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A STUDY OF THE MECHANICAL RESPONSE OF REINFORCED CONCRETE TO CYCLIC SHEAR REVERSALS

B. MARTÍN PÉREZ and S.J. PANTAZOPOULOU

Department of Civil Engineering, University of Toronto,
Toronto, ON M5S 1A4, Canada.

ABSTRACT

The participation of concrete in the mechanism of shear resistance of reinforced concrete (r.c.) members subjected to cyclic load reversals is investigated in this paper with the objective to study the relationship between shear strength and deformation demand. Recent experimental studies have illustrated that the concrete contribution to shear resistance, taken as a fixed quantity in the current design practice, degrades with increasing ductility demand. The parametric dependence of the mechanics of cyclic-shear behaviour of r.c. elements is assessed here by means of a nonlinear analytical model of cyclic plane stress states. The formulation establishes tensorial, constitutive and equilibrium relations, all expressed in terms of average stresses and average strains. The goal of this study has been to quantify the components of a recently proposed conceptual model which is intended to reflect in the design process the dependence of concrete shear resistance on ductility demand.

KEYWORDS

Shear strength; concrete contribution; deformation demand; ductility; loading cycles.

INTRODUCTION

Recent experimental studies (Ghee *et al.*, 1989; Aschheim *et al.*, 1992; Wong *et al.*, 1993; Priestley *et al.*, 1994) have led to empirical formulations that express the concrete contribution to shear resistance as a degrading quantity with increasing displacement ductility (d.d) demand. These approaches provide a novel outlook in earthquake design of r.c., wherein it is necessary to assess deformability demand as well as strength requirements, and to link resistance supply to deformation demand. The objective of this paper is to investigate the mechanical problem of cyclic shear in r.c., and to contribute to the development of a simplified ductility-based estimation of shear resistance for earthquake design applications.

Current Design Guidelines

In estimating the nominal shear strength of r.c. members, V_n, current design guidelines consider a portion of the shear to be resisted by the concrete, V_c, while the remainder is resisted by the steel, V_s, i.e., $V_n = V_c + V_s$ (ACI, 1989; simplified method in CSA, 1984). The force resisted by the shear reinforcement, V_s, is obtained from the 45°-truss model formulated by Ritter (1899) and Mörsch (1902) and corresponds to yielding conditions of the steel.

The shear strength provided by the concrete, V_c, taken as the shear force corresponding to initiation of diagonal cracking, has been assessed empirically from experimental data and is given in terms of stress units (MPa) as

$$\textbf{ACI:} \quad v_c = 0.17\sqrt{f'_c}\left(1 + \frac{P}{14\,A_g}\right) \tag{1}$$

$$\textbf{CSA:} \quad v_c = 0.20\sqrt{f'_c}\left(1 + \frac{3\,P}{f'_c\,A_g}\right) \tag{2}$$

where P is the factored axial load (positive in compression) and A_g is the gross cross-sectional area of the element. For conservatism, for r.c. members designed to withstand earthquake loading, the ACI and CSA codes take the concrete shear contribution stress v_c to be zero if the factored axial compressive stress (based on gross section dimensions and including earthquake effects) is less than a small fraction of f'_c (5% for ACI and 10% for CSA, respectively). For cases where the axial compressive stresses exceed the above limiting values, v_c is given by Eq. 1 and Eq. 2. These values have been shown to give conservative estimates of shear strength at low levels of d.d. demand, whereas they are unconservative at high values of d.d., since they fail to recognize the degradation of concrete compressive strength with increasing crack widths.

A completely different approach has been recently adopted by the AIJ code (1994). The Japanese design guidelines, based on the superposition of a truss and an arch-action shear resisting mechanisms, use a lower bound plasticity approach, in which both the concrete and the reinforcement are assumed to be under yielding conditions. In the same line as the above guidelines, the shear force carried by the truss mechanism is expressed in terms of the yielding stress of the shear reinforcement. However, the shear force carried by the arch mechanism, i.e., the contribution of concrete to shear strength, is reduced linearly with increasing inelastic rotation by introducing an effectiveness factor for the concrete compressive strength f'_c.

