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DYNAMIC NONLINEAR ANALYSIS OF 3-D RC SHEAR WALL BY FINITE ELEMENT METHOD

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ABSTRACT

A dynamic nonlinear model by finite element method (FEM) is proposed to analyse 3-dimensional (3-D) reinforced concrete (RC) shear wall structures subjected to earthquake motions. The proposed constitutive model is based on the nonlinearity of concrete and reinforcement. It considers the tension stiffening in tension and the degradation of stiffness and strength in compression of concrete after cracking. Reinforcement is modeled by a bi-linear relationship. To solve the nonlinear dynamic equations and obtain the responses of structures the Impulse Acceleration Method is employed. The damping effect is considered by assuming equivalent viscous damping. This analytical method was applied to a test specimen of the wall with H-shaped section which is one of the dynamic model tests for evaluation of the seismic behavior of reactor buildings. The calcutions were performed continuously from the elastic range to the vicinity near failure. The comparison with the test results shows that this approach had good accuracy.

KEYWORDS

Dynamic responses; nonlinear analysis; reinforced concrete; three dimensional shear walls; finite element method; impulse acceleration method; constitutive model.

INTRODUCTION

During the last decade, in order to get a proper model for finite element analysis available for the structural engineer, many research works have been done for RC finite element method which are surely very important. One of these famous works is the modified compression field theory for RC (Collins, *et al.*, 1982 and Vecchio, *et al.*, 1986). Its application was made to static nonlinear finite element analysis of RC shear walls (Inoue *et al.*, 1985 and Vecchio, 1989). Another one of these works is dynamic nonlinear finite element analysis of RC plane structures (Song and Maekawa, 1991). Their approach is based on path-dependent constitutive model of RC elements and the dynamic equilibrium of structures.

Based on the above studies, a simple model under cyclic loading is proposed (Yang, Inoue and Shibata, 1995) toward formulating a more rational model which can be used in a dynamical nonlinear FEM analysis of 3-D RC shear walls. The constitutive model is formulated on the basis of the nonlinearity of concrete and reinforcement. It considers the tension stiffening in tension and the degradation of stiffness and strength in compression after cracking based on the Collins' theory. Reinforcement is modeled by a bi-linear relationship.

In this paper, a dynamic FEM analysis procedure for RC 3-D shear walls is given according to above model. Furthermore, the analysis of a dynamic test is presented.

FEM MODEL AND MATERIAL CONSTITUTIVE LAW

FEM Model

To simplify the complex 3-D RC shear wall-slab structures with the nonlinearity of RC materials, the proposed model is developed by assembling the iso-parametric plane elements with 4-node for walls and iso-parametric 3-D solid elements with 8-node for a slab on the walls. The joint nodes, which tie a wall to its orthogonal wall and the slab, are defined to be 3-D nodes in order to model the 3-D shear wall structures (see Fig. 1). The RC wall is considered as inelastic material, while the slab is considered as relatively rigid elastic material.

