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Wave Attenuation in Wavenumber-Frequency Domain

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Abstract

To solve the problem of wave attenuation in ground, the authors have presented 2 papers [1], [2]. One is on the wave attenuation in time-domain using Fading Memory theory [1], and the other one is on the wave attenuation in spatial domain using the attenuating neighborhood theory [2]. Because in a real case, an earthquake wave will attenuate in time-spatial domain in the same time during propagation, this paper is aimed to solve such problem by presenting suitable model. Using the constitutive law, the Q value can be certainly obtained, and in turn if the Q value is obtained by experiments the way to calculate the influence function has been shown.

Keywords

Wave Attenuation, Time-Spatial Domain, Fading Memory, Attenuation Neighborhood

Introduction

From our previous papers it is clear that the wave attenuation is divided into two kinds and has both been solved in general, as follows. The first is that the wave attenuation in time domain, mainly representing the viscosity of the soil ground. The other one is the the wave attenuation in spatial domain, mainly representing the inhomogeneousity of the soil ground.

It is the fact that the real earthquake wave will attenuate in time and spatial domain in the same time, because the soil ground really has viscousity together should be inhomogeneous. Thus the wave attenuation problem should be studied in time-spatial domain, which needs one to present the Q value in wavenumber-frequency domain

Following our previous papers, it is reasonable to use the fading memory theory together with the attenuating neighborhood theory to treat such attenuation problem, where the viscousity has been assumed to follow the constitutive law of fading memory while the inhomogeneousity has been assumed to follow the attenuating theory.

In other words the experimental data of Q value for wave attenuation should be regarded as the data in time-spatial domain. The relationship between our theory and the real data has been shown, where the method to obtain the memory function and/or the influence function has been discussed.

Derivation of Constitutive Law

The general constitutive law for viscoelastic material with fading memory can be written in the following [1].

$$\sigma = A\varepsilon + \int_{-\infty}^{t} m(t-s)\frac{\partial \varepsilon(s)}{\partial s}ds \tag{1}$$

where σ is stress of the soil ground, A is is the Lamè constant, ε is the strain, $m(t)$ is the so-called memory function.

In the above equation, if the ground is elastic inhomogeneous, the elastic constant and the density should be considered as follows.

$$A = A_0 + \delta A, \qquad \rho = \rho_0 + \delta\rho \tag{2}$$

the above equation is the often used one in scattering theory for earthquake wave, where

$$\begin{aligned} A_0 &= < A >, & < \delta A > &= 0 \\ \rho_0 &= < \rho >. & < \delta\rho > &= 0 \end{aligned} \tag{3}$$

Using the above equations the motion equation can be obtained as:

$$A_0\frac{\partial^2 u}{\partial x^2} + \int_{-\infty}^{t} m(t-s)\frac{\partial^3 u}{\partial x^2 \partial s}ds - \rho_0\frac{\partial^2 u}{\partial t^2} = -\frac{\partial}{\partial x}(\delta A\varepsilon) + \delta\rho\ddot{u} \tag{4}$$

For simplicity of derivation and explanation, $m(t) = 0$ is assumed. Thus the motion equation becomes

$$\frac{\partial^2 u}{\partial t^2} = -\frac{1}{\rho_0}\Phi(x,t) + c^2\frac{\partial^2 u}{\partial x^2}, \qquad c^2 = \frac{A_0}{\rho} \tag{5}$$

where

$$\Phi(x,t) = \frac{\partial}{\partial x}(\delta A\varepsilon) - \delta\rho\ddot{u} \tag{6}$$

Because:

$$\Phi(x,t) = \int_{-\infty}^{\infty} d\tau \int_{\xi} \Phi(\xi,\tau)\delta(x-\xi)\delta(t-\tau)d\xi \tag{7}$$

In general, the solution of the motion equation is.

$$u = \frac{1}{4\pi c^2 \rho} \int_\xi \frac{\Phi(\xi, t - \frac{|x-\xi|}{c})}{|x - \xi|} d\xi \tag{8}$$

From the wave scattering theory, it is understood that the scattering wave decrease the primary wave. The above solution can be written as:

$$u = u(x, t, x - \xi) \tag{9}$$

And thus it is reasonable to assume that

$$\Phi(x,t) = \int_{-\infty}^{\infty} b(x-\xi)\varepsilon(\xi,t)d\xi \tag{10}$$

which obeys the attenuating neighborhood theory.

