3 + 0.7 e^2} \sqrt{P_a \sigma_m} \quad (2)$$

where OCR is the overconsolidation ratio, e is the void ratio, P_a is atmospheric pressure and σ_m is the mean effective stress, all in consistent units.

Equations 1 and 2 show that for all soils the shear modulus is a function of the soil type and strain level multiplied by the square root of the mean effective stress. For the elastic waves used with the Vf-Vs method, the strain levels in the soil are very low. The shear modulus has its maximum value at low strains as given by Equation 2. In Equation 1 the value of K_2 is also a decreasing function of the strain level with the maximum at low strain level values. Therefore, for the Vf-Vs measurements the form of Equations 1 and 2 is in general:

$$G = F_1 \sqrt{\sigma_m} \quad (3)$$

where F_1 is a function of the soil type. Substituting the expression for the shear modulus as related to the shear wave velocity, the mean effective stress in geostatic conditions, and σ_v , the vertical effective stress, obtained as the product of the unit weight and the depth Z, we have:

$$V_s = \sqrt{\frac{g}{\gamma} F_1 \sqrt{\frac{1 + 2 K_o}{3} \gamma_{avg} Z}} = F_2 Z^{0.25} \quad (4)$$

Equation 4 shows that the shear wave velocity is a function of depth. For a uniform granular, recently deposited soil F_2 is approximately constant. However, F_1 , K_o and γ are functions of Z and therefore a more general expression of equation 4 is:

$$V_s = V_{os} Z^{ms}$$

(5)

Equation 5 has the same form as experimental data obtained from the CXW measurements (USC, 1993), (VIC, 1995). The parameter V_{os} depend on the type of soil, while ms reflects the combined effect of soil properties variation with depth and the SWV change with the confining stress.

Based on typical values of soil parameters form laboratory tests, the range of V_{os} values in Equation 5 is between 50 and 300 m/s. The parameter ms accounts for V_s variation with respect to the confining pressure and the variation of soil parameters with depth. If soil parameters do not change, the only effect is the variation with the confining pressure, and ms should be close to 0.25. However, it is most likely that parameters such as void ratio and unit weight also vary with depth. Assuming that the combined effects of the different parameters in F_2 were proportional to the square root of Z , then the value of ms would be 0.50. If the combined effect were proportional to Z , ms would be 0.75. It does not appear that ms in soils could be higher than 0.5 to 0.75. For a uniform velocity soil layer, ms would be close to 0.0. Typical ms values range between 0.025 and 0.50. In rock or shallow residual soil sites with partially weathered rock where SWV increases rapidly with depth ms could be higher.

Results of a large number of SASW measurements in a wide variety of soils (USC, 1993), (VIC, 1995) are consistent with the range of values and the form of the V_s reference profile. Moreover, the data show that the general variation of phase velocity with wave length in soil profiles can also be defined by a simple function of two parameters. The function that best fit the experimental data is defined as the reference phase velocity profile. The reference profile in a uniform soil site is defined by two parameters, V_{of} and mf which relate the phase velocity V_f with the wave length λ by the equation:

$$V_f = V_{of} \left(\frac{\lambda}{2} \right)^{mf} \quad (6)$$

Equations 5 and 6 show that both the general variation of the SWV and the dispersion curve for soil profiles can be described by equations of similar form, one in terms of the depth and the other in terms of the wave length. These equations are refered to as the reference phase and shear wave velocity profiles and give the general behavior of the SWV and V_f at a site.

INVERSION OF THE REFERENCE PROFILE

The reference profiles concept can be used to obtain a first approximation for the interpretation of V_f - V_s measurements. If the relationship between V_f and the corresponding V_s profile could be found, it would be possible to establish a direct method for inversion of the reference profile. A comprehensive parametric analysis was conducted to obtain V_s profiles over the range of values of V_{of} and mf that occur in actual soil deposits. The parametric analysis is described in the following.