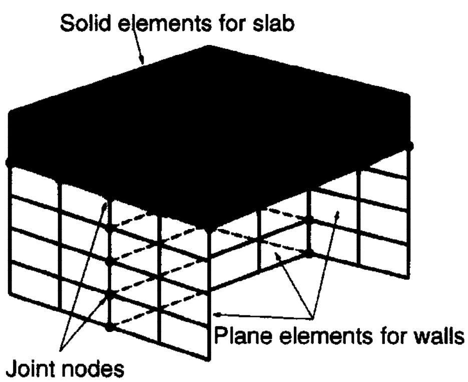


Fig. 1 FEM Model

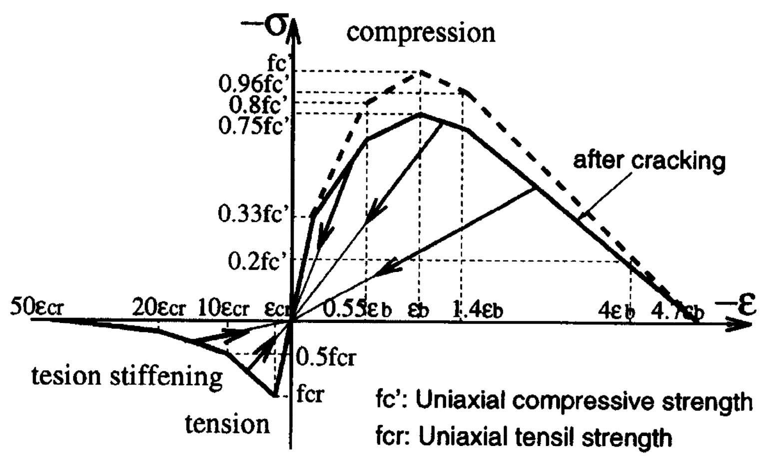


Fig. 2 Hysteretic Rule of Concrete

Material Constitutive Model

The constitutive model is formulated on the basis of the nonlinearity of concrete and reinforcement. The nonlinear RC model is based on the smeared appoach for both concrete cracks and reinforcements in an element. The constitutive equations of concrete under biaxial stresses are defined for two different concrete states as follows:

1. Uncracked concrete
The isotropic material stiffness matrix is used before concrete cracking, where the tangential isotropic modulus Ei is determined by the curve (Fig. 2) of the larger absolute value of the principal stresses in uncracked Gauss points of every element. Here ν is the Poisson's ratio.

$$[D]_c = \frac{E_i}{1-\nu^2}\begin{pmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & (1-\nu)/2 \end{pmatrix} \tag{1}$$

2. Cracked concrete
Based on the Collins' theory, cracked concrete is treated as a kind of new material with modified stress-strain characteristics. Thus, the cracked concrete material stiffness matrix is as follows

$$[D]_c = \begin{pmatrix} E_{c1} & 0 & 0 \\ 0 & E_{c2} & 0 \\ 0 & 0 & G_c \end{pmatrix} \tag{2}$$

where E_{c1} and E_{c2} are the tangential moduli as evaluated using the stress-strain curve of Fig. 2 in the compression and tension respectively. G_c is the tangential shear stiffness after cracking.

Basic assumptions for cracked concrete can be summarized as follows
(1) After cracking, the concrete was treated as an orthotropic material with respect to the crack direction and its orthogonal direction.
(2) The tension stiffening effects of concrete was considered after cracking.
(3) The degradation of stiffness and strength of concrete in compression was considered after cracking. The degraduation ratio of strength was assumed to be 0.75.
(4) Under cyclic loading, unloading hysteretic rule was assumed an origin oriented model (See Fig. 2). The hysteretic energy will be taken into considerition by equivalent viscous damping in the dynamic analytical procedure to overcome the shortcomings that this model only gives small area surrounded by the hysteretic loop.
(5) The new crack direction under the negative load was assumed to be orthogonal to the direction of the old crack under the positive load.

The hysteretic rule of reinforcement is modeled by a bi-linear relationship.

DYNAMIC ANALYTICAL PROCEDURE

Free Vibration

Because the weight of shear walls is considerably lighter than that of the lead masses and the slab of the specimen, there is almost no affection if the weight of shear walls is neglected in a dynamic analysis. In this study, only the weight of the lead and the slab is considered as concentrated masses in the nodes of the top slab, so that the reduction of freedoms must be done in a dynamic analysis.

In order to determine the circular frequencies ω and modes $\{X\}$ of free vibrations of a structure, it is necessary to solve the linear eigenproblem as follows

$$[K]\{X\} = \omega^2[M]\{X\} \tag{3}$$

where, $[K]$ is the elastic stiffness matrix of a structure. $[M]$ is the mass matrix of a structure.

The $[K]$ and $[M]$ can be rearranged as

$$\begin{pmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{pmatrix} \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = \omega^2 \begin{pmatrix} M_1 & 0 \\ 0 & 0 \end{pmatrix} \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} \tag{4}$$

where
$\{X_1\}$ is the displacement vector with respect to the degrees of freedom with concentrated masses.
$\{X_2\}$ is the displacement vector with respect to the degrees of freedom without concentrated masses.

