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A SIMULATION TECHNIQUE BASED ON SPATIAL INTERPOLATION OF GREEN'S FUNCTIONS

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ABSTRACT

High frequency numerical strong ground motion prediction for large earthquakes (magnitude > 7) requires extremely heavy calculations. In order to minimize the computing time of the simulation, we propose here a hybrid method using the discrete wavenumber technique (Bouchon, 1981) to derive numerical Green's functions, and using the philosophy of the empirical Green's functions method (Hartzell, 1978; Irikura, 1983). Considering a big fault surface, it is necessary to divide it into a large number of small subfaults if we want to simulate correctly the high frequency radiation, especially in the near field. As it would be very long to calculate the theoretical Green's functions at many points, the method presented here is based on a technique of spatial interpolation of numerical Green's functions for point sources. The theoretical Green's functions are calculated for only a few points on the fault plane, while they are interpolated for all the other points. The source is described as the composite source model proposed by Zeng *et al.* (1994). Using this method it is possible to calculate the theoretical Green's functions for every point of a thin discretization grid on the fault plane, in a very reasonable computer time. It makes it possible to make strong motion simulation even in the near field domain, and in the whole frequency range of interest for engineering seismology (up to about 20Hz).

KEYWORDS

Strong motion simulation; theoretical Green's functions, spatial interpolation of Green's functions, seismic source.

INTRODUCTION

Strong ground motion simulation is now more and more used to infer the type and amplitude of ground shaking that a given area would suffer in case of a big earthquake. For this kind of simulation, it is necessary to account for site and source effects. This means that geological information is to be known such as the local geological structure and the fault geometry.

One technique which is now used quite often is the empirical Green's functions method (Irikura, 1983). Using the recording of a small event, one can simulate a big earthquake from the same fault plane. In practice, only one aftershock record is used to reproduce the main shock.

This technique using a single recording as subevent to simulate the big earthquake is only valid in the case of far field simulations. As soon as the distance between source and receiver is of the same order as the fault dimensions, it is necessary to take into account separately the contributions of the different parts of the fault. But it is very rare to have many good recordings of small events well distributed on the fault plane, especially for future earthquakes. Frankel (1995) gives examples of simulations using only one aftershock, considering a site situated just above the fault plane. It appears that the use of different aftershocks produces strong motions that are quite different in amplitude particularly.

Another way to account for the contribution of many points on the fault plane is to calculate the theoretical Green's functions for a regular grid of point sources on the fault plane. But this technique basically faces two problems: detailed enough knowledge of the geological structure, and an awfully long computing time. This time will mainly depend on the maximum frequency that is needed for the simulation. But considering the aim of such a simulation, for engineering point of view, it is useful to calculate the simulated motion up to a frequency of 20 Hz. This makes it necessary to have a very thin discretization of the fault plane. A very simple calculation shows that an interval of about 20m is needed between two points, which leads to 250000 points for a 10km*10km fault plane. The calculation of the theoretical Green's functions for each points would take a very long time on a small classical computer station. In order to strongly reduce this computing time, we propose here a procedure of spatial interpolation of Green's functions on the fault plane, coupled with a composite source model described by Zeng *et al.* (1994) for the spatial distribution of subevents on the fault plane. The computer time is then reduced to a reasonable time of a few hours. We describe here the entire procedure used to obtain a simulation of strong motion valid in the near field domain, and up to a frequency of 20Hz, in order to be used for engineering purposes; and a few preliminary examples are presented.

SOURCE MODEL

The source model used here is the composite source model proposed by Zeng *et al.* (1994), which can be described as follows. This model is based on a random distribution of circular subevents on the fault plane. The size and number of subevents follow a power law distribution.

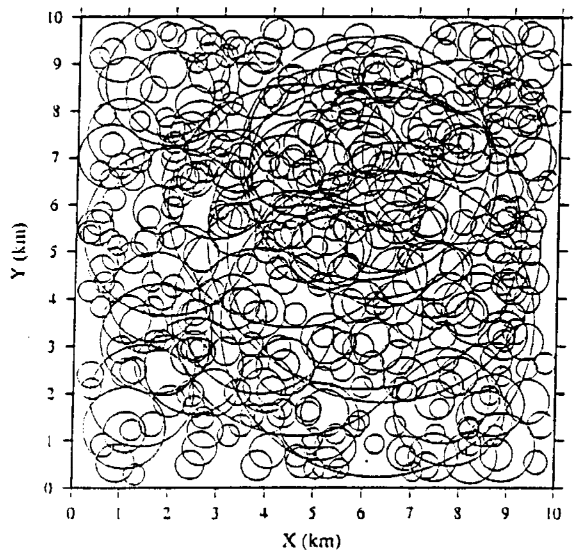

Fig. 1. Distribution of the subevents on the fault plane.

