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# SITE-DEPENDENT SEISMIC DEMANDS FOR NONLINEAR SDOF SYSTEMS

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## ABSTRACT

The effect of site conditions on the seismic demands on single-degree-of-freedom systems is investigated. The paper summarizes recent studies on seismic demands to be used in strength-based seismic design criteria as well as in displacement-based seismic design criteria. In strength-based design it is necessary to estimate the lateral strength required to avoid displacement ductility demands larger than the ductility capacity, while in the displacement-based design it is necessary to estimate the lateral displacement demands that can provide guidelines to the lateral stiffness and deformation capacity required in the structure in order to avoid collapse and to control the level of damage in the structure. Special emphasis is given on the direct estimation of inelastic strength and deformation demands and on factors that permit the estimation of inelastic strength and deformation demands as a function their corresponding elastic demands.

## KEYWORDS

Response spectra; site conditions; strength-reduction factors; displacement amplification factors; seismic criteria; building codes.

## INTRODUCTION

Current seismic design provisions oversimplify a complex problem by using simplified procedures that have no transparent relation to many demand and capacity parameters that dominate the seismic design of structures (Nassar and Krawinkler, 1991; ATC-34, 1993). Improvement to current seismic provisions can be obtained by an explicit incorporation in the design process of relevant demand and capacity parameters which control the response of structures during earthquakes. Since the concept of the response spectrum was introduced into earthquake engineering by Biot (1941), this technique has been widely used to estimate force, deformation and energy demands imposed by earthquake ground motions on structures. Today, response spectra form the basis of seismic design forces on most seismic codes (*Earthquake resistant, 1992*). The objective of this paper is to summarize some recent developments on the estimation of strength and displacement demands on single-degree-of-freedom (SDOF) systems by means of inelastic demand spectra. Energy demands on SDOF demands and modifications to seismic demands on SDOF systems to be used in the design of multi-degree-of-freedom (MDOF) systems are presented elsewhere (Fajfar, 1996; Krawinkler, 1996, respectively).

STRENGTH DEMANDS

## Elastic Strength Demands

The elastic strength demand, $F_{y,e}$ , defines the minimum yield strength required in the structure in order to remain elastic during a given ground motion. For SDOF systems, the elastic acceleration response spectra provide the needed information for this parameter.

$$F_{y,e} = m \cdot S_a = \frac{W \cdot S_a}{g} \qquad (1)$$

where $m$ is the mass, $W$ is the weight, $g$ is the acceleration due to gravity and $S_a$ is the acceleration response spectral ordinate of an elastic SDOF system.

One of the better known studies on the effect of site conditions in the elastic response spectrum is that of Seed et al. (1976) who studied spectral amplifications on 104 horizontal ground motions recorded on different soil conditions classified into four groups. Strength demands required to maintain the system elastic normalized by maximum ground acceleration for firm alluvium sites from Seed's study are compared to two recent studies (Miranda, 1993a; Riddell, 1995) in Fig. 1. It can be seen that although these studies are based on different sets of ground motions, the results are very similar.

In contrast to previous studies on the effect of site conditions in the elastic response spectrum Seed et al. (1976), recent studies on ground motions recorded on soft soil have concluded that elastic spectral amplifications can be much larger in soft soils than for rock or firm alluvium sites (Miranda, 1993a; Rahnama and Krawinkler, 1993). These studies have concluded that elastic strength demands are strongly dependent on the predominant period of the soft soil site, $T_s$ , thus, elastic as well as inelastic strength demands should be specified as a function of the ratio of the fundamental period of the structure to the predominant period of the soft soil site, $T/T_s$ .