According to above equations, the linear eigenproblem equation of a structure can be obtained as

$$[K'] = [K_{11}] - [K_{12}][K_{22}]^{-1}[K_{21}] \tag{5}$$

$$[K']\{X_1\} = \omega^2[M_1]\{X_1\} \tag{6}$$

Numerical Procedure

The dynamic analysis of structures is based on the equations of motion of the systems,

$$[M]\{\ddot{X}\} + [C]\{\dot{X}\} + [K]\{X\} = \{F\} \tag{7}$$

in which, $[M]$: mass matrix. $[C]$: viscous damping matrix. $[K]$: secant stiffness matrix. $\{\ddot{X}\}$: acceleration vector. $\{\dot{X}\}$: velocity vector. $\{X\}$: displacement vector. $\{F\}$: external force vector.

The Impulse Acceleration Method (*i. e.* Newmark-β method in case of $\beta = 0$) is employed here to solve the nonlinear dynamic equations and obtain the responses of structures. The basic idea of this method is to predict the displacements at next step $(n+1)$ from the displacements at former two steps n and $(n-1)$, and restoring forces at step n by equations of equilibrium directly. The basic assumptions of Impulse Acceleration Method can be summarized as follows.
(1) The velocity is assumed constant within a time interval Δt.
(2) The displacement is assumed changing linearly within a time interval Δt.
(3) The acceleration is assumed changing uncontinuously something like an impulse action within a time interval Δt.

Considering the proposed constitutive model and reduction of freedoms in this study, the basic formula to predict the displacements can be derived from the above assumptions as

$$\begin{aligned}\{X_{n+1,1}\} &= ([I] + 0.5[M_1]^{-1}[C_1]\Delta t)^{-1}(2\{X_{n,1}\} - \{X_{n-1,1}\} \\ &\quad +0.5[M_1]^{-1}[C_1]\Delta t\{X_{n-1,1}\} - [M_1]^{-1}\{Q_1(X_n)\}\Delta t^2 \\ &\quad -\{\ddot{X}_{0n} + g\}\Delta t^2)\end{aligned} \tag{8}$$

where
the subscript 1 in $\{X_{n+1,1}\}$ means the parameters with respect to the degrees of freedom with concentrated masses. $\{\ddot{X}_{0n}\}$ is the acceleration vector of ground motion which acted as external loads $\{F_n\} = -[M]\{\ddot{X}_{0n}\}$. $\{g\}$ is the acceleration vector of gravity. $\{Q_1(X_n)\}$ is the restoring force vector of a structure equal to the $[K]\{X\}$ in Eq.7. $[I]$ is a unit matrix.

The advantages of using the Impulse Acceleration Method in the form given in Eq.8 are as follows. Since the displacements are calculated from equations of equilibrium directly at every incremental step, no iterations need to be performed and furthermore, it can restrain accumulated errors. Another merit of this approach is that the negative tangential modulus in the downword sloping portion of stress-strain curve as tension stiffening part in tension and softening part in compression shown in Fig.2 can be dealt with smoothly because no tangential stiffness matrix appears in Eq.8. The method is simple and effective for highly nonlinear problems.

The velocity and acceleration can be calculated by following equations respectively.

$$\{\dot{X}_{n,1}\} = 0.5(\{X_{n+1,1}\} - \{X_{n-1,1}\})/\Delta t \tag{9}$$

$$\{\ddot{X}_{n,1}\} = (\{X_{n+1,1}\} - 2\{X_{n,1}\} + \{X_{n-1,1}\})/\Delta t^2 \tag{10}$$

The incremental displacements with respect to the degrees of freedom without concentrated masses are calculated by the assumption shown in Eq.11.

$$\{\Delta X_{n+1,2}\} = -[K_{22}]^{-1}[K_{21}]\{\Delta X_{n+1,1}\} \tag{11}$$

where the subscript 2 in $\{X_{n+1,2}\}$ means the parameters with respect to the degrees of freedom without concentrated masses, and $[K_{22}]$ and $[K_{21}]$ are tangential stiffness matrices.