As conclusion of this section, the constitutive law is written as.

$$\sigma = A\varepsilon + \int_{-\infty}^{\infty} b(x-\xi)\varepsilon(\xi,t)d\xi + \int_{-\infty}^{t} m(t-s)\frac{\partial \varepsilon(s)}{\partial s} ds \tag{11}$$

and the wave equation becomes.

$$A_0 \frac{\partial^2 u}{\partial x^2} + \frac{\partial}{\partial x}\int_{-\infty}^{\infty} b(x-\xi)\frac{\partial u(\xi,t)}{\partial \xi} d\xi + \int_{-\infty}^{t} m(t-s)\frac{\partial^3 u}{\partial x^2 \partial s} ds = \rho_0 \frac{\partial^2 u}{\partial t^2} \tag{12}$$

Q Value

After Fourier transform, the constitutive law in wavenumber-frequency domain is

$$\Sigma(\kappa,\omega) = AE(\kappa,\omega) + B(\kappa)E(\kappa,\omega) + i\omega M(\omega)E(\kappa,\omega) \tag{13}$$

where the memory function and influence function after Fourier transform can be divided into real and imaginary parts as follows.

$$\begin{aligned} B(\kappa) &= B_R(\kappa) + iB_I(\kappa), \\ M(\omega) &= M_R(\omega) + iM_I(\omega) \end{aligned} \tag{14}$$

Then the constitutive law can be rewritten as.

$$\Sigma(\kappa,\omega) = [A + B_R(\kappa) - \omega M_I(\omega) + iB_I(\kappa) + i\omega M_R(\omega)]E(\kappa,\omega) \tag{15}$$

Using the definition of the Q value, we obtained

$$\frac{1}{Q} = \frac{B_I(\kappa) + \omega M_R(\omega)}{A + B_R(\kappa) - \omega M_I(\omega)} \tag{16}$$

The simulation results to obtain the Q value from memory function and influence function are shown in Fig.1 and Fig.2.

