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RANDOM PROCESSES AND RANDOM FIELDS IN EARTHQUAKE ENGINEERING

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ABSTRACT

Methodologies for earthquake ground motion modeling based on random process and random vibration theory have heretofore not accounted for the inherent variability of the ordinates of Fourier amplitude spectra, peak motion amplitudes and response spectra stemming from the limited duration of motion. We show herein that a major component of the variability of peak ground motion amplitudes, which can also explain observed fluctuations of response spectra, is analytically tractable in terms of the *square* of the ground motion spectral density function $G(\omega)$ and the duration of strong shaking s_0. Data from dense arrays indicates that this component of the uncertainty of ground motion parameter also reflects the inherent *spatial* variability of ground motion parameters at different locations within a region, called a "local field", in which ground motion properties are *nominally* the same and to which one might assign a (single) Mercalli Intensity after an earthquake.

KEYWORDS

Accelerograph arrays; attenuation laws; earthquake ground motion; Mercalli Intensity; probability; random fields; random processes; random vibration; response spectra; seismic analysis; seismic design criteria; spatial variation; spectral density function.

INTRODUCTION

It seems fitting, in this Symposium honoring Emilio Rosenblueth, with before me the task of speculating about future developments of random process and random field theory in earthquake engineering, to briefly quote from his comments (Rosenblueth 1977) about questions as to the appropriate degree of sophistication in probabilistic modeling: "The first time you hear Schoenberg you do not like his music. The 17th time maybe, maybe you appreciate it if you have studied it. Answers to our questions hinge on the familiarity of those who are to define design earthquakes with probability theory." From which you might infer he thought studying random fields is not unlike listening to Schoenberg. But arguing that strong medicine may be good for us (and the profession), he wrote (Rosenblueth 1985): "The simpler the approach the more we sacrifice optimality of the ensuing design", bringing to mind Einstein's famous dictum about theoretical models: "A model should be as simple as possible, but not simpler." Emilio Rosenblueth often stressed the imperative of incorporating probabilistic theory in a framework of engineering decision making, as exemplified by this (1985) quote: "Today the literature blooms with methods for computing reliabilities ever more accurately and efficiently. Bless the literature! Yet we shall remain stranded until we build that next span of the bridge, the span that tells for which reliabilities we ought to design." The random field approach to describing loads on structures corresponds to "Level 4" in Rosenblueth's ranking of methodologies according to complexity, where "Level 1", in elementary safety assessment, treats load and resistance as deterministic quantities (using point estimates); "Level 2" as semi-probabilistic in terms of their point estimates and coefficients of variation; and "Level 3" as statistically independent random variables.

DIRECT STOCHASTIC REPRESENTATION OF GROUND MOTION

We first reassess the rationale for the direct stochastic representation of earthquake ground motions in terms of their one-sided ($\omega \geq 0$) spectral density function $G(\omega)$ and the duration of strong ground motion s_0 (Vanmarcke 1976, 1986; Vanmarcke & Lai 1980), according to which strong-motion accelerograms are idealized as limited-duration segments—in effect, "sample functions"—of stationary Gaussian random processes with s.d.f. $G(\omega)$. This simple representation can of course be extended, most notably by expressing how the ground motion's frequency content evolves with time, but the simple model suffices (and is preferable) for the purpose of introducing the new theory about fluctuations of response spectra and its relation to the local spatial variation of ground motion parameters.

Limitations of Response Spectra

The most common representation of earthquake ground motion for seismic analysis and design is in terms of response spectra, plots of maximum seismic response of a simple linear oscillator versus natural frequency ω_n for different damping ratios ζ. They permit the designer to assess the severity of ground shaking directly in terms of the response of different alternative (simple linear) systems. They reflect the frequency content and the duration of the ground motion, as well as the way the motion is filtered by a single-degree linear oscillator, but their value may be much reduced when the system of interest does not act as a simple linear oscillator. For linear multi-degree systems, one resorts to approximate rules of modal combination of response spectra ordinates at the different modal frequencies. Since the time interval during which strong ground shaking lasts is not explicitly accounted for, phenomena sensitive to motion duration tend to be poorly predicted by procedures based directly on response spectra, e.g., when inelastic action, low-cycle fatigue or soil failure due to liquefaction dominate behavior. And it may be cumbersome to modify response spectra where local soil effects or local spatial variation of earthquake ground motion need to be accounted for.