Shear Strength Models

Recent experimental studies on r.c. members, mainly columns, have led to several researchers to propose conceptual models with the objective of reflecting the reduction in the shear carrying capacity of concrete with increasing levels of d.d. demand (Ghee *et al.*, 1989; Aschheim *et al.*, 1992; Wong *et al.*, 1993; Priestley *et al.*, 1994). In general, the concrete contribution V_c to the shear strength of r.c. members is assumed to be independent of the level of deformation at low d.d.. Most of the proposed models suggest a constant initial value for V_c up to a d.d. of 2 (Ang *et al.*, 1989; Aschheim *et al.*, 1992; Priestley *et al.*, 1994). At large d.d. (usually in excess of 4), V_c is assigned a fixed residual value. The shear contribution of concrete is assumed to degrade linearly for d.d. values that fall in between those two limits (see Fig. 1). These conceptual relationships reflect the reduction in shear strength with increasing d.d. demand and identify the increase in shear capacity with increasing amount of transverse reinforcement and level of axial load.

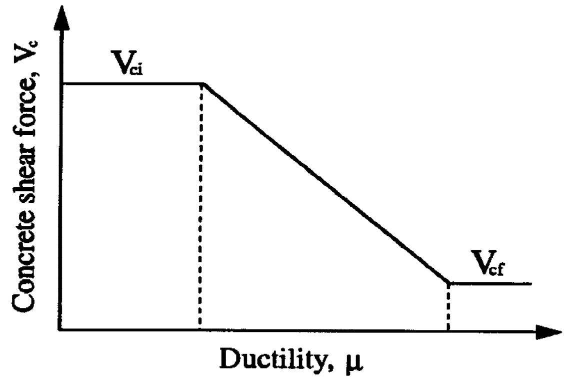

Fig. 1: *Model for shear strength degradation.*

ANALYTICAL EVALUATION OF V_c

The relation between shear strength and deformation demand was studied by defining the mechanical behaviour of a simple r.c. element whose deformation is primarily characterized by in-plane shear distortion. A nonlinear analytical model was developed to assess the response of such an element subjected to reversed cyclic in-plane stresses.

In the analytical model developed, material stresses are approximated by a state of plane stress. The model assumes adequate detailing so that cracks are smeared out across the r.c. element and the reinforcing bars are uniformly distributed in two orthogonal directions. Because of these two assumptions, the behaviour of the material is formulated in terms of average stresses and average strains. It is further assumed that the crack planes coincide with planes of maximum normal tensile strain, and that cracked concrete behaves as an orthotropic material with the material axes being oriented along the directions of principal stresses. The direction of principal strains is assumed to coincide with the direction of principal stresses for cracked concrete. Average stresses and average strains are related by a secant-material stiffness, in which dilatancy associated with the principal compressive stress direction is neglected. Concrete and reinforcement are assumed to develop compatible (equal) amounts of average deformation, i.e., bond slip can occur only locally.

The material stiffness of the composite material is obtained by superposition of the material stiffnesses of the individual material components, allowing to treat the constitutive relations for concrete and steel independently (see Fig. 2).

