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SEISMIC RESISTANCE OF A CLASS OF SLAB-ON-GIRDER STEEL HIGHWAY BRIDGES

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ABSTRACT

The seismic performance of a class of existing slab-on-girder steel highway bridges is studied, first considering that damage to bearings is unacceptable, then by releasing this constraint. When anchor bolts are severed, and friction forces between the disconnected bridge components are exceeded, the bridge superstructure may slide. Linear elastic and non-linear inelastic analyses are conducted to respectively investigate force and sliding demands on bearings. Quantitative results are provided for single span bridges, and continuous bridges having steel columns.

KEYWORDS

Steel bridges; seismic evaluation; bearings; steel columns; highway bridges; friction; sliding bearings; quantitative evaluation; simply supported bridges; continuous bridges.

INTRODUCTION

Extensive research has already been conducted to identify the structural components and parameters which significantly affect the seismic response of reinforced concrete bridges. By contrast, little information is found on the seismic behavior of steel bridges in the existing literature, partly because few have been exposed to severe earthquakes in the past, at least until the Loma Prieta earthquake of 1989. Some research findings exist on the seismic performance of steel box-sections often used in Japan for bridge piers (e.g. Macrae et al., 1990) and the seismic resistance of some types of long span steel bridges (e.g. Astaneh 1993), but the seismic performance of many other types of steel bridges has received little attention to date. As most steel bridges in the North American inventory have been designed without any seismic resistance considerations, their seismic adequacy is legitimately questionable. This is further justified in light of the extensive damage suffered by steel bridges during the recent Kobe earthquake (Bruneau et al. 1996). Obviously, some of the general research on the behavior of generic bridges (e.g. Tseng and Penzien, 1973; Kawashima and Penzien, 1976; Douglas, 1979; Imbsen and Penzien, 1984) is broadly applicable to any bridge type and can be useful, but much research is still needed to reliably assess the seismic resistance of steel bridges deficiently or never designed to resist earthquakes.

Upon first examination, in slab-on-girder steel bridges commonly used throughout the North American highway system, the bearings and columns appear to be the weakest links of structural resistance to the seismic loads.

In this paper, the seismic performance of this class of highway bridges is studied, first considering that damage to bearings is unacceptable, then by releasing this constraint. While in both cases, damage to columns is deemed not acceptable, the latter case is of particular interest since there exists many short to medium span old steel bridges supported by sliding-bearings whose anchor bolts were not designed to resist seismic forces and which are vulnerable to earthquakes. When these anchor bolts are severed, and friction forces between the disconnected bridge components are exceeded, the bridge deck may slide to a point where insufficient bearing resistance (or brittle detailing) exists to resist the suddenly abnormally located gravity reaction forces, or may even fall off its support. Due to excessive sliding displacements, columns in continuous bridges may also suffer damage before the bridge deck falls off its support.

Therefore, for the purpose of this study, both linear elastic and non-linear inelastic dynamic analyses are performed. The objective in performing linear elastic analyses is to find the seismically-induced bearings' forces and elastic column moments. The non-linear inelastic analyses are performed (i) to find the maximum transverse sliding displacement at the bearings as a function of various parameters, considering that the bridge deck may slide in both directions and collide with the abutment walls or fall off its support, and (ii) to determine the effect of sliding at the bearings on the seismic capacity of continuous bridges considering the capacity of the columns.

PROPERTIES OF THE BRIDGES STUDIED

A large percentage of the existing slab-on-girder steel highway bridges were constructed in the 1960's. Accordingly, to study typical old steel highway bridges and better understand their expected seismic performance, 2 and 3-lane single span simply supported bridges with spans of 20, 30, 40, 50, 60 metres, and continuous bridges with two spans of 20, 30, 40, 50, 60 metres each, are designed in compliance with the 1961 edition of the American Association of State Highways Officials (AASHO) code (American..., 1961). The 2 and 3-lane bridges have respectively 8 and 12 metres widths and girders spaced at 2 metres intervals. The deck of these bridges is generally supported by fixed bearings on one abutment and by expansion bearings on the other abutment. The deck of continuous bridges is also supported by steel columns of 6 metres height as typically found in most highway bridges, rigidly connected to each steel girder at midspan to make a moment resisting steel frame, simply connected at their base. These columns are oriented such that strong-axis bending is mobilized by movement in the longitudinal direction of the bridges, and bents are unbraced transversely. Slab thickness is taken as 200 mm.