Further complications arise owing to the fact that, in design situations, seismic analysis typically proceeds based on "design" response spectra which, in a sense, are envelopes of response spectra corresponding to different types of potential ground motions (with different magnitudes and distances, and hence durations and spectral parameters). An unknown degree of conservatism enters into the analysis of multi-degree linear systems when the modal ordinates of "design" response spectra, unlikely to occur simultaneously, are combined. Another significant weakness stems from the fact that the maximum ground acceleration—the "zero-period" acceleration—is widely used as scaling factor for acceleration time histories and response spectra. This has led to the development of "standard" response spectra shapes, obtained from statistical analysis of a suite of recorded accelerograms scaled to a common maximum acceleration. It is well known, however, that the maximum acceleration is a somewhat unreliable indicator of ground motion severity for many kinds of systems, as it is highly sensitive to (poorly predictable details of) the high frequency content of ground motions.

Simple Stochastic Representation of Earthquake Ground Motion "at a Point"

In light of the limitations just mentioned, a strong case can be made for the direct stochastic representation of earthquake ground motion, in terms of the spectral density function of the ground motion $G(\omega)$ and the duration of strong shaking s_0. While essentially equivalent to response spectra in reference to single-degree linear systems, it leads to improved predictions of the response of linear multi-degree systems and the behavior of a variety of nonlinear systems, including those sensitive to low cycle fatigue and liquefaction. It also provides a tractable format for: dealing with the effect of local spatial variation of ground motion; accounting for the influence of local geology; relating ground motion frequency content (and duration) to basic earthquake source parameters and source-to-site distance; performing "overall" seismic safety analysis, incorporating both seismicity and vulnerability of structures; and simulating sets of "time histories" suitable for use in seismic analysis and design. The information about spectral density functions comes from: (a) recorded accelerograms, including sets of records at accelerograph-array stations; (b) geophysical models (e.g., the Brune and related source spectra, which, combined with information about how the different sinusoidal components of source spectra decay with distance, yield spectral densities of ground motion at bedrock outcropping); (c) information about local geology, enabling filtering of the bedrock motion; and (d) inverting known or specified smooth site response spectra (using random vibration theory).

Local Fields of Ground Motion

Short-range spatial variation of ground motion is of interest to engineers and seismologists alike, but their perspectives differ. Seismologists seek to describe seismic wave composition, polarization, and source and path properties, while engineering interest lies in what is needed for response prediction, namely information about the energy-rich strong-motion phase which is often dominated by relatively high frequency components. Studies on spatial variation of ground motion aim to understand, describe and predict the local variability of ground motion parameters and local damage patterns. Such variation may be important in the design of structures with wide foundations such as dams, tall offshore structures, or nuclear plant facilities; structures with widely-spaced multiple supports such as bridges; and all kinds of "lifelines" carrying oil, gas, water, or traffic. A related critical question is how well a single (recorded) time history represents the ground motion at points in its vicinity.

The term "local field", as used herein, refers to surface areas that are small enough so that the (internal) variation of motion amplitudes as predicted by (epicentral-distance-dependent) attenuation laws, is negligible; specifically, within the confines of a "local field", peak accelerations estimated in function of magnitude and distance differ negligibly compared to the differences in peak accelerations as measured by an (actual or hypothetical) dense array of strong-motion accelerographs; the latter may differ by factors of 2 and more, even when separation distances are of the order of tens of meters. Each "local field" exists in a particular seismic setting, characterized by faults or other seismogenic zones, and the seismic threat at the extended site—the "local field"—can be assessed by means of standard (site) seismic risk analysis.

Empirical data about spatial variation comes from dense arrays of strong motion accelerographs covering areas with typical dimensions ranging from several meters to several kilometers. The first productive accelerograph array was the SMART-1 array located in Lotung, Taiwan (Bolt *et al.* 1982). The array proper, consisting of 37 instruments synchronously measuring three ground acceleration components, has recorded many earthquakes generating array-site ground motion levels severe enough to damage structures. Array recordings of a seismic event may be thought of as incomplete observations of a space-time random field, with partially predictable phase lags; the "aligned" motions—from the time lags owing to wave-front propagation have