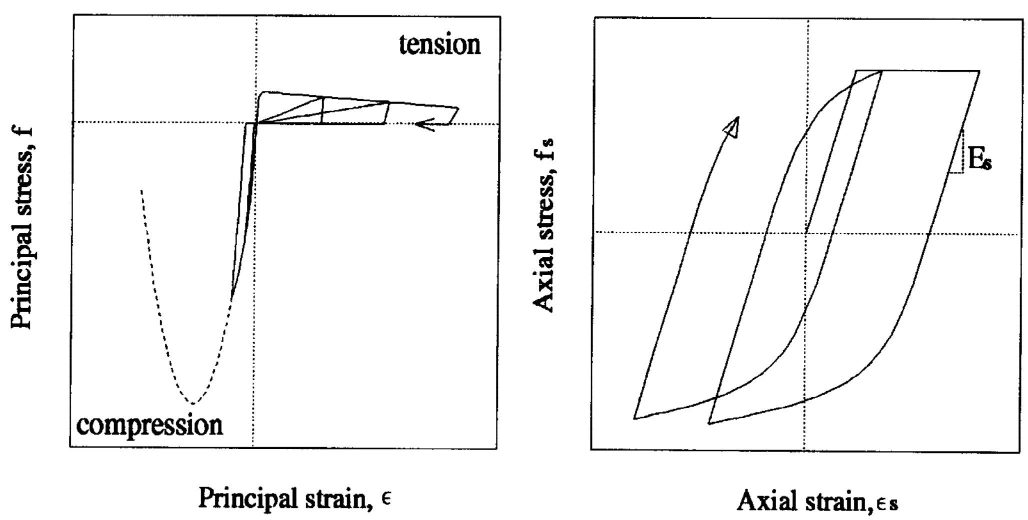

Fig. 2: *Material models:* **a)** *concrete in compression and tension;* **b)** *steel.*

Model Formulation

Given the boundary axial stresses n_x and n_y, the problem at hand is to find the resulting shear stress v_{xy} required to maintain equilibrium in the r.c. element for a given angle of imposed shear distortion γ_{xy} with cyclic history over a specified number of cycles (Fig. 3). To solve the problem, the formulation establishes tensorial, constitutive and equilibrium relations expressed in terms of average stress and deformation values.

Kinematic and Equilibrium Considerations. The states of deformation and stress of the r.c. element are given by the second-order tensors ϵ_{ij} and σ_{ij}, respectively, i.e.,

$$\epsilon_{ij} = \begin{pmatrix} \epsilon_x & \frac{\gamma_{xy}}{2} \\ \frac{\gamma_{xy}}{2} & \epsilon_y \end{pmatrix} \qquad \sigma_{ij} = \begin{pmatrix} f_{cx} & v_{xy} \\ v_{xy} & f_{cy} \end{pmatrix} \tag{3}$$

where ϵ_x and ϵ_y are the average strains in the x and y-directions, respectively, γ_{xy} is the average angle of shear distortion, f_{cx} and f_{cy} are the average concrete stresses in the x and y-directions, respectively, and v_{xy} is the shear stress. Expressions that relate the entries of the stress tensor are

$$f_{c1} + f_{c2} = f_{cx} + f_{cy} \tag{4}$$
$$f_{c1} - f_{cx} = v_{xy} \tan\theta \tag{5}$$
$$f_{c1} - f_{cy} = \frac{v_{xy}}{\tan\theta} \tag{6}$$

where f_{c1} and f_{c2} are the average principal tensile and compressive stresses in the concrete, respectively, and θ gives the direction of average principal tensile strain measured from the x-axis. Note that the convention used here is tension positive.

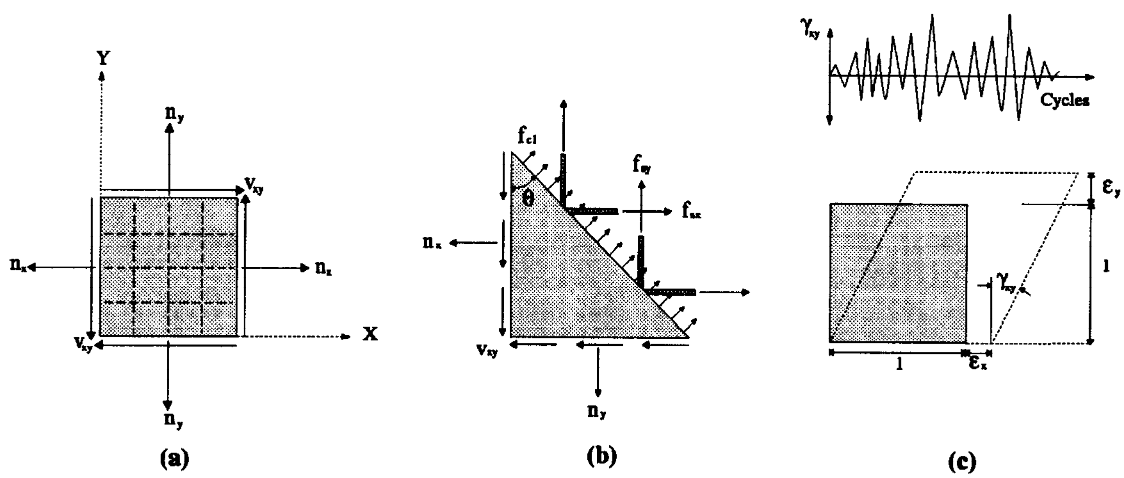

Fig. 3: *Membrane element:* **a)** *externally applied stresses;* **b)** *average stresses;* **c)** *average strains.*