For each particular case, defined by the values of V_{os} and ms , a shear wave velocity profile was generated to a depth of 100 m. Many layers were used to approximate the actual variation of V_s . The layer thicknesses were computed so that no layer would have a thickness of more than 0.125 of the typical wavelength corresponding to that depth. This discretization of the SWV profile guarantees that the computed dispersion curve is virtually identical to that obtained by using the actual continuous variation of SWV as described by the reference profile equation.

The range of V_{os} covered in the parametric analysis was between 10 and 300 m/s at 10 m/s increments. The

range of m_s considered was between 0.05 and 0.8 at 0.025 intervals. Other soil parameters were kept constant. A Poisson ratio of 0.2 and unit weight of 17 kN/m^3 were used.

The dispersion curve for the fundamental mode of plane Rayleigh waves was computed for the assumed soil profile by using a transfer matrix method (Rodriguez, 1994). A nonlinear curve-fitting procedure was used to obtain the reference phase velocity profile parameters from the computed dispersion curve. This formulation minimizes the deviations between Equation 6 and the target dispersion curve to obtain the parameters V_0f and m_f that produce the best fit by a least squares criteria. The NOLSOL computer routines based on the method by Dennis et al. (1981) were used for this computation. The curve-fitting of Equation 6 with the computed dispersion curve was very good for all cases, with average relative variations within $\pm 1\%$ for all cases.

Based on the results of the first step in this parametric analysis, the relationship between V_s and V_f reference profiles were determined. The variation of V_f may be described by the following equation:

$$V_0f = \frac{0.9382 - 0.3637 m_s^{1.1695}}{1.0352 - 0.044 m_s} V_0s \quad (7)$$

The corresponding equation obtained for m_f is as follows:

$$m_f = 0.08582 \log(1.4752 V_0s) m_s \quad (8)$$

With equations 7 and 8 the shear wave velocity reference profile may be obtained from the phase velocity reference profile. The inversion of the V_f - V_s method is reduced to solving for V_0s and m_s the non linear system of equations 7 and 8. This is accomplished easily using the Newton method with analytical Jacobian.

EFFECT OF SINGLE LAYERS ON THE DISPERSION CURVE

Once a reference profile is obtained, the effect of single layers with SWV that is different from the reference profile should be considered. A detailed analysis of the single layer effect on the dispersion curve was performed. Approximate equations were developed to establish the relationship between layer parameters (depth, thickness, and shear wave velocity), and their effect on the dispersion curve. The results from this section provide useful data to evaluate the resolution of the V_f - V_s method for such soil profiles.

A parametric study was performed to evaluate the relationship between the parameters of a single layer and the dispersion curve, for a reference profile described by Equation 5. Particularly, the focus of this evaluation was on the relationship between the change in shear wave velocity in the layer and the corresponding change in phase velocity, the wave length range that is affected by the single layer, and the effect of layer thickness. The layer parameters which were investigated included the mid point depth, thickness, and relative change of velocity with respect to the reference profile. The dispersion curves for the parametric study of the single layer were obtained by using a transfer matrix method (Rodriguez, 1994). The parameters of the reference profile were also varied over the range of values found in soils as discussed before. These parameters are shown in Fig. 1.

The reference shear wave velocity profile is defined by the parameters V_0s , m_s ; the layer is defined by its mid-depth D_l , thickness W_l , and maximum shear wave velocity V_{smax} with respect to the reference V_{sref} . For the analysis, the SWV difference was defined by the ratio $dv = V_{smax}/V_{sref}$, and the relative thickness of the layer, defined as $w = W_l/D_l$. These ratios were used for characterization of the results.

For each profile the dispersion curve was computed (as shown in the right side of Fig. 1) and the parameters defining the effect of the layer were determined including the maximum phase velocity value V_{fmax} and the corresponding wave length where it occurs λ_{max} ; $D_{fmax}=1/2\lambda_{max}$. Another value of interest determined from the dispersion curve was D_{f0} (the wave length at which the effect of the layer becomes evident).