The total displacements with respect to the degrees of freedom without concentrated masses can be calculated by the following equation.

$$\{X_{n+1,2}\} = \{X_{n,2}\} - [K_{22}]^{-1}[K_{21}]\{\Delta X_{n+1,1}\} \tag{12}$$

The equivalent viscous damping is assumed by the Eq.13.

$$[C_1] = 2h_i[K'_{si}]/\omega_i = 2h_i\sqrt{\beta_i}[K']_e/\omega_e \tag{13}$$

where
i is the $i-$th vibration step. h_i is the coefficient of viscous damping evaluated from test data. $[K'_{si}]$ is the secant stiffness matrix of a structure. ω_i is the corresponding circular frequency of a structure in nonlinear procedure. β_i is the corresponding degradation ratio between secant stiffness and elastic stiffness of a structure in nonlinear procedure which can be evaluated from test data or from a static nonlinear analysis of a structure. ω_e is the elastic circular frequency of a structure. $[K']_e$ is the elastic stiffness matrix of a structure which can be calculated by the following equation.

$$[K']_e = [K_{11}]_e - [K_{12}]_e[K_{22}]_e^{-1}[K_{21}]_e \tag{14}$$

NUMERICAL EXAMPLE

"Model Dynamic Response Tests for Evaluation of the Seismic Behavior of Reactor Building" was carried out using a large-scale, high-performance shaking table for H-shaped RC shear walls. From these tests, many data, useful for verification of seismic response analysis codes, were obtained and their dynamic response behaviors were studied (Nagashima, Shibata and Akino *et al.*, 1995). The dynamic analysis of this test by the above proposed method was done and its comparisons with test results were presented as following figures.

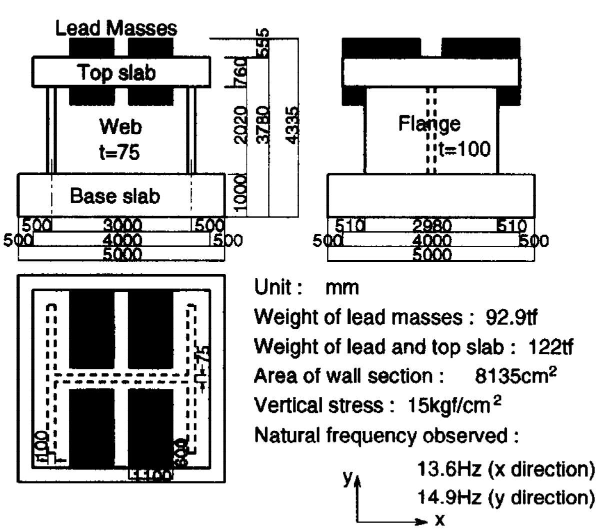
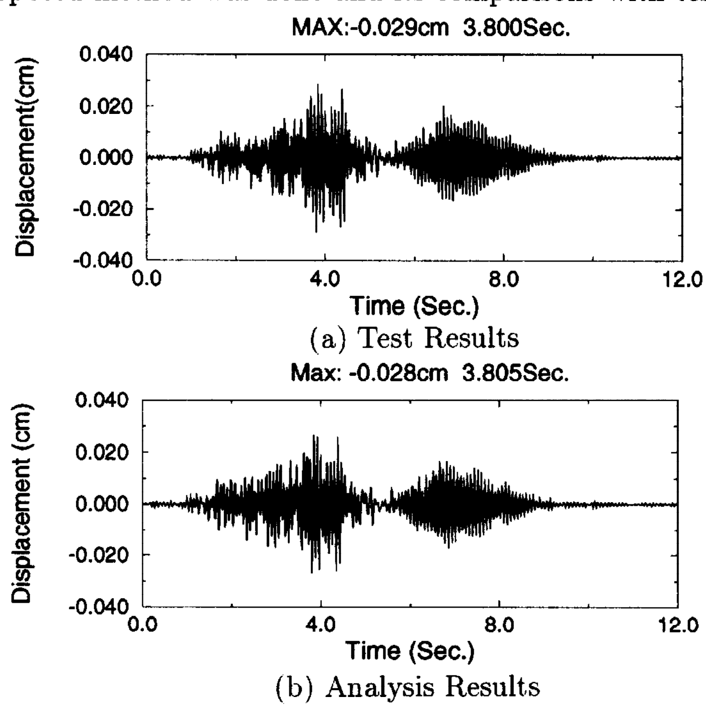