The maximum possible size for the subevents is the largest that fits inside the fault plane. The radius of the smallest subevent is chosen arbitrary and will determine the total number of subevents N_{tot}. The distribution law chosen by Zeng *et al.* (1994) is based on a self similar model proposed by Frankel (1991). The number of circular subevents with radius between R and R+dR is given by:

$$\frac{dN}{d(\ln R)} = p\, R^{-D} \tag{1}$$

where D is the fractal dimension, N the number of subevents and p a proportionality constant given by:

$$p = \frac{7\, M_0}{16\, \Delta\sigma} \frac{3-D}{\left(R_{max}{}^{3-D} - R_{min}{}^{3-D}\right)} \tag{2}$$

with M_0 the total moment; $\Delta\sigma$ the stress drop, which is taken to be independent of the subevent radius. R_{min} and R_{max} are the minimum and maximum radii of the subevents respectively. Frankel (1991) predicted that the high frequency part of the displacement spectrum will be proportional to $\omega^{-(3-D/2)}$, so taking a fractal dimension D=2, we get a ω^{-2} spectrum. Integrating equation (1), we get the number of subevents with radius larger than R:

$$N(R) = \frac{p}{D}\left(R^{-D} - R_{max}^{-D}\right) \tag{3}$$

Then we can deduce the total number of subevents, depending on the size of the smallest subevent:

$$N_{tot} = \frac{p}{D}\left(R_{min}^{-D} - R_{max}^{-D}\right) \tag{4}$$

In the case of our study, the minimum radius for the subevents is set to 200m. For a 10km*10km fault, R_{max}=5000m, and then we get about 450 subevents. Practically we generate N_{tot} real random numbers N comprised between 0 and N_{tot}, to each of which will correspond one subevent of radius R(N) given by the following expression, using equation (3):

$$R(N) = \left(\frac{D\,N}{p} + R_{max}^{-D}\right)^{\frac{-1}{D}} \tag{5}$$

The position of the center of each subevent on the fault plane is also determined randomly. Then overlapping of subevents on the fault plane is allowed. Figure 1 shows the distribution the subevents for one realization for a 10km*10km fault plane.

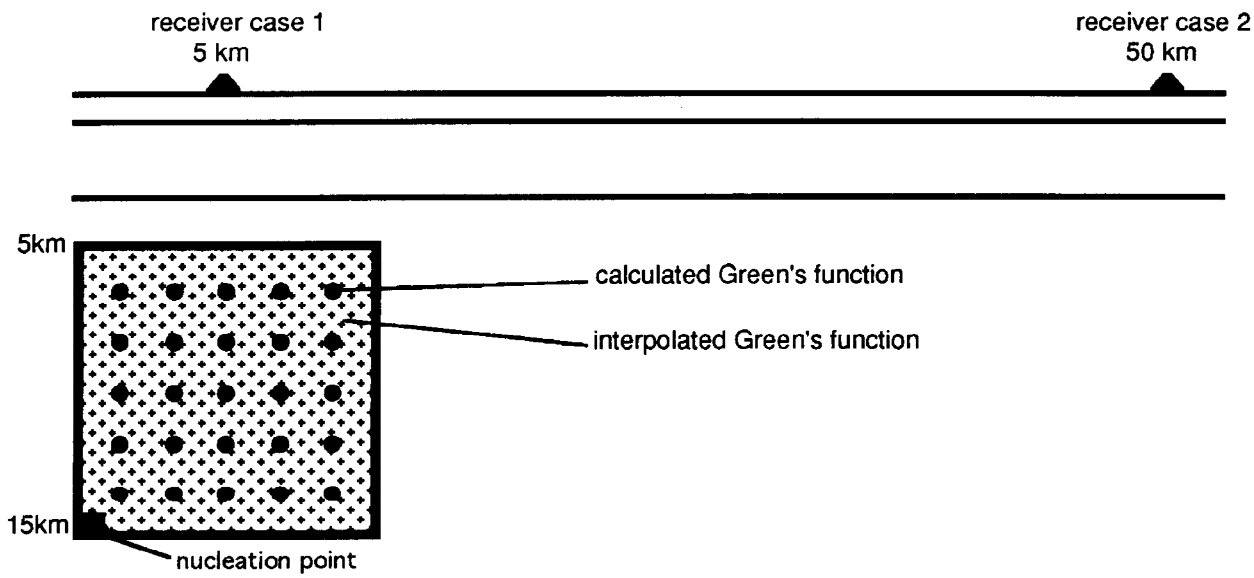

Fig. 2. Geometry of the fault and receivers in the horizontally layered structure