The contributions from concrete and steel are calculated separately and then added together to satisfy equilibrium. The equilibrium of horizontal and vertical forces is established by integrating the stresses over the cross-sectional area. By ignoring the small reduction in the area of concrete due to the presence of the reinforcing bars, the equilibrium equations in terms of average stresses are simplified to

$$n_x = f_{cx} + \rho_x f_{sx} \tag{7}$$
$$n_y = f_{cy} + \rho_y f_{sy} \tag{8}$$

where ρ_x and ρ_y are the reinforcement ratios and f_{sx} and f_{sy} are the average stresses of the reinforcement in the x and y-direction, respectively.

Solution Procedure. The non-linear solution of the system of equilibrium equations (Eq. 7 and Eq. 8) was controlled by means of a step-wise increment (either positive or negative) in the value of shear strain γ_{xy} according to a prescribed deformation history. An iterative procedure is performed for each value of γ_{xy} until tensorial and equilibrium relations are simultaneously satisfied.

Concrete Shear Contribution

Following the currently adopted design code approach, the shear stress v_{xy} has been estimated in this study as the sum of three contributions: the shear stress carried by the concrete-resisting mechanisms v_c (such as residual tensile capacity of concrete and aggregate interlock), the shear stress carried by the reinforcement v_s, and the shear stress resisted by the presence of axial loads v_p. From tensorial and equilibrium relations, the shear stress v_{xy} in terms of average stresses is evaluated from

$$v_{xy} = \frac{1}{\tan\theta}\left(f_{c1} + \rho_x f_{sx} - n_x\right) \tag{9}$$

By identifying the contribution to shear resistance of the steel v_s as $\cot\theta\,\rho_x f_{sx}$, and of the axial load v_p as $-\cot\theta\, n_x$, the concrete contribution to shear resistance v_c is given by

$$v_c = v_{xy} - v_s - v_p = \frac{f_{c1}}{\tan\theta} \tag{10}$$

PARAMETRIC STUDY

To identify the critical variables that control the mechanism of cyclic-shear resistance, a sensitivity study of the concrete shear contribution to different design parameters was conducted using the analytical model described in the preceding. The parameters selected were the applied load history (Fig. 4(a)), the concrete compressive strength f'_c, ranging from 14 to 40 MPa, the yielding stress of the reinforcement in the x and y-direction f_{yx} and f_{yy}, ranging from 250 to 450 MPa, the corresponding reinforcement ratios ρ_x and ρ_y, ranging from 0.25% to 2.00%, and the normal boundary stresses n_x and n_y, taken as 5% and 10% of f'_c. The effect of each parameter was studied by varying its value while maintaining the remainder input variables constant. A total of 22 cases were evaluated.

Results

Influence of Load History. Two different sequences of imposed displacements were applied to one of the study cases ($f'c = 20$ MPa, $f_{yx} = f_{yy} = 350$ MPa, $\rho_x = \rho_y = 1.00\%$, $n_x = n_y = 0$ MPa) in order to determine the effect of load history on v_c. The resulting concrete contribution normalized to $\sqrt{f'_c}$ versus ductility μ_γ, defined herein as the ratio of the attained shear strain at a specific point in the displacement history to the shear strain corresponding to first yielding of the reinforcement in either direction, i.e., $\mu_\gamma = \gamma_{xy}/\gamma_{xy,yld}$, is plotted in Fig. 4(b) for both load histories. The solid and dashed lines correspond to the values of v_c given by the design codes (ACI, 1989; CSA, 1984). The model does not reflect a reduction of v_c as the number of cycles at a fixed ductility level increase since the envelope curves used for the various constitutive relations do not degrade with increasing number of cycles at a fixed deformation level.