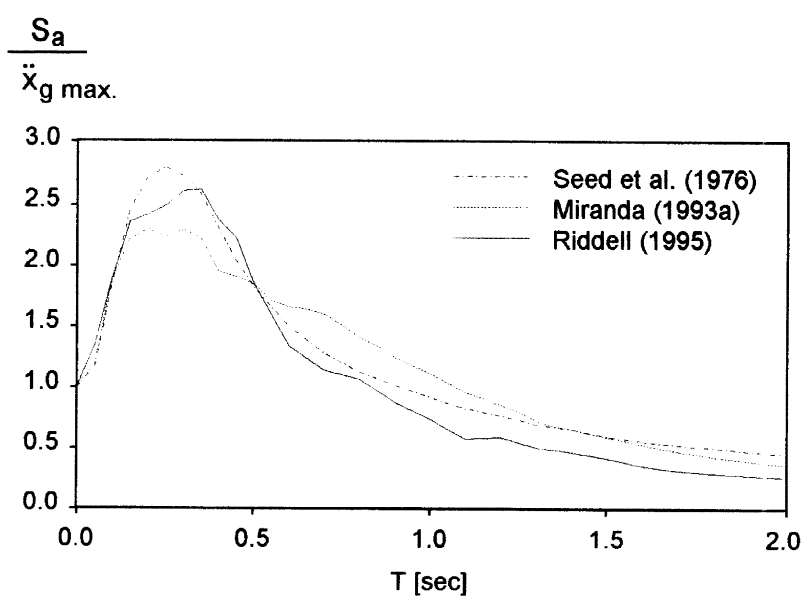


Fig. 1. Comparison of mean normalized elastic strength demand spectra for firm alluvium sites.

Inelastic Strength Demands

Due to economic reasons, present seismic design philosophy allows buildings and other types of structures to undergo inelastic deformations in the event of strong earthquake ground motions. As a result of this philosophy, the design lateral strength prescribed in seismic codes is lower and in some cases much lower, than the strength required to maintain the structure in the elastic range. Current seismic design criteria are associated with strength-based concepts in which the structure is designed to have a lateral strength equal or larger to that required to control the displacement ductility demands below a certain level. For SDOF systems the *inelastic strength demand spectra* provide this required lateral strength. An inelastic strength demand spectrum is a plot of the yield strength in a SDOF system, $F_y(\mu_i)$ , required to limit the displacement ductility ratio to specified maximum tolerable displacement ductility ratios, $\mu_i$ (i.e., target ductility). Examples of strength demand spectra for firm alluvium sites from two recent studies are shown in Fig. 2. A complete literature review of previous studies on inelastic response spectra has recently been presented (Miranda, 1993a).

Although, recent studies have concluded that a rational design may be attained through smoothed inelastic design strength demand spectra that are derived directly from statistical and probabilistic analyses of response spectra (Bertero et al., 1991; Miranda, 1991, 1993b; Riddell, 1995), the lateral strength required by current seismic design procedures are the result of smoothed linear elastic response spectra (SLERS), which are then reduced to take into account the inelastic behavior in the structure.

Reductions in forces produced by the hysteretic energy dissipation capacity of the structure (i.e., reduction in forces due to inelastic behavior) are typically accounted for through the use of *strength-reduction factors*. Recently, Miranda and Bertero (1994) have presented a summary of previous studies on strength-reduction factors.

The strength-reduction factor (i.e., reduction in lateral strength demand due to inelastic behavior), $R_\mu$ , is defined as the ratio of the elastic strength demand to the inelastic strength demand

$$R_\mu = \frac{F_{y,e}}{F_y(\mu_i)} = \frac{C_y(\mu=1)}{C_y(\mu=\mu_i)} \qquad (2)$$