Each subevent on the fault is chosen to be a circular crack as defined by Madariaga (1976) and Papageorgiou and Aki (1983). Then, as given by Papageorgiou and Aki (1983), we calculate the average slip for a circular crack of radius R as:

$$\overline{\Delta u} = \frac{16}{7\,\pi}\,\frac{\Delta\sigma}{\mu}\,R \tag{6}$$

μ being the rigidity.
According to Brune (1970, 1971) and a correction of the original relation given by Madariaga (1976), Papageorgiou and Aki (1983) used the following relation between the corner frequency f_0 and the radius R of the circular crack:

$$2\pi f_0 = \frac{1.32}{R}\,V_s \tag{7}$$

with V_s being the S wave velocity, and assuming a constant rupture velocity $V_r = 0.9\ V_s$.
Then according to Hanks (1979), a estimation of the rupture duration T_0 is given by:

$$T_0 = \frac{1}{f_0} \tag{8}$$

Finally we get the relation between the rupture duration T_0 and the radius R of the circular crack:

$$T_0 = \frac{2\pi R}{1.32\ V_s} \tag{9}$$

Zeng *et al.* (1994) use the composite source model they proposed to make the convolution of the slip function on the fault with a synthetic Green's function. Because of the big number of subevents, they do not compute all the synthetic Green's functions, but they divide the fault into square subfaults and calculate one synthetic Green's function for each square and make the convolution with the equivalent composite source time function. Examples of simulations obtained with this technique are given for the Northridge earthquake by Anderson and Yu (1995) and Zeng and Anderson (1995), and for the Uttarkashi earthquake by Yu *et al.* (1995).
But if we want to produce a simulation of strong motions that takes into account the whole frequency range of interest for engineering purposes, we must have, as said before, a very thin discretization of the fault plane, and take into account the difference between the Green's functions at each point. This is the reason why we propose a method of spatial interpolations of the synthetic Green's functions on the fault plane.

SPATIAL INTERPOLATION OF GREEN'S FUNCTIONS

The source model used here is then the composite source model of Zeng *et al.* (1994) as shown before. Figure 2 shows the fault and receiver disposition in space. The fault plane is covered by a set of points sources. We want to get the simulated motion for frequencies up to 20Hz. So considering that we need to have 5 to 10 points per wavelength we can choose the spacing between two points. The minimum wavelength is:

$$\lambda_{min} = \frac{V_s}{f_{max}} \tag{10}$$

with f_{max} = 20Hz and V_s being the S wave velocity. For a S wave velocity of 3000 m/s, we get d_{min} comprised between 15m and 30m. So for our simulation we chose a distance of 20m between two points. As said before, for a 10km*10km fault plane this leads to 250000 points.
In order not to calculate the synthetic Green's functions at each point, we propose this technique of spatial interpolation of the Green's functions. We calculate the theoretical Green's functions for only 25 points regularly spaced every 2km on a 10km*10km fault plane. We use the code Axitra (Coutant, 1989) based on the discrete wavenumber method (Bouchon, 1981) to calculate these theoretical Green's functions. We use here a simple horizontally layered structure described in Table 1. As shown in Figure 2, the Green's functions for all the other points will be interpolated using the calculated ones. Figure 3 gives a detailed view of the fault plane to show the parameters used for the spatial interpolation.

Table 1. Description of the flat layered structure.