Influence of f'_c. The effect of the compressive strength f'_c on the concrete contribution to shear resistance is illustrated in Fig. 5(a), which plots the calculated values of v_c versus ductility for f'_c equal to 14 and 40 MPa. Shear strength degradation takes place at lower ductility levels for higher

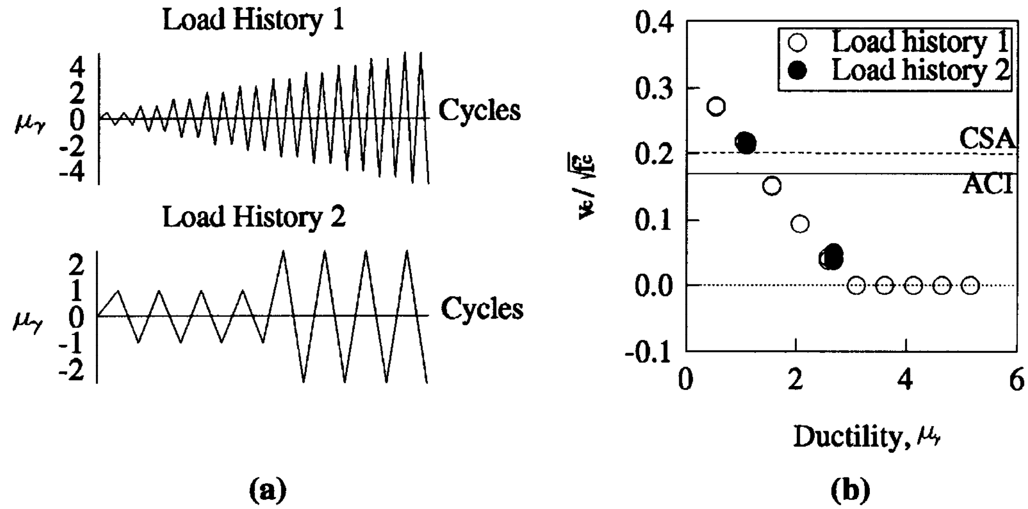

Fig. 4: **a)** *Load histories considered;* **b)** *Influence of load history on* v_c.

compressive strengths. For high compressive strengths, the load-deformation response of plane-stress elements tends to be governed by yielding of the reinforcement. Because the shear strain corresponding to steel yielding decreased with increasing compressive strength f_c', the estimated deformation of the r.c. element at a given level of ductility decreased as f_c' increased.

Influence of f_{yx} and f_{yy}. The influence of the reinforcement yield stresses f_{yx} and f_{yy} in determining the contribution of concrete to shear resistance was also evaluated by considering values in the range of 250 MPa to 450 MPa. Figure 5(b) shows the relationship between normalized concrete shear strength and ductility levels resulting from changing both yielding stresses simultaneously to 250 MPa and 450 MPa. The concrete contribution is higher and the rate of strength degradation is lower for lower yielding strengths. Lower yielding strengths correspond to lower magnitudes of principal tensile strain, ϵ_1, which is an indirect measure of the state of damage due to cracking in the concrete. At high values of ϵ_1 the concrete loses its ability to carry tensile stresses and thus a lower contribution to shear resistance results. When only one of the two yielding stresses is varied, it was observed that the reduction in v_c with increasing yield strength is less pronounced.

Influence of ρ_x and ρ_y. The parametric influence of the reinforcement ratios ρ_x and ρ_y on the resulting value of v_c was also investigated by increasing simultaneously both reinforcement ratios first, followed by the case where only ρ_y was varied while ρ_x was maintained at constant magnitude (1.00%). Figure 5(c) illustrates the effect on v_c as only one of the two reinforcement ratios is varied ($\rho_y = 0.25\%$, $\rho_y = 2.00\%$). The results are in agreement with reported experimental observations (Ang *et al.*, 1989; Aschheim *et al.*, 1992); Wong *et al.*, 1993) wherein an increase in the transverse reinforcement amount delays the onset of shear strength degradation to higher ductility levels.