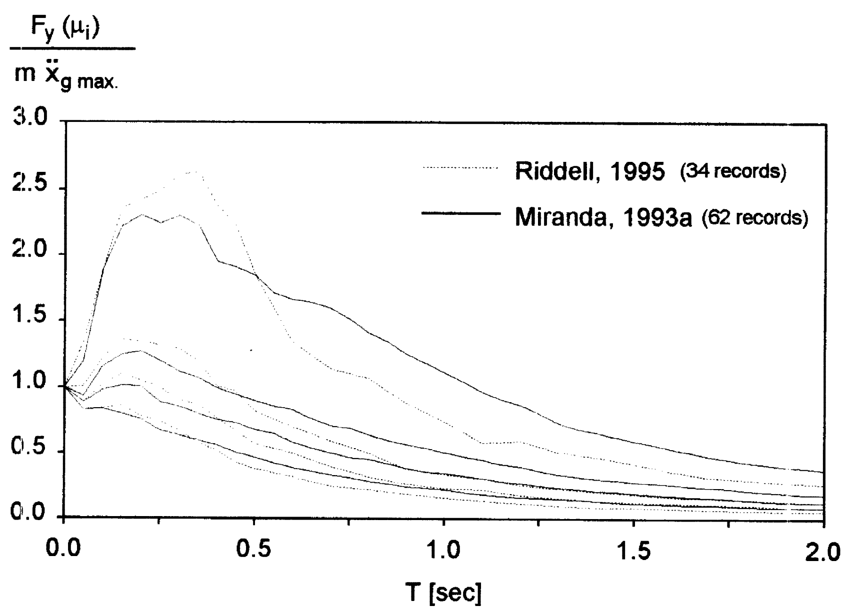


Fig. 2. Examples of mean normalized inelastic strength demand spectra for firm alluvium sites and for ductilities of 1, 2, 3 and 5.

where $C_y(\mu=1)$ is the seismic coefficient (yielding strength divided by the weight of the structure) required to avoid yielding; and $C_y(\mu=\mu_i)$ is the minimum seismic coefficient required to control the displacement ductility demand to $\mu_i$. The strength-reduction factor, $R_\mu$, permits the estimation of inelastic strength demand from elastic strength demand.

$$F_y(\mu_i) = \frac{F_{y,e}}{R_\mu} = \frac{C_y(\mu=1)}{R_\mu} W \tag{3}$$

For design purposes, $R_\mu$, corresponds to the maximum reduction in strength that can be used in order to limit the displacement ductility to $\mu_i$ in a structure that will have a lateral strength equal to the design strength.

A comparison of mean strength-reduction factors for systems subjected to ground motions recorded on firm alluvium sites from three different studies (Nassar and Krawinkler, 1991; Miranda, 1993c; Riddell, 1995) is shown in Fig. 3. Although these studies are based on different sets of ground motions, the similarity of the results is remarkable. Based on the results of a comprehensive statistical study on strength-reduction factors of SDOF systems undergoing different levels of inelastic deformation when subjected to 124 ground motions, Miranda (1993c) proposed the following simplified expressions to obtain analytical estimates of the strength-reduction factors:

$$R_\mu = 1 + \frac{\mu_i - 1}{\Phi} \tag{4}$$

where $\Phi$ is a function of $\mu$, $T$ and the soil conditions at the site and is given by:

*For rock sites:*

$$\Phi = 1 + \frac{1}{10T - \mu T} - \frac{1}{2T} exp\left[-\frac{3}{2}\left(Ln\,T - \frac{3}{5}\right)^2\right] \tag{5}$$

*For alluvium sites:*

$$\Phi = 1 + \frac{1}{12T - \mu T} - \frac{2}{5T} exp\left[-2\left(Ln\,T - \frac{1}{5}\right)^2\right] \tag{6}$$

*For soft soil sites:*

$$\Phi = 1 + \frac{T_s}{3T} - \frac{3T_s}{4T} exp\left[-3\left(Ln\frac{T}{T_s} - \frac{1}{4}\right)^2\right] \tag{7}$$

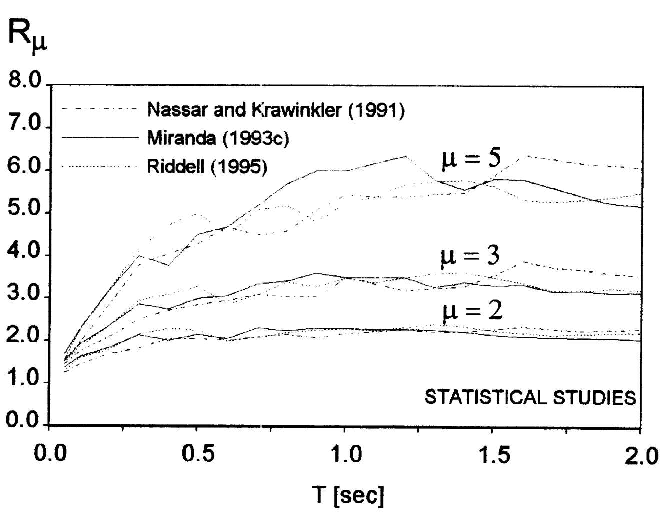


Fig. 3. Comparison of mean strength-reduction factors for firm alluvium sites.