Thickness (m)	P wave velocity (m/s)	S wave velocity (m/s)	Density (g/cm3)
700	2600	1500	2.65
2000	4700	2710	2.65
half space	6000	3460	2.65

The interpolation procedure is described as follows. Let $U_k(t)$ denote the k calculated Green's functions. Let us now consider a point S_{ij} of the dense grid, for which we want to estimate the Green's function $U_{ij}(t)$. This point is characterized by its coordinates on the source area: $x_i=i\delta x$, $y_j=j\delta y$. We give the expression of $U_{ij}(t)$ as:

$$U_{ij}(t) = \sum_k \alpha_{ijk}\, U_k(t - \delta t_{ijk}) \tag{11}$$

where δt_{ijk} is the difference in S wave travel time between point k and the receiver and between point (i,j) and the receiver, and α_{ijk} is a weighting coefficient for each calculated Green's function, defined as follows:

$$\delta t_{ijk} = \frac{r_{ij} - r_k}{V_s} \tag{12}$$

where r_{ij} and r_k are the distances between point S_{ij} and receiver and point k and receiver, respectively. The weights are based on the following principle: the closer the source (point k), the larger the weight:

$$\alpha_{ijk} = \frac{C_{ij}}{(d_{ijk})^{\eta}} \tag{13}$$

with d_{ijk} being the distance between point k and point S_{ij}, and C_{ij} is just a proportionality constant allowing to have $\Sigma_k \alpha_{ijk} = 1$, thus:

$$C_{ij} = \left(\sum_{k=1}^{K} (d_{ijk})^{-\eta} \right)^{-1} \tag{14}$$

The value of η can be described as the power of decrease with distance to the point k considered. Two values have been tested here: $\eta=1$ and $\eta=2$. After comparison of the results, we chose the value $\eta=2$ which gives a better agreement between calculated and interpolated signals. It is then possible to use this interpolation based on a -2 decrease with distance to every calculated Green's function, in order to approximate the theoretical Green's function at every grid point on the fault plane. The quality of the interpolation technique has first been tested for 5 different points on the fault plane and for 5 receiver positions. Figure 4 shows for one point-receiver pair the calculated theoretical Green's function (left) as long as the equivalent interpolated Green's function (right) for the same point. The interpolated and the calculated Green's functions for these 25 point-receiver pairs are also compared in terms of envelope and spectrum of the signal. We found an overall good correlation between the calculated and the interpolated Green's functions.

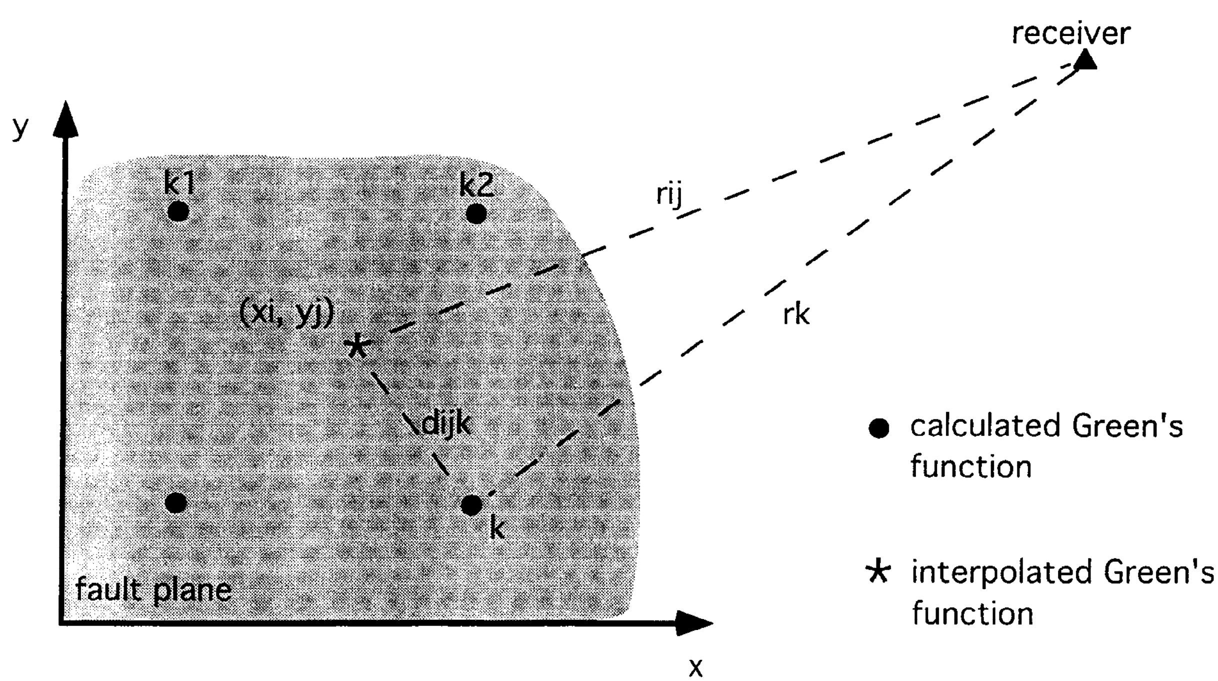

Fig. 3. Detailed view of the fault plane.