All the bridges are assumed to have sliding-bearings, but elastomeric bearings are also considered in the case of single span bridges. Longitudinal stiffness is calculated to vary between 400,000 kN/m and 800,000 kN/m for span ranges considered in this research, but the theoretical cases of bearings with zero and infinite rotational stiffness are also considered in the case of simply supported bridges. Only sliding bearings with 800,000 kN/m stiffness are considered in the case of continuous bridges. Infinite rotational stiffness about an axis perpendicular to the deck occurs when the fixed bearing is infinitely rigid in the longitudinal direction. Zero rotational stiffness occurs when the fixed bearing has negligible longitudinal stiffness (a purely theoretical case) or when the bearings are damaged. In both cases transverse stiffness is assumed to be infinitely rigid. Also, for simply supported bridges, a 4,000 kN/m stiffness per elastomeric bearing is assumed for the span ranges considered (Heins and Firmage, 1979).

EARTHQUAKE LOADING

Ground motions can be characterized by the peak ground acceleration to peak ground velocity ratio, A_p/V_p, where A_p is expressed in units of the gravitational acceleration and V_p is expressed in metres per second. Ground motions with higher frequency content have higher A_p/V_p ratios. In Eastern North America, generally, $A_p \geq V_p$, and in Western North America, $A_p \leq V_p$. As the A_p/V_p ratio gets larger, higher spectral acceleration values are obtained in the long period region of the spectrum while spectral acceleration values almost remain the same in the short period region for all A_p/V_p ratios.

For the elastic analyses, the Eastern Canada design spectrum with $Z_a = Z_v$ is taken as representative of Eastern North America seismicity and as a conservative envelope of all cases for which $Z_a \geq Z_v$, where Z_a and Z_v are respectively the peak acceleration and velocity zone parameters (Associate..., 1990) and directly related to A_p and V_p. Similarly, for Western North America, the Western Canada design spectrum with $Z_a < Z_v$ is taken as a conservative envelope to study the response of steel bridges.

For the nonlinear time history analyses, four Western USA earthquakes all recorded on rock or stiff soil are chosen as representative of the strong motion data in Western North America. Two Eastern Canada earthquake records, one on bedrock and another on alluvium deposit are also considered. The smoothed acceleration spectrum which matches the mean plus one standard deviation (MP1SD) of the spectra of the Western USA earthquakes for 5 percent damping was generated and found to be in close agreement with the Uniform Building Code (UBC) spectrum (International..., 1991).

MODELING

For linear elastic analyses, the stiffness of the fixed type of sliding-bearings in the longitudinal direction is formulated considering only the contributions from bending of the bearing bar and elongation of the anchor bolts (Imbsen and Penzien, 1984). Knowing that the bearing bar and the anchor bolts are acting as springs connected in series, the longitudinal spring coefficient, k_{bL}, for each bearing is obtained as;

$$k_{bL} = \frac{1}{\dfrac{1}{\dfrac{3\,E\,I_{bb}}{h_{bb}^{3}}} + \dfrac{1}{\dfrac{E}{h_{bb}} \displaystyle\sum_{i=1}^{n_{ab}} \left(\dfrac{l_p\, A_{ab}}{l_{ab}} \right)_i}} \tag{1}$$

where, E is the modulus of elasticity of steel, I_{bb} is the moment of inertia of the cross-section of the bearing bar parallel to the deck about an axis in the bridge's transverse direction, h_{bb} is the height of the bearing bar, l_p is the length between an anchor bolt in tension and tip of the bottom plate, A_{ab} and l_{ab} are respectively the area and the length of the anchor bolt and n_{ab} is the number of anchor bolts. By using this spring coefficient for each bearing located under each girder, the longitudinal effect of the bearings-set is transformed into one translational spring parallel to the span and one rotational spring about a vertical axis perpendicular to the bridge deck. Both springs are located at the centre-line of the bridge deck and at the one end of the bridge where fixed bearings are present. In the transverse direction sliding-bearings are very rigid, hence, the bearing stiffness, k_{bT}, in this direction need not be calculated.