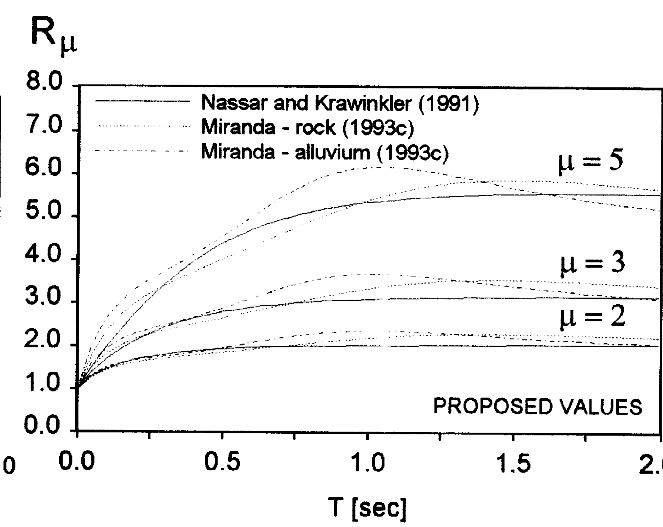


Fig. 4. Comparison of strength-reduction factors proposed by Nassar & Krawinkler with those proposed by Miranda.

Using mean strength-reduction factors computed with a different set of ground motions (consisting of 15 ground motions recorded on rock or alluvium sites) Nassar and Krawinkler (1991) proposed an approximate expression to estimate $R_\mu$. A comparison of $R_\mu$'s computed with their expression with $R_\mu$'s computed with the equations proposed by Miranda for rock and firm alluvium sites is shown in Fig. 4. It can be seen that the differences among the proposed equations is relatively small. For periods smaller than abut 1.0 s strength-reduction factors computed for systems on alluvium sites are slightly larger than those for systems on rock sites, while for systems with periods larger than 1.5 s the opposite is true.

For very soft soil sites, the shape of the $R_\mu$ spectrum is strongly dependent on the value of $T_s$ thus, mean $R_\mu$ spectra must be plotted against the ratio of the fundamental period of the structure to the predominant period of the soft soil site, $T/T_S$. Mean $R_\mu$ versus $T/T_S$ spectra for 22 ground motions recorded on the soft zone of Mexico City are shown on Fig. 5. As shown in this figure, strength-reduction factors for ground motions recorded on soft-soil sites exhibit strong variations with changes in the $T/T_S$ ratio. For periods closer than the predominant period of the site (i.e., $T/T_S \approx 1$) $R_\mu$ is much larger than the target ductility. For systems with periods shorter than two thirds of the predominant period of the soil site, the strength-reduction factor is smaller than the target ductility, whereas for systems with periods longer than two times $T_S$ the strength-reduction factor is approximately equal to the target ductility.

The influence of soil conditions on strength-reduction factors can be seen in Fig. 6 where mean $R_\mu$ spectra are plotted for systems undergoing displacement ductilities demands of five when subjected to ground motions recorded on rock, on alluvium, and on soft soil sites. For soft soil sites, the mean $R_\mu$ are plotted assuming a predominant period of the soil site of 2.0 s. As shown in this figure, strength-reduction factors corresponding to ground motions recorded on rock are similar to those corresponding to ground motions recorded on firm alluvium sites, but significantly different from those recorded on very soft soil sites. For periods shorter than 1.3 s ($T < 2T_S/3$), the strength-reduction factors are considerably smaller than those corresponding to systems on rock sites or those corresponding to systems on alluvium sites. This observation has very important design implications. Mainly, that the use of strength-reduction factors derived from studies of systems subjected to ground motions recorded on rock and firm alluvium sites can lead to unconservative designs if used in the design of short-period structures located on soft-soil sites. For periods between 0.85 and 1.5 times the predominant period of the soil site (between 1.7 s and 3.0 s in this example), the strength-reduction factors corresponding to soft soil sites are significantly larger than those corresponding to systems on rock sites or those corresponding to systems on alluvium sites.