Fig. 3 Test Specimen



(a) Test Results

(b) Analysis Results

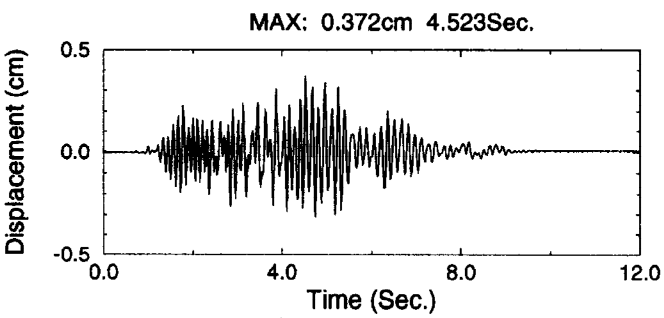
Fig. 4 Time History of Lateral Displacement Response in RUN1

The test specimen was shown in Fig. 3, and the parameters of specimen were described in Tab. 1 where ρ_x and ρ_y are the reinforcement ratios in lateral and vertical direction respectively, ε_b is the compressive strain at maximum compressive stress of concrete, f_c' is the compressive strength of concrete, f_{cr} is the tensile strength of concrete which is evaluated by $\sqrt{f_c' kgf/cm^2}$ (Vecchio and Collins, 1986), f_y is the yield strength of steel, E_c and E_s are the Young's moduli of concrete and steel respectively. In the test, the specimen was subjected to excitations in the X-direction. The vibration test steps were set corresponding to a series of five target response levels which were sequentially performed as RUN1 to RUN5. For each step, the same artificial wave was inputted and its amplitude was enlarged gradually with consideration given to clarifying the response properties of the specimen ranging from the elastic to the ultimate state. In RUN2 of the test step, the observed response level was smaller than that of the target response. For this reason, an additional RUN2' was carried out, so this test had six vibration steps. The vibration procedure observed from the test was presented in Tab. 2. Because the specimen was only subjected to excitations in the X-direction, the top slab was modeled by using rigid plane elements in which half of the thickness of the top slab was chosen as the height of those elements. The ratio of stiffness between the top slab and walls was assumed to be 1000. Concentrate masses were arranged on five nodes at the rigid elements symmetrically according to the condition of the total weight $122 tonf$ and the inertia moment $1.259 \times 10^6 tonf \cdot cm^2$ of the specimen.

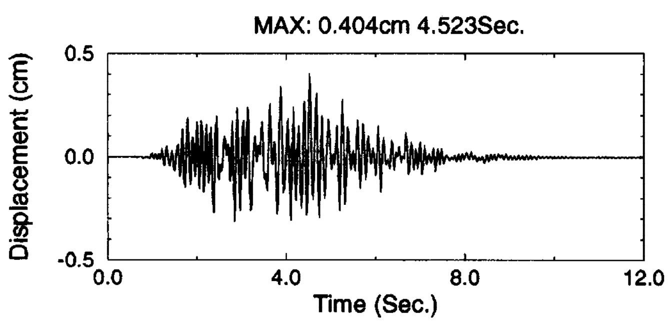
Fig. 4-5 show the time history of displacements at the middle point of the top slab in RUN1 and RUN4 respectively. Fig. 6 shows the time history of acceleration at the same point in RUN5 which only gives the stable results till the vicinity near failure of the specimen. Fig. 7 gives the comparison