Figures 2 through 4 show typical results from the parametric study for a particular reference profile. Fig. 2 shows the shear wave velocity profile and corresponding dispersion curves for eight cases. For each case the profile follows the reference profile except for a single layer that departs from it. All the cases shown have the same value of $w=0.15$, and dv values of either 0.7 or 1.3. The figure shows the range of variation of the dispersion curve due to the presence of these layers. It appears that when the change in the layer shear wave velocity was $\pm 30\%$, the respective change in the phase velocity was only $\pm 3\%$. In each case the single layer forms a large variation in the shear wave velocity profile, and the respective change in the dispersion curve extends over a wide range of wave lengths. The Figure demonstrates that the Vf-Vs has low sensitivity and resolution.

Figure 3 shows results of various relative thicknesses and velocities of a single layer. The Figure shows the dispersion curves of three single layer cases and the reference profile. For the case of a single, relatively thin ($w=0.05$) layer with low velocity ($dv=0.85$), the difference with respect to the reference dispersion curve is very small. For practical purposes it would be impossible to resolve this layer even if a very good measurement is obtained. Two other cases were considered where both w and dv were increased. The Figure shows again that the change in phase velocity is much smaller than the SWV change in the layer. However, it is evident that the changes in the dispersion curve depend on the velocity and thickness of the single layer. Another observation is that for all the cases shown, the effect of the layer is negligible up to a depth $D=7$ m. It appears that deeper than this point there is a noticeable difference between the reference dispersion curve and the one obtained with the layer. This was a consistent observation for all the cases in the parametric study. This depth, defined as D_{f0} , corresponds to values that are between $0.4 D_1$ to $0.6 D_1$.

The range of influence of the layer on the longer wave length range is much larger than toward the lower end (defined by D_{f0}). Significant differences with respect to the reference profile are observed even for depths reaching $5 D_1$.

Additional results from the same cases that were considered in Fig. 3 are shown in Fig. 4. In this Figure, three shear wave velocity profiles were considered. The layer relative thicknesses were $w=0.2, 0.1$ and 0.05 , and the relative SWV changes were $dv=1.15, 1.25$ and 1.40 respectively. The dispersion curves for these three profiles are shown in the Figure. The small differences between these curves are well within the normal range of variation of the experimental data. For all practical purposes it would be impossible to differentiate between these three cases if the data were to be obtained by an actual measurement. Furthermore, the difference between the dispersion curves shown in Fig. 4 and the reference dispersion curve for this case (shown in Fig. 3) is also very small. Based on these limitations, the layers shown in Fig. 4 could not be resolved from a Vf-Vs measurement, no matter what method is used for the test or the interpretation.

The results in Figs. 3 and 4 indicate that the change in the dispersion curve depends on the combined effect of the layer velocity and thickness. The results shown in Figs. 2 to 4 consider the effect of only one layer in the profile. If there are several such layers, their combined effect may overlap over certain wavelength ranges. In such cases the layers' resolution is even more difficult. Therefore, it is concluded that for practical purposes, there are no unique interpretation of SWV given a dispersion curve. Only the general variation of SWV given by the reference can be reliably obtained from the Vf-Vs method interpretation.