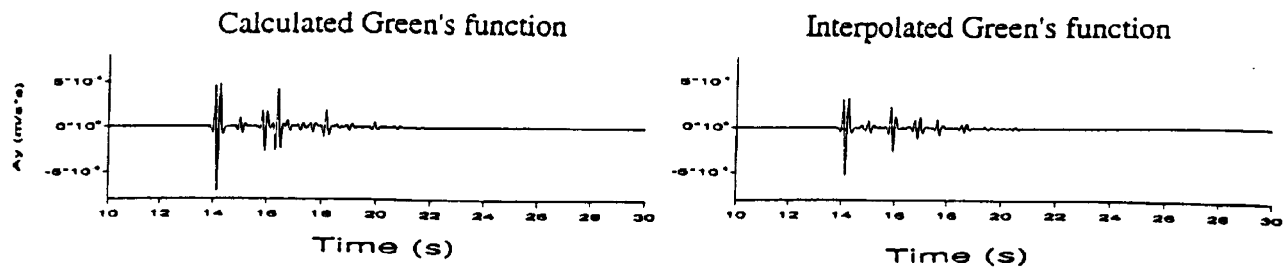

Fig. 4. Comparison between a calculated Green's function (left) and the corresponding interpolated one (right).

RESULTS

This technique is used here for two theoretical cases, for the two source-receiver configurations shown in Figure 2. The source is a vertical 10km*10km fault situated between 5km and 15km depths, we used the geological structure described in Table 1, for two positions of the receiver. In case 2, the receiver is 50km far from the fault, while in case 1, we are in a situation of very near field with the receiver placed above the middle of the fault. The receivers are aligned with the fault surface trace. For this simulation, the seismic moment M_0 as been chosen equal to 1.6×10^{25} dyne-cm. This value corresponds to the moment of a circular crack with a surface equal to the surface of the considered fault plane (10km*10km). The value of the stress drop as been chosen equal to 4 MPa.