Tab. 1 Parameters of Specimen

ρ_x ρ_y (%)	ε_b	fc' (kgf/cm²)	fcr (kgf/cm²)	fy (kgf/cm²)	Ec (kgf/cm²)	Es (kgf/cm²)
1.2	-0.0025	292	17.1	3.91×10^3	2.34×10^5	1.88×10^6

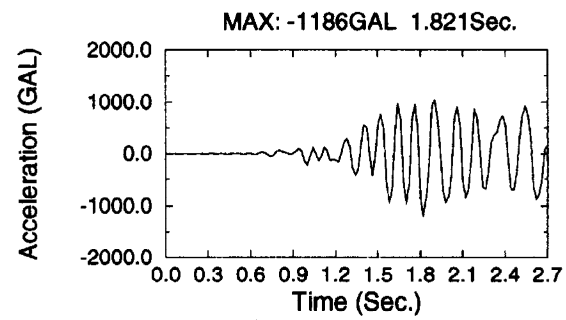


(a) Test Results

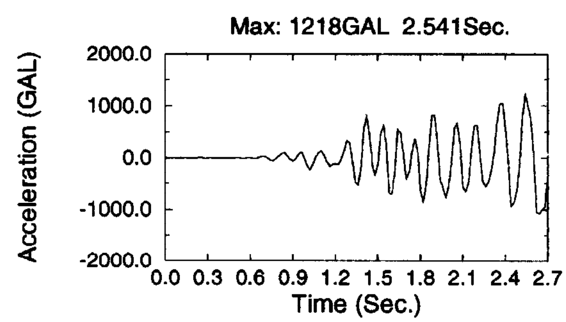


(b) Analysis Results

Fig. 5 Time History of Lateral Displacement Response in RUN4



(a) Test Results



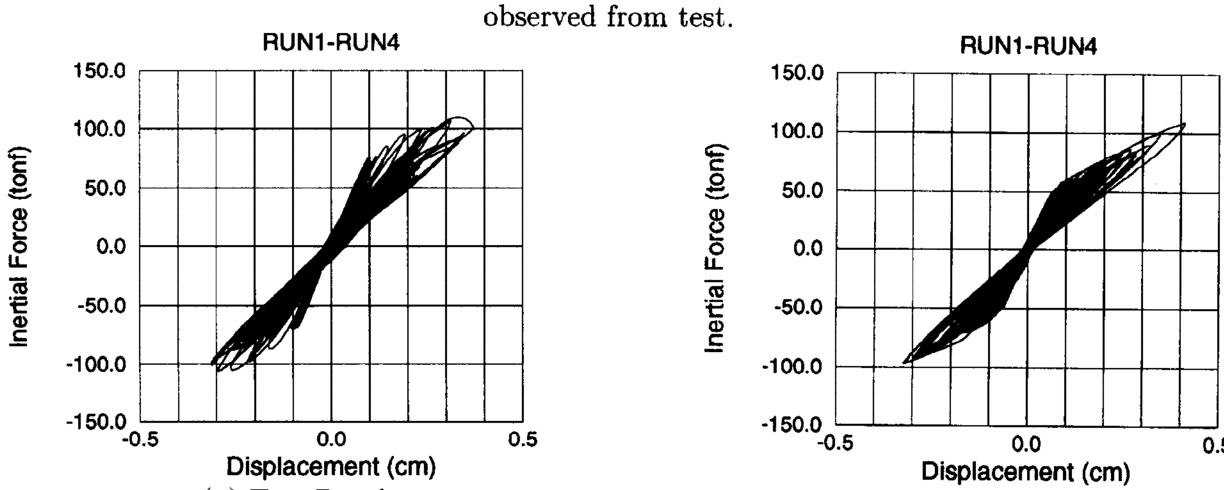
(b) Analysis Results

Fig. 6 Time History of Lateral Acceleration Response in RUN5 of inertial force-displacement relationship with test results in RUN1 to RUN4. The analytical results simulated well the overall behavior of the test ones without estimating the lower cracking loads. Fig. 8-9 present the acceleration response spectra in RUN1 and RUN4. It is obvious that the periods and the amplitudes at peaks of response spectra are coincided well with observed one. The comparisons