Influence of n_x and n_y. Numerical examples were run by applying normal boundary stresses equal to 5% and 10% of the compressive strength f_c'. The resulting normalized concrete contribution $v_c/\sqrt{f_c'}$ versus ductility μ_γ is plotted in Fig. 5(d). It has been observed from experimental data that the load at which cracking occurs increases with the level of applied axial load (Ang *et al.*, 1989). The reason for this is the reduced magnitude of the tensile strains resulting from load confinement, which enhances the aggregate interlock capacity along the crack planes and the transfer of compression loads across the cracks. This experimental evidence is implied in the design code equations for cases where the applied compression stresses are greater than 5% and 10% of the concrete compressive strength f_c'. According to Fig. 5(d), the initial shear stress carried by the concrete-resisting mechanisms does not differ much for different uniaxial stress states. However, note that the axial load contribution to shear v_p has not been taken into account in calculating v_c as the current design equations do.

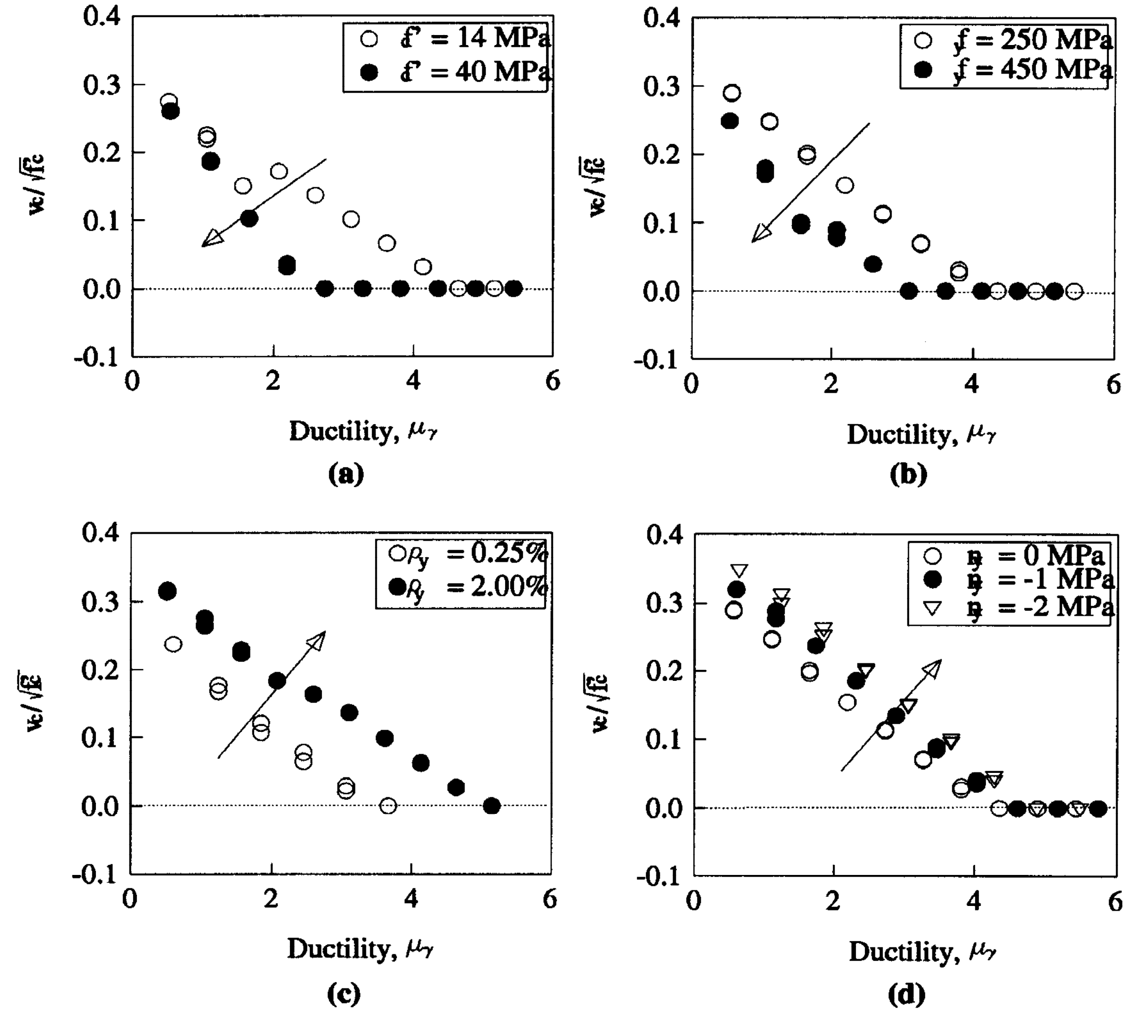

Fig. 5: *Influence of various parameters on v_c. (The arrows indicate trends given by increasing the particular parameter.)*