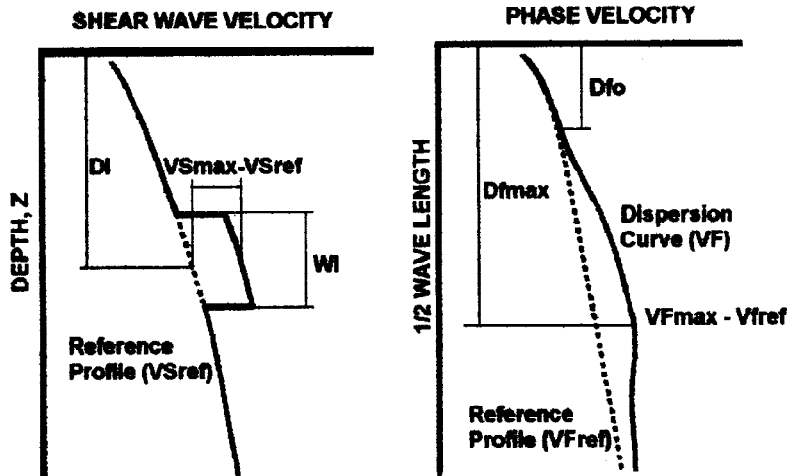


Fig. 1. Parametric Study Definitions

The parametric study included the following values: D_I values of 2.5, 5.5, 12.5 and 30.0 m; d_v values of 0.6, 0.75, 0.85, 1.15, 1.25 and 1.4; and, w values of 0.05, 0.1, and 0.20. This resulted in the analysis of 72 cases for each reference profile. Ten reference profiles were considered in total with m_s values varying between 0.1 and 0.6, and V_{os} values varying between 100 and 170 m/s.

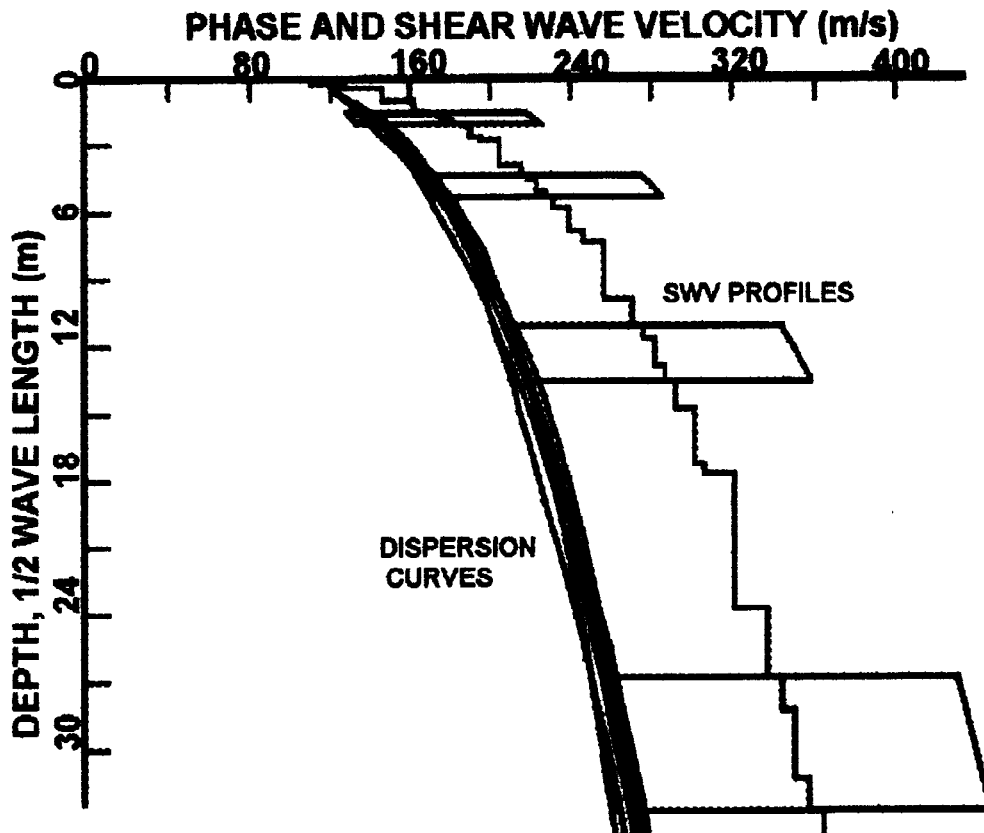


Fig. 2. Typical parametric Study results.