Tab. 2 Vibration Procedure

Step	Name of vibration step	Input level observed	Damping* h(%)	Comments
1	RUN1	58GAL	1.1	Small-amplitude level in elastic range
2	RUN2	122GAL	1.1	Strain level at shear crack initiation
2'	RUN2'	317GAL	2.5	Strain level at shear crack initiation
3	RUN3	361GAL	3.0	Strain level at 3 times the above step 2 value
4	RUN4	583GAL	4.0	Shear deformation angle about 2/1000 rad.
5	RUN5	1256GAL	4.0	Vicinity of ultimate strength

* The values of h were used in the analysis as equivalent viscous damping coefficients which were observed from test.



(a) Test Results (b) Analysis Results

Fig. 7 Relationship between Inertial Force and Lateral Displacement in RUN1 to RUN4

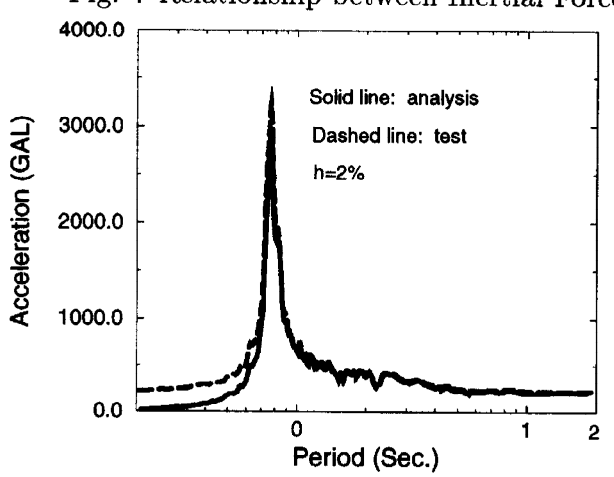


Fig. 8 Spectrum of Acceleration Response in RUN1

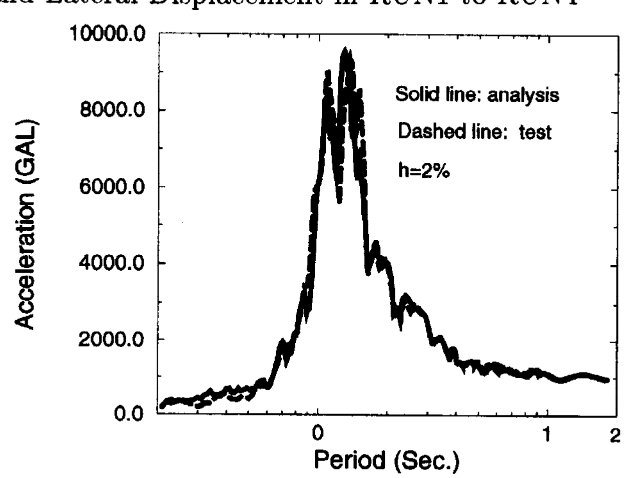


Fig. 9 Spectrum of Acceleration Response in RUN4

with experimental results demonstrated that the analytical results by the simple model proposed herein can give a quite good simulation of the test under dynamic loading up to the shear deformation angle of about 2/1000. But further studies are needed to simulate the final failure behavior well.

CONCLUSIONS

A dynamic nonlinear model by finite element method is proposed to analyse 3-D RC shear wall structures subjected to earthquake motions. The analytical results of a H-shaped shear wall are compared with the observed ones in the time history of displacement and acceleration and inertial force-displacement relationship. Furthermore the response spectra are investigated. The comparison shows that this approach had good accuracy up to the range before the final failure of the structure. In conclusion the model proposed herein is effective for dynamic nonlinear analysis of 3-D RC shear wall structures subjected to earthquake motions. It is applicable to not only reactor buildings but also other structures such as 3-D shear walls with L-shaped section or T-shaped section and the shear core walls of buildings.

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