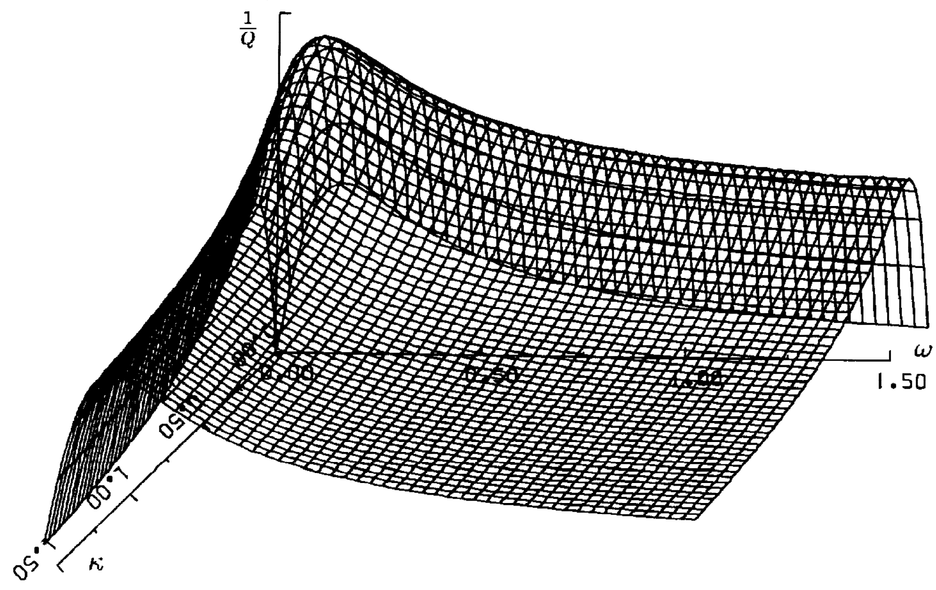

Fig.1. The Q Value when $m(t) = e^{-t}$ and $b(x) = e^{-x}$

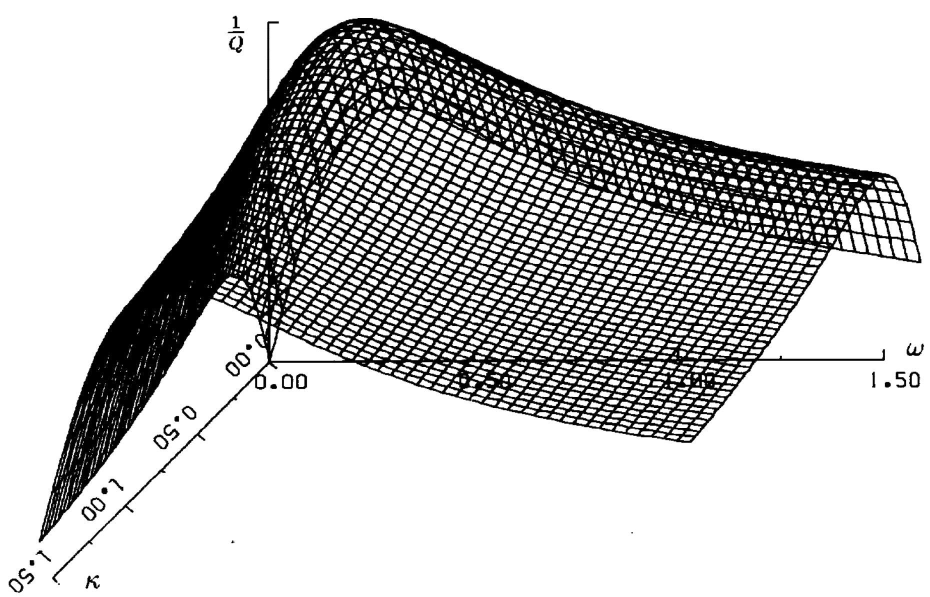

Fig.2. The Q Value when $m(t) = 1.5e^{-2t}$ and $b(x) = 1.5e^{-2x}$

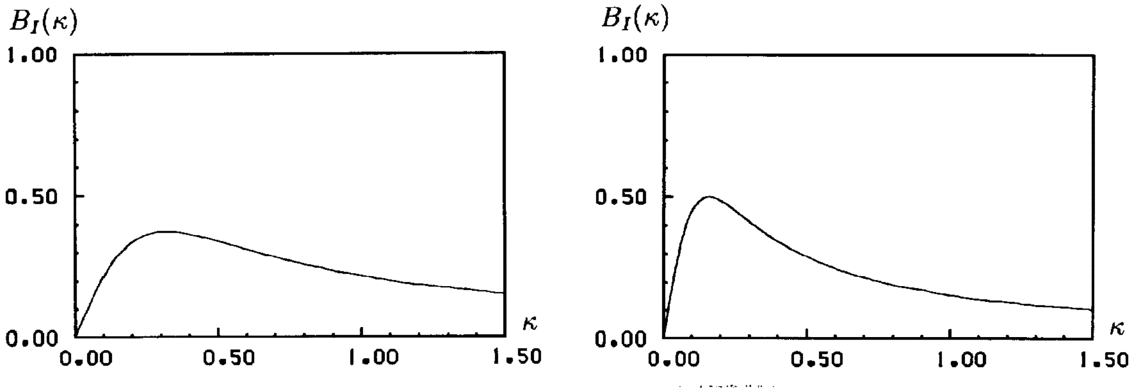

Fig.3. The Imaginary Part of Influence Function Calculated from Q Value

Influence Function from Q Value

In real applications, generally the Q value can be investigated from experiments. Thus to grasp the wave propagation it is important to determine the $m(t)$ and $b(x)$. Because until now the authors have not developed a method to determine a memory function together with an influence function, using only one group of Q data.