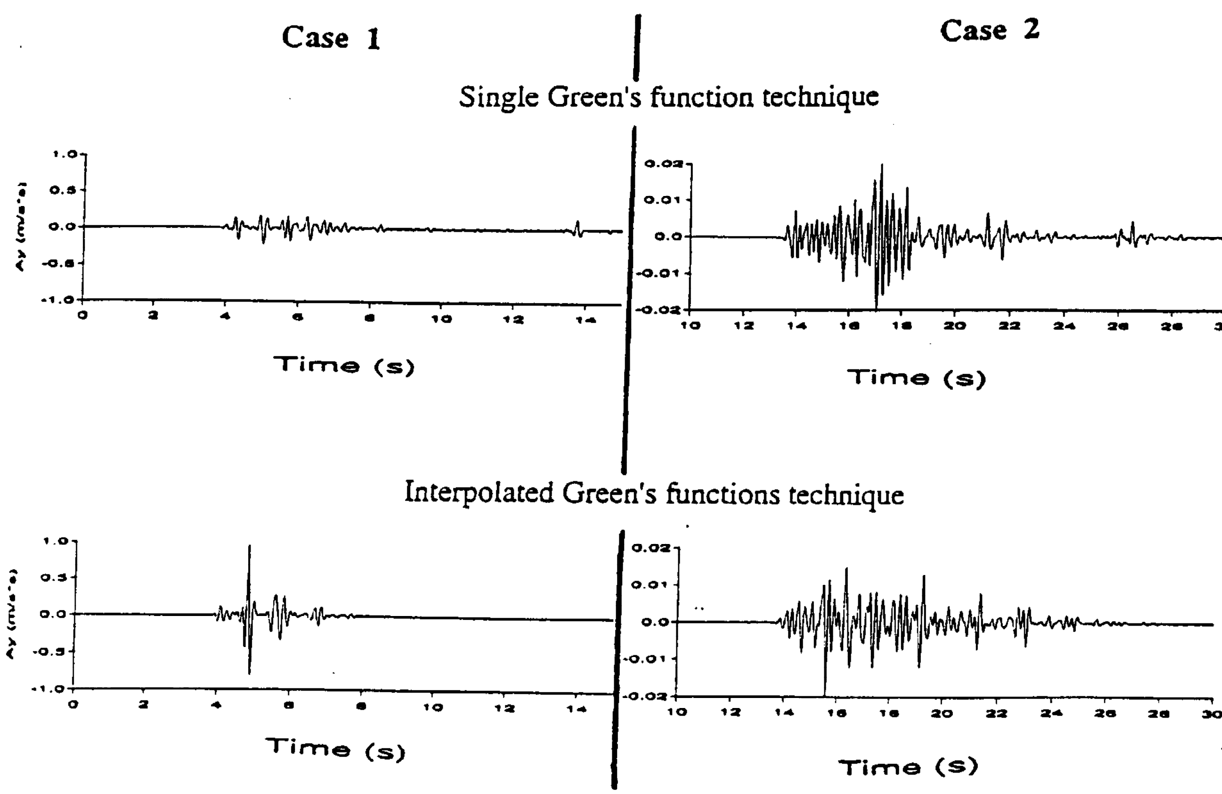

Fig. 5. Simulated strong motion using one single Green's function for case 1 (top left) and for case 2 (top right); and simulated strong ground motion using the interpolated Green's functions at each grid point on the fault for case 1 (bottom left) and for case 2 (bottom right).

To show the advantage of this method, we compare the results for the simulation considering first only one single Green's function for all the points, and secondly the whole calculation with the interpolated Green's functions at every point on the fault plane.
Figure 5 shows the simulated strong motion using only one single Green's function (top) for case 1 (top left) and for case 2 (top right); and the simulated strong motion obtained with the complete technique using spatial interpolation of Green's functions at each point (bottom) for case 1 (bottom left) and for case 2 (bottom right). We can see that for case 2, where the receiver is quite far from the source (50km), the simulated motion is not very different using one single Green's function or the whole interpolation procedure; this confirms the fact that the technique of empirical Green's functions is very suitable in the case of quite big distance between source and receiver. On the contrary, we observe for case 1 (very near field) that the use of only one Green's function can not be sufficient to synthesize the corresponding strong ground motion. We observe a big difference in the maximum acceleration and also in the duration of the signal. In this case, the technique proposed here can be a very useful tool for strong motion simulation.
For the case of a 10km*10km fault plane, the simulation procedure takes between 10 and 15 hours of CPU time on a classical work station. It is then possible, using this technique, to simulate strong motion for various fault geometries and various site conditions, with a reasonable computer time, even up to frequencies of about 20Hz and in the very near field domain. The use of this method might then be of great interest for engineering seismology purposes.

CONCLUSIONS

The method developed here is intended to constitute a practical tool for strong motion simulation. It is based on the principle of the empirical Green's functions technique, but it is applied here using theoretical Green's functions. This provides the advantage that there is no need for good recordings of small events to be considered as empirical Green's functions. However, for the computed Green's functions to be realistic, this method requires to have a reliable information on the crustal structure in the considered area. In some areas of moderate seismicity it is indeed difficult to obtain such recordings; problems due to bad signal to noise ratio are also often a big drawback of the empirical Green's functions technique, when applied for urban sites for example.
Furthermore, the method proposed here takes into account the contribution of the different parts of the fault plane, making it possible to use it in the case of near field simulations, where only one Green's function is not sufficient.
Finally, most of the simulations done with the Green's functions techniques are limited to a low frequency range, because of the discretization of the fault plane. In the case of this study, we introduce a method of spatial interpolation of the Green's functions that allows to calculate the theoretical Green's functions for a very thin grid of points on the fault, without the problem of very high computer time. This makes it possible to consider simulated strong motions up to frequencies as 20Hz. The frequency range of interest for engineering seismology is then covered with this simulation technique.
This paper was aimed at the presentation and the description of the method in purely theoretical cases; but it is clear that some practical application is now needed to validate the use of this technique in the case of real sites. This is the reason why the code developed here will be now applied to the case of the 1995 Hyogo-Ken-Nanbu Earthquake. Simulations will be conducted using this method for different sites in the area of Kobe city, using the fault parameters inferred after the earthquake. Results will then be compared with real recordings obtained during the main shock.

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