The bridge span is divided into 3-D beam segments and full composite action between the slab and girders is assumed for the response in both orthogonal directions. For the continuous bridges, a rigid bar connected to the beam element's end node at the columns' location and oriented in the transverse direction, is used to model the interaction between the columns and the bridge deck. The program SAP90 (Wilson and Habibullah, 1990) is used for the response spectrum analyses of the elastic models.

For non-linear inelastic analyses, considered when bearing forces calculated as per the above elastic analyses exceed the bearings' resistance, the sliding of the bridge deck in the transverse direction is investigated using the program NEABS (Penzien et al., 1981). Bearings are assumed to be immediately damaged and their initial elastic contribution to response is conservatively ignored. Thus, a rigid-plastic hysteresis model is assumed to simulate sliding of the bridge since the peak of ultimate bearing resistance prior to anchor bolt failure occurs only once and does not affect significantly the total amount of energy dissipated through sliding. This is a reasonable simplification considering that results obtained from inelastic analyses would only be consulted if the seismically-induced forces in bearings calculated using elastic analyses were found to exceed their resistance. Mass proportional damping and five percent damping are considered. Due to space constraints, other modeling details are presented elsewhere (Dicleli and Bruneau 1995a). Moreover, for the same reason, only results due to seismic excitation in the direction perpendicular to the bridge axis are presented here.

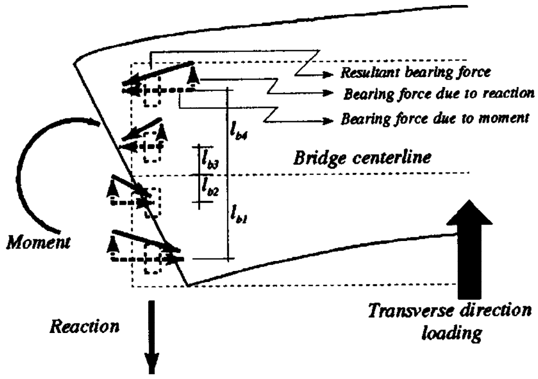


Figure 1: Bearing forces due to loading in the transverse direction

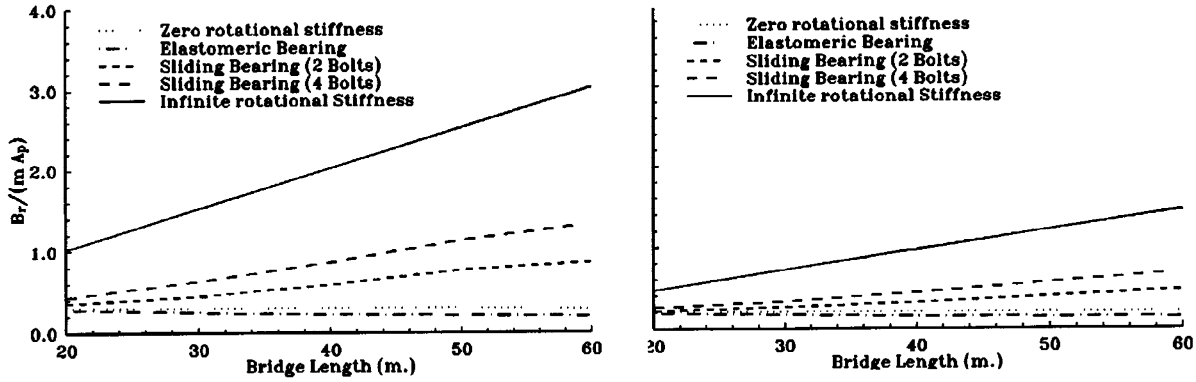


Figure 2: Effect of bearing's stiffness on transverse bearing force: 2-Lane simply supported bridges on the left, 3-lane simply supported bridges on the right.