CONCLUSIONS

The numerical analyses performed here indicate that the concrete contribution to shear resistance of r.c. elements degrades with increasing deformation demand. It is also observed that the concrete shear contribution is enhanced by an increase in the amount of reinforcement along one direction, which has been experimentally observed and quantified by Aschheim *et al.* (1992), and an increase in the level of axial load applied, already taken into account in the current design equations.

The current design practice tends to give conservative estimates of shear strength for low levels of deformation demand; however, the fact that this approach is independent of the attained ductility level results in unconservative values of v_c at high levels of deformation, critical for r.c. structures that undergo inelastic deformation under earthquake loading. It is the objective of this work to propose an alternative yet simple design equation that reflects the experimental fact that the shear contribution of concrete diminishes with increasing ductility demand and is enhanced with increasing amount of transverse reinforcement and axial load. An equation that takes into account these phenomena is given

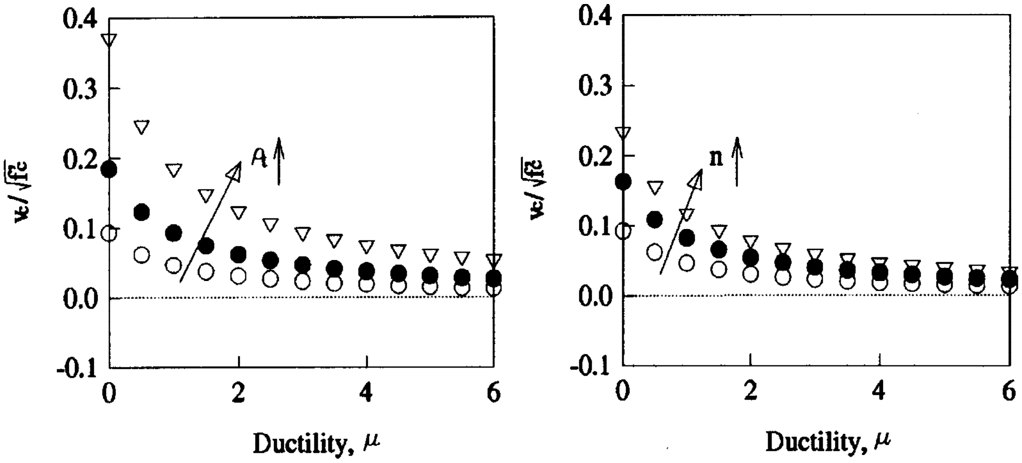

Fig. 6: *Model for concrete shear strength degradation (Eq. 11) of r.c. members subjected to seismic loading.*

in terms of stress units (MPa) as

$$v_c = \frac{\alpha\,\rho_t}{(1+\mu)}\sqrt{f_c'}\left[1-\beta\frac{n}{\sqrt{f_c'}}\right] \tag{11}$$

where ρ_t is the amount of shear reinforcement, μ is the attained ductility index, and n is the axial stress applied (positive in tension). The values of the constants α and β have been assessed analytically and assigned to be 37 and 7.6, respectively. Note that the term $\alpha\sqrt{f_c'}(1-\beta\,n/\sqrt{f_c'})$ is similar to the existing code expressions (Eq. 1 and Eq. 2). The difference is in the additional factor $\rho_t/(1+\mu)$, which is meant to reflect the dependence of v_c on the transverse reinforcement amount and ductility demand (see Fig. 6). The conservative estimates of v_c at low levels of deformation given by the current design equations (Eq. 1 and Eq. 2) are valid for linear elastic design of r.c. members. However, it is suggested that the concrete contribution to shear strength of r.c. elements subjected to seismic loading be evaluated from an expression such as the one proposed herein (Eq. 11). These simple guidelines aiming to reflect in the design process the experimental fact that the concrete contribution to shear resistance diminishes with increasing ductility demand contribute towards rationalizing of the shear design methods in earthquake resisting elements.

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