Thus at this point we have to make some assumptions for the purpose to calculate the functions. Because in general the viscosity of the ground can be easily obtained which determines the memory function. In other words from the previous studies, it is understood that the wave attenuation due to vibration is generally easily determined while the attenuation due to inhomogeneousity is difficultly determined.

Thus the problem becomes to present a method to calculate the $b(x)$ while the $m(t)$ and $Q(\kappa, \omega)$ are known.

The relation between the real part and the imaginary part of the influence function is as follows.

$$B_R(\kappa) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{B_I(\tau)}{\kappa - \tau} d\tau \tag{17}$$

also we have the following equation from last section.

$$\frac{B_I(\kappa) + \omega M_R(\omega)}{A + B_R(\kappa) - \omega M_I(\omega)} = \frac{1}{Q(\kappa, \omega)} \tag{18}$$

Using the above two equations we obtained

$$B_I(\kappa) - \frac{1}{\pi Q(\kappa,\omega)}\int_{-\infty}^{\infty} \frac{B_I(\tau)}{\kappa - \tau} d\tau = \frac{A}{Q(\kappa,\omega)} - \frac{\omega}{Q(\kappa,\omega)} M_I(\omega) - \omega M_R(\omega) \tag{19}$$

In the above equation the $Q(\kappa, \omega)$ and $\{M_{Ii}, M_{Ri}\}$ are known, and the unknowns are $B_I(\kappa)$ and $\{B_{Ii}(\kappa)\}$. Thus the equation to obtain the influence function is as follows.

$$Q_i(\kappa,\omega) B_{Ii}(\kappa) - \frac{1}{\pi}\sum_{i=1}^{n} a_i \frac{B_{Ii}(\tau)}{\kappa - \tau} = A - \omega M_{Ii}(\omega) - \omega M_{Ri}(\omega) Q_i(\kappa,\omega) \tag{20}$$

Using this equation, the imaginary part of the influence function has been calculated from the Q value of last section. The simulated results are shown in Fig.3, where is is clear that the calculated influence function is same with the original one.

Theoretical Solution of Wave Propagation

We have the wave equation.

$$A_0\frac{\partial^2 u}{\partial x^2} - \rho_0\frac{\partial^2 u}{\partial t^2} = -\frac{\partial}{\partial x}\int_{-\infty}^{\infty} b(x-\xi)\frac{\partial u(\xi,t)}{\partial \xi}d\xi - \int_{-\infty}^{t} m(t-s)\frac{\partial^3 u}{\partial x^2 \partial s}ds \tag{21}$$

Because the wave equation is a linear one, the solution can be obtained by superposing the two solution of the following two equations.

$$A_0\frac{\partial^2 u}{\partial x^2} - \rho_0\frac{\partial^2 u}{\partial t^2} = -\frac{\partial}{\partial x}\int_{-\infty}^{\infty} b(x-\xi)\frac{\partial u(\xi,t)}{\partial \xi}d\xi \tag{22}$$

$$A_0\frac{\partial^2 u}{\partial x^2} - \rho_0\frac{\partial^2 u}{\partial t^2} = -\int_{-\infty}^{t} m(t-s)\frac{\partial^3 u}{\partial x^2 \partial s}ds \tag{23}$$

The solutions to the above two equations have been shown in [1] and [2], which clearly gives the following solutions for eq.(21), as follows.

$$u(x,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} c_1 e^{-iD(\kappa)t - i\kappa x}d\kappa + \int_{-\infty}^{\infty} c_2 e^{-B(\omega)x - i\omega t}d\omega \tag{24}$$

Conclusion

The way to solve the problem of earthquake wave attenuation in wavenumber-frequency has been presented and some simulation results have been shown.

References

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