LINEAR ELASTIC ANALYSES RESULTS - TRANSVERSE DIRECTION

Bearing Forces

Response spectrum analyses of 2 and 3-lane simply supported and continuous bridges are conducted in the transverse direction. As illustrated in Figure 1, the forces, B_r, in fixed type of sliding-bearings are calculated first by dividing the transverse reaction force at the abutment by the number of bearings, then, summing vectorially this force and the longitudinal force produced by the resistance to rotation at the bearings. As seen in that figure, bearings farthest to the bridge centre-line attract larger seismic forces, and are therefore the most critical ones. Obviously, the expansion type of sliding-bearings does not attract as much forces as the fixed type since the deck can freely rotate about a vertical axis. Therefore, the bearing forces in that case are calculated simply by dividing the reaction force on the abutment by the number of bearings.

The effect of end fixity on the transverse bearing forces is illustrated in Figure 2 for 2- and 3-lane simply supported bridges respectively. In these figures, the maximum transverse bearing force coefficient (TBFC) is plotted as a function of span length for various bearing stiffnesses. TBFC is obtained by dividing the maximum of the resultant bearing forces due to seismic loading in transverse direction, by the bridge mass and the peak ground acceleration. Eastern Canada design spectrum is arbitrarily chosen for the analyses. It is observed that as the stiffness of the bearings increases, the TBFC becomes larger. For all types of bearings, except for elastomeric and those with zero rotational stiffness, the TBFC increases with span length. This is mostly due to the corresponding increase of the in-plane end moment at the support.

In the case of bearings with zero or very small longitudinal, but infinite transverse stiffness, the TBFC is almost constant but absolute bearing forces actually increase with span length largely on account of the increased reactive mass of longer bridges. In the case of bridges with elastomeric bearings, due to their much longer

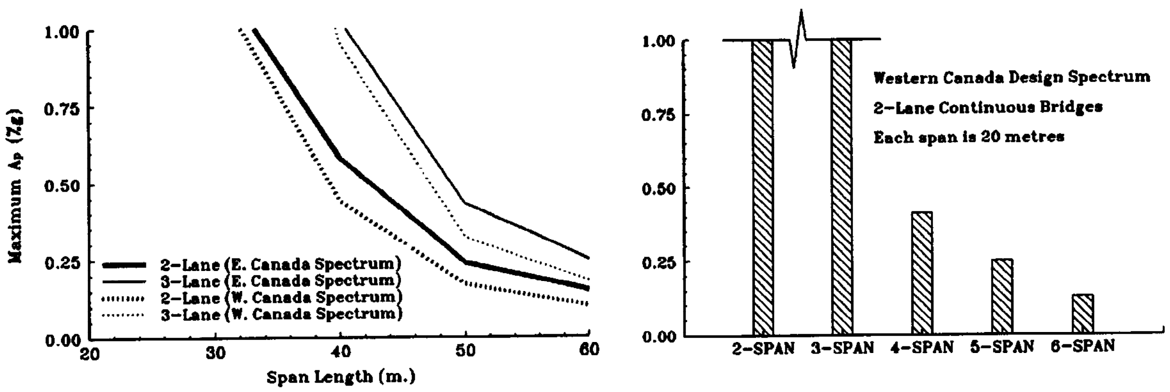


Figure 3: Maximum resistible peak ground acceleration as a function of span (left), and effect of number of spans on seismic capacity (right), for linear elastic analysis (undamaged bearings).

periods that fall in the declining part of the response spectra, the structures attract relatively less seismic forces and therefore very small bearing forces are obtained.

For the continuous bridges, results similar to those for simply supported bridges are observed for the bearing forces. However, in this case, the difference between the absolute bearing forces of 2 and 3-lane continuous bridges is not very large and is about 30% for spans up to 30 metres and 10% for longer spans. As an overall trend, transverse bearing forces are proportional to the mass of the bridge, hence to the span length, and are larger in 2-lane bridges.