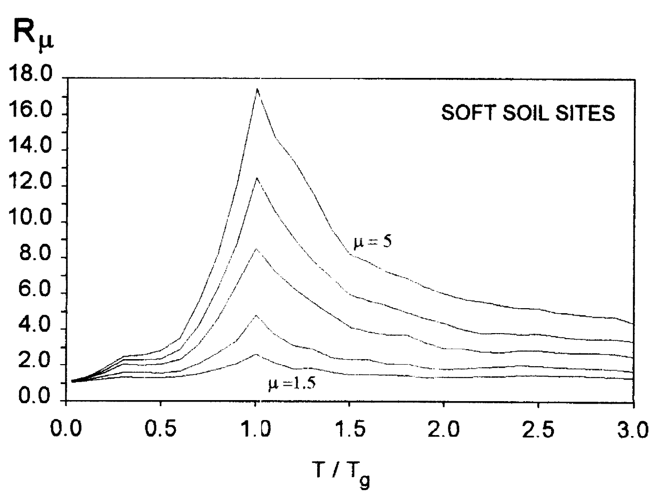


Fig. 5. Mean strength-reduction factors for systems subjected to ground motions recorded on very soft soil sites for ductilities of 1.5, 2,3, 4 and 5.

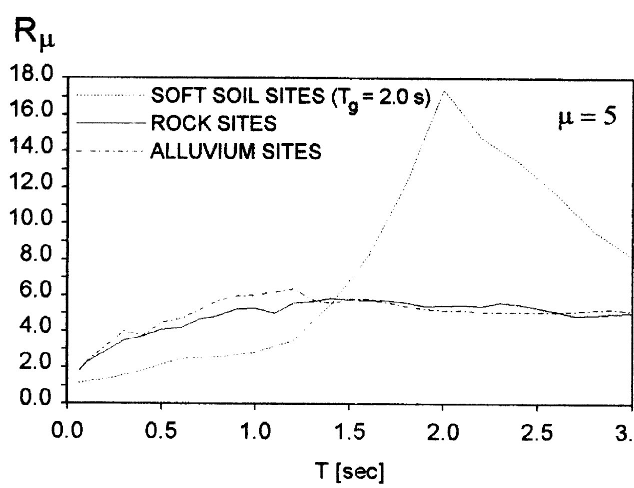


Fig. 6. Influence of local site conditions on strength-reduction factors ($\mu$ = 5).

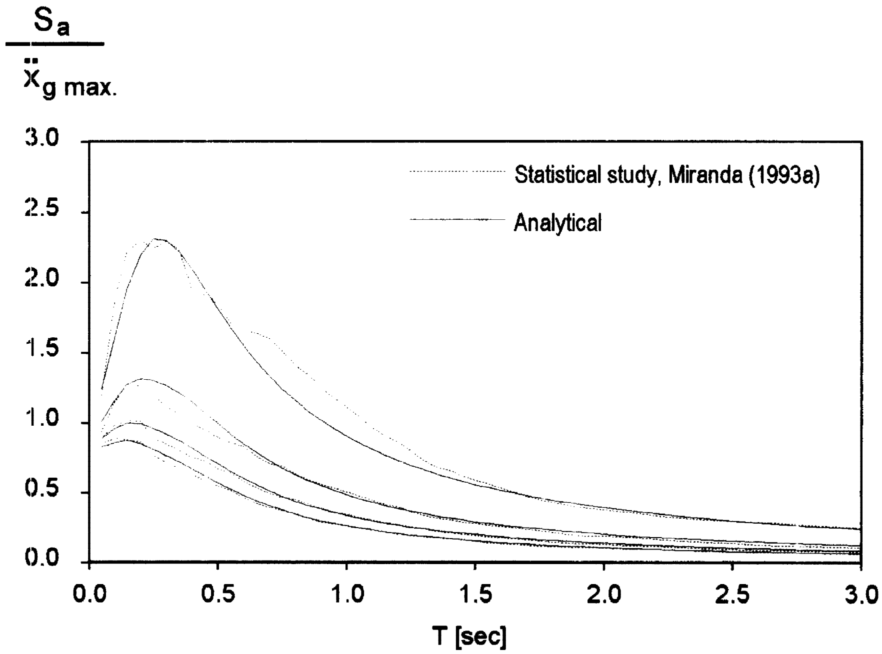


Fig. 7. Comparison of mean strength demand spectra with combined use of a smoothed elastic spectrum reduced by analytical strength-reduction factors ($\mu$ = 1, 2, 3 and 4).