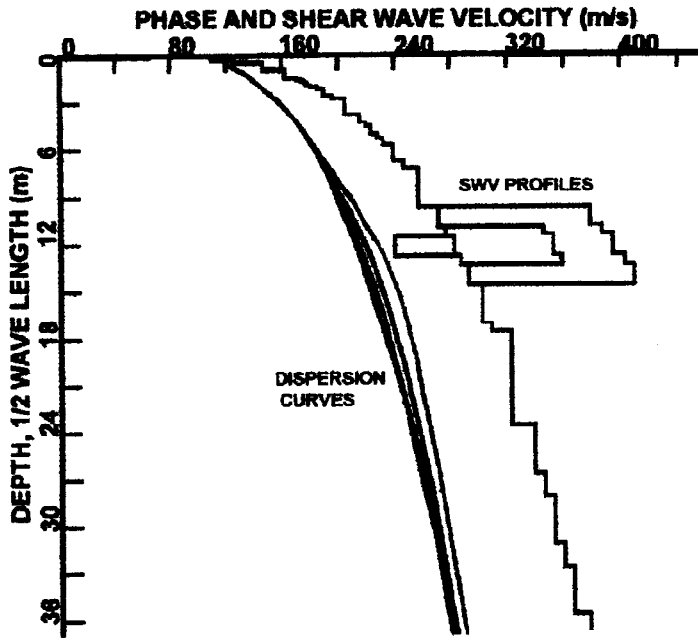


Fig. 3. Effect of layers having distinct SWV variation with respect to the reference profile on the dispersion curves.

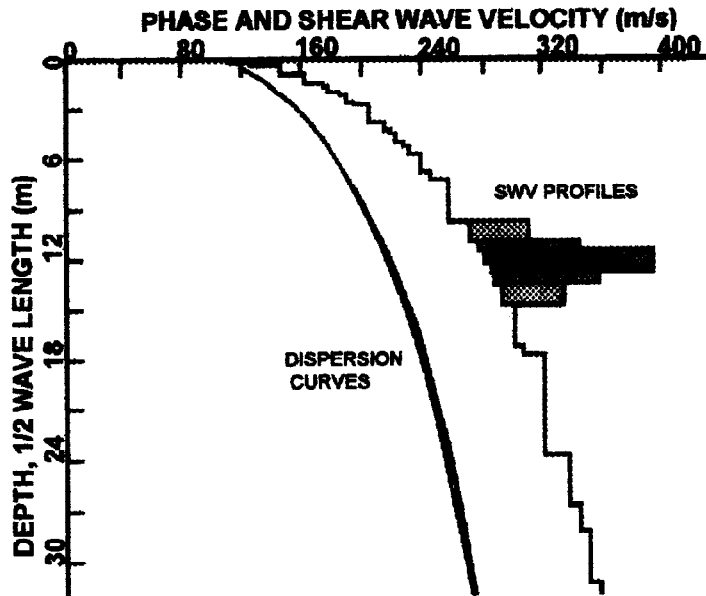


Fig. 4. Very similar dispersion curves for three profiles with large variations on a single distinctl ayer with varying proportions of V_s and thickness.

CONCLUSIONS

The results of this study show that both the dispersion curves and the SWV can be described by an equation of the form used for the reference SWV profile. A relationship between the SWV profile and the dispersion curve was obtained that show the relative importance of the soil parameters. An analysis of these equations show that the resolution of the SASW method decreases with increased wave length.

The results obtained from considering the SWV variations with respect to the reference profile show that the SASW method is very insensitive to local variations in the SWV profile. Furthermore, it is shown that, within finite numerical accuracy, infinite SWV profiles can be obtained that produce the same dispersion curve. This proves that no unique inversion can be obtained in the interpretation of SASW tests for the identification of distinct features in the soil profile.

The results of this study show that SASW tests can be used to obtain a good estimate of the average SWV profile at a site in terms of the reference profile. However, the method is very poor in resolving distinct features, or single layers with properties different from the reference profile in a soil site. Within these limitations, SASW results can be used for site response and foundation design. At the same time, these results rule out the practical use of SASW tests for applications such as liquefaction potential evaluation.

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