The effect of deck width and girder spacing on the above results was also investigated using 2-lane bridges having various combinations of deck width and girder spacing. Analyses results indicate that the bearing forces of bridges with decks wider than 4 metres per lane are smaller, but the difference is not very large. Furthermore, narrower bridges can have larger bearing forces, but a difference of no more than 12% was obtained for the worst case considered. Finally, note that collision of the deck with the abutment wall as a result of displacements at the corners of the bridge decks due to support rotations resulting from transverse direction seismic excitation were calculated to be unlikely.

Column Response For Continuous Bridges

Most steel columns in old steel bridges were not designed to absorb energy through cyclic inelastic deformations and therefore are often non-compact sections. Also, for the type of bridges considered here, columns are not braced laterally, and depending on their slenderness, lateral torsional buckling is possible under strong axis bending. Considering these factors and the lack of information about the behavior of steel bridge columns, they are conservatively assumed to fail as soon as the capacity delimited by statically derived interaction curve is reached. The stability interaction equations proposed by Duan and Chen (1989) are used for this purpose, and second-order effects were considered.

Using the maximum weak axis column moment considering stability and the magnified seismic moments as a function of A_p, the maximum resistible peak ground acceleration (MRPGA) prior to column failure is calculated for the ranges of spans considered. The results are plotted for 2 and 3-lane bridges as a function of span length in Figure 3. As seen in that figure, the MRPGA is larger for 3-lane continuous bridges. It is noteworthy that as the number of lanes increases the bridge deck gets wider and stiffer in the transverse direction. Accordingly, the calculated displacements and forces in columns are much smaller in wider bridges, and therefore they can resist larger seismic forces as seen in Figure 3. On the contrary, increasing span length has a negative impact on the seismic capacity. The lateral displacement of the deck, which is proportional to the fourth power of the span length, is much larger in longer bridges. Therefore, the calculated first and second order forces in columns are larger in longer bridges, which, as a result, can only resist smaller seismic forces, as seen in Figure 3. It noteworthy that 2-lane bridges up to 40 metres and 3-lane bridges up to 50 metres can survive rather strong

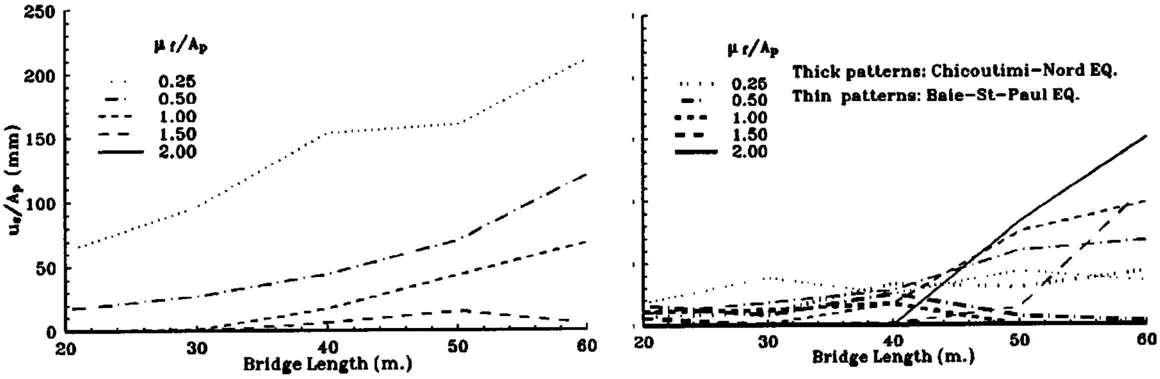


Figure 4: Transverse sliding displacement per peak ground acceleration (%g) for various friction coefficient to peak ground acceleration ratios for 2-lane bridge for average of western North America earthquakes (left), and selected eastern North American earthquake records (right).

earthquakes of up to 0.4g peak ground acceleration, without any damage to columns provided that bearings are not ruptured at the abutments.

Response spectrum analyses of two lane continuous bridges of 3, 4, 5 and 6 spans of each 20 metres in length are conducted using the Western Canada design spectrum. In Figure 3 the MRPGA is also plotted as a function of number of spans. It is observed that, as the number of spans increases, the seismic capacity rapidly decreases. This is mainly due to the increased mass and flexibility of the structure which subsequently results in higher displacements at the column locations for a given peak ground acceleration.

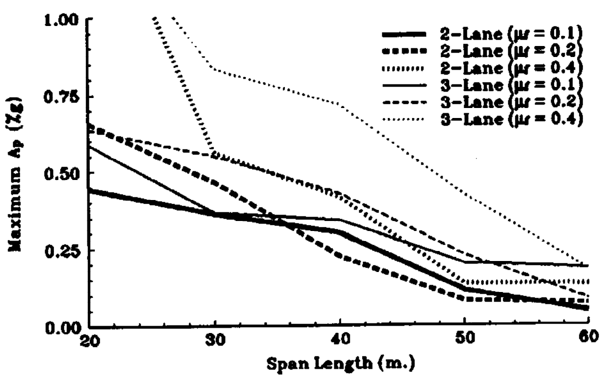
NON-LINEAR TIME HISTORY ANALYSES

Sliding of Single Span Bridges in the Transverse Direction

Using the program NEABS, the transverse sliding displacements at the support of the previously described 2- and 3-lane simply supported bridges is obtained as a function of span length for various friction coefficient to peak ground acceleration ratios, μ_f/A_p (A_p is expressed as a percentage of g), using four Western USA and two Eastern Canada earthquakes. Results are shown in Figure 4. In these figures, the vertical axis is the support sliding displacement, in mm per unit peak ground acceleration, and the horizontal axis is the span length. It is found that, for the same μ_f/A_p ratio, the sliding displacement is linearly proportional to the amplitude of the peak ground acceleration. This dependency can be explained by an energy formulation of the sliding problem (Dicleli and Bruneau, 1995b). As seen in Figure 4, for Western North America earthquakes, the sliding displacement increases with increasing span length and decreasing μ_f/A_p ratio. Obviously, larger forces are applied on the bridges with longer span. This subsequently results in a higher amount of energy to be dissipated by friction and consequently larger sliding displacements. It was also found that, for the range of spans considered, the attracted seismic forces per unit mass are smaller for wider bridges than those for narrower bridges of the same length and therefore their sliding displacements are smaller (Dicleli and Bruneau 1995a).

As for Eastern North America type of earthquakes, almost the same trend as for Western USA earthquakes is observed in the case of Baie-St-Paul earthquake. However, in the case of Chicoutimi-Nord earthquake, sliding displacements start decreasing for bridges of 50 metres span and longer ($T_1 > 0.20$ sec.). This is logical according to the earthquake spectrum of this record which shows, for structures with fundamental periods longer than 0.2 second, a decrease in the force applied on the structure, and hence a decrease in sliding displacement.

For the same bridges, the average of sliding displacements obtained using Western North America earthquakes are larger than those obtained using Eastern North America ones. Eastern North America earthquakes, which have very high A_p/V_p ratios, contain very high frequency pulses. Therefore, once the structure slides, this sliding motion cannot be sustained for a long time since the force applied on the system remains above the threshold



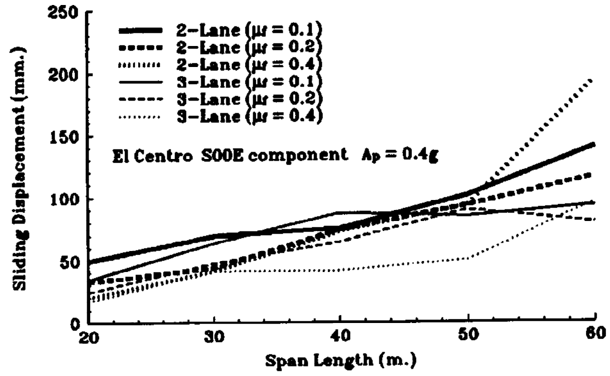


Figure 5: Maximum resistible peak ground acceleration as function of span length, for average of Western U.S. earthquakes, non-linear analyses, and damaged bearings.

Figure 6: Sliding displacement at support of 2- and 3-lane bridges (non-linear inelastic analysis, damaged bearings, for El Centro S00E scale to 0.4g PGA).

of friction only for a short duration. Western North America earthquakes, which have intermediate A_p/V_p ratios, contain frequency pulses longer than those of Eastern North America earthquakes. Thus, the sliding motion can be sustained for a longer time. Consequently, ground motions with high A_p/V_p ratio generally produce smaller sliding displacements than those with relatively lower A_p/V_p ratios.

Sliding of Continuous Bridges in the Transverse Direction

Assuming that the anchor bolts in sliding-bearings are severed, inelastic dynamic analyses of the continuous bridges are conducted to investigate the effect of sliding on the seismic capacity of the structure. More than 400 analyses were performed using the program NEABS to establish the relationship between the MRPGA and span length for various friction coefficients. This relationship is determined using the average of results obtained from the 4 Western USA earthquakes and considering the stability of columns in the transverse direction. The results are presented in Figure 5. As seen in that figure, the MRPGA's are larger for 3-lane bridges than those for 2-lane bridges. Also, bridges with higher friction coefficient at the bearings, can obviously sustain bigger earthquakes and have a larger MRPGA. In fact, based on the results obtained, should a friction coefficient of 0.4 exist at the supports, 3-lane continuous bridges of spans shorter than 40 metres and 2-lane continuous bridges of spans shorter than 30 metres may survive high intensity earthquakes even if their bearings are damaged, provided that during the earthquake other structural elements preserve their integrity.

Neglecting column instability, sliding displacements of 2 and 3-lane continuous bridges are plotted in Figure 6 as a function of span length for various friction coefficients, using the S00E component of 1940 El Centro earthquake scaled to 0.4g. These sliding displacements are nearly twice those of simply supported bridges of identical total length. This behavior is attributable to lower gravity forces acting on the abutment bearings in the continuous bridges which provide a smaller frictional resistance, and thus larger sliding displacements.

Effect of Number of Spans on the Transverse Sliding Displacement

As the number of spans increases the proportion of the total gravity load transferred to the abutments decreases. Additional analyses revealed that this results in larger sliding displacements compared to those of the bridges of equivalent total length but smaller number of spans. These large sliding displacements may reduce the seismic capacity of the structure since the columns may not be capable to accommodate that much displacement.

CONCLUSIONS

Within the realm of elastic response, bearing forces due to seismic excitation in the transverse direction are proportional to the mass of the bridge, hence, to the span length. Also, their magnitude decreases as the number of lanes increases. Bearings with higher longitudinal stiffness and closer to the edge of the bridge deck attract forces larger than those in other bearings. However, bearings with negligible longitudinal stiffness, attract almost equal forces regardless of the number of lanes of bridges. If bearings in continuous bridges are not damaged, 2-lane bridges up to 40 metres and 3-lane bridges up to 50 metres span are found capable to survive earthquakes with 0.4g peak ground acceleration without any damage to columns.

As the number of spans of continuous bridges increases, seismic capacity decreases. This can be explained by the relative insignificant contribution of steel columns to resist loads transversely applied to the bridge's deck; the resistance of a column is therefore dictated by its displacement compatibility with the deck, and consequently by its longitudinal location along the bridge.

When bearings are stable and can slide freely once their anchors are ruptured by earthquakes, damage to the bearings may be acceptable. For the same friction coefficient to peak ground acceleration ratio, μ_f/A_p, the sliding displacement of a structural system is linearly proportional to the peak ground acceleration. The sliding displacement increases with increasing span length and decreasing μ_f/A_p ratio, but decreases as the bridge gets wider. Ground motions with high frequency content, or high A_p/V_p ratio, may produce smaller sliding displacements than ground motions with relatively lower A_p/V_p ratios. Hence, steel bridges located in Eastern North America will be subjected to smaller sliding displacements than their identical counterparts in Western North America. Nevertheless, the displacements are not considerable for the earthquakes and range of spans of simply supported bridges considered.

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