The dispersion on strength-reduction factors have been recently studied (Miranda, 1993c; Riddell, 1995). These studies have concluded that with the exception of very short periods ($T < 0.2$ s), the coefficient of variation (COV)of $R_\mu$ is approximately period independent and that the dispersion increases with increasing displacement ductility ratio. COV's vary from 0.2 for ductility ratios of 2 to 0.5 for ductility ratios of 6.

Nassar and Krawinkler (1991) and Miranda (1993c) studied the influence of earthquake magnitude and epicentral distance on the strength-reduction factors. Both studies concluded that the effect of both parameters is negligible.

Approximate reduction factors like those computed with equations 4 to 7 can be used together with a SLERS to estimate design inelastic strength demand spectra. An example is shown in Fig. 7, in which the mean normalized elastic strength demand spectrum corresponding to ground motions recorded on firm alluvium sites has been approximated by a function similar to that proposed by Riddell (1995)

$$\frac{F_{y,e}}{m\,\ddot{x}_{g\,max.}} = \frac{1 + 18T^{1.4}}{1 + 20T^{2.6}} \tag{8}$$

and has been reduced by $R_\mu$ factors (i.e., by using eq. 3) computed with equations 4 and 6. The resulting inelastic strength demand spectra are compared with mean inelastic strength demand spectra from a statistical study of 62 ground motions recorded on firm alluvium sites (Miranda, 1993a). It can be seen that the agreement is good and thus adequate to be used for practical purposes.

## DISPLACEMENT DEMAND

Although the use of lateral displacement demands directly in earthquake resistant design was proposed many years ago, it is not until recently that displacement-based seismic design procedures have been proposed (Moehle, 1992; Bertero et al., 1991; Wallace, 1995; Calvi and Kingley, 1995).

Elastic Displacement Demands

The elastic displacement demand, $\Delta_e$ , defines the minimum yield displacement required in the structure in order to remain elastic during a given ground motion. For SDOF systems, the elastic acceleration response spectra provide the needed information for this parameter.

$$\Delta_e = \frac{S_a \cdot T^2}{4 \cdot \pi^2} = \frac{F_{y,e} \cdot T^2}{4 \cdot m \cdot \pi^2} \tag{9}$$

Inelastic Displacement Demands

The inelastic displacement demand $\Delta_i(\mu_i)$ is defined as the maximum displacement of an inelastic SDOF system whose displacement ductility demand is equal to $\mu_i$. The inelastic displacement demand can be written in terms of the inelastic strength demand, $F_y(\mu_i)$, and the displacement ductility demand, $\mu_i$ , as follows

$$\Delta_i(\mu_i) = \frac{\mu_i \cdot F_y(\mu_i) \cdot T^2}{4 \cdot \pi^2} \tag{10}$$

A typical *inelastic displacement demand spectra* corresponding to ground motions recorded on firm alluvium sites is shown in Fig. 8. It can be shown that inelastic displacement demand increase with increasing periods. Just as inelastic strength demands can be obtained from elastic strength demands through the use of strength reduction (i.e., using equation 3), inelastic displacement demands can be obtained from elastic displacement demands through the use of *inelastic displacement ratios* with the following equation

$$\Delta_i(\mu_i) = \frac{F_{y,e} \cdot T^2}{m \cdot 4 \cdot \pi^2} \frac{\Delta_{inelastic}}{\Delta_{elastic}} = \Delta_e \frac{\Delta_{inelastic}}{\Delta_{elastic}} \tag{11}$$

where the inelastic displacement ratios, $\Delta_{elastic}/\Delta_{inelastic}$ , are a function of the period of the system, $T$, the displacement ductility ratio, $\mu$ , and the site conditions. The inelastic displacement ratios have recently been studied by Miranda (1993a), who computed mean spectra of inelastic displacement ratios for three different soil conditions and for five levels of displacement ductility ratio. Results of this study for firm alluvium sites and for soft soil sites are shown in Figs. 9 and 10.

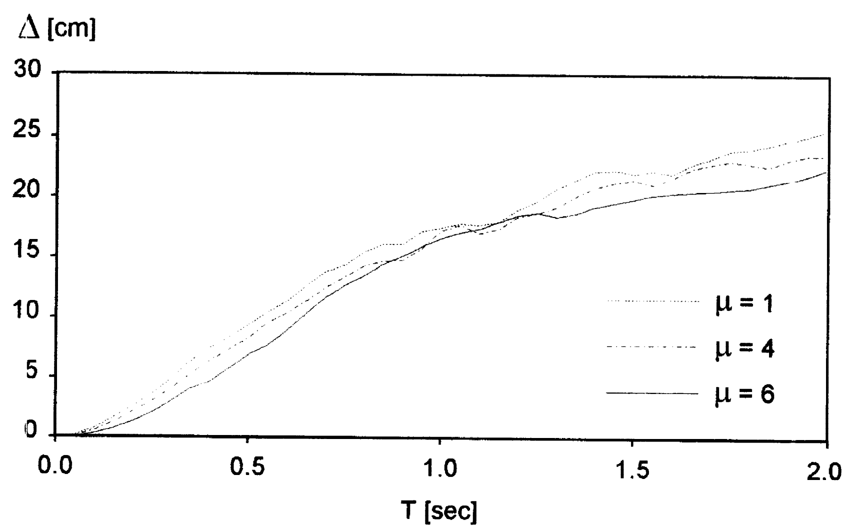


Fig. 8. Inelastic displacement demand spectra for systems subjected to ground motions recorded on alluvium.

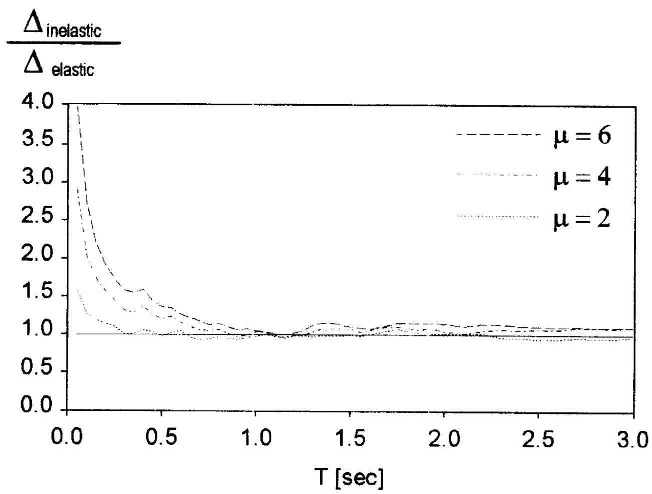


Fig.9. Mean inelastic displacement ratios for systems subjected to ground motions recorded on alluvium soil sites.

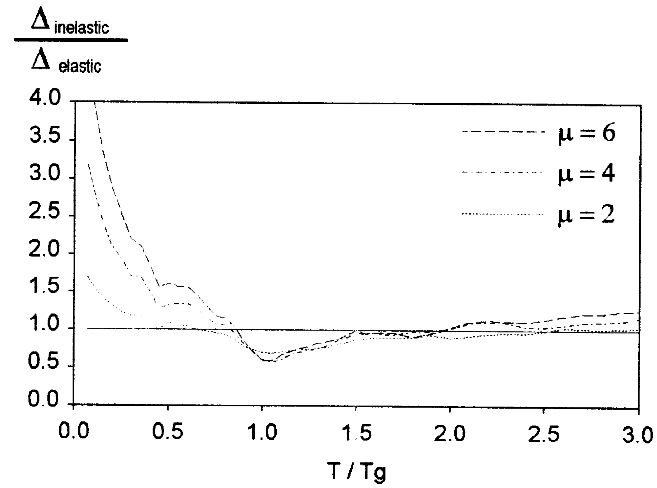


Fig.10. Mean inelastic displacement ratios for systems subjected to ground motions recorded on alluvium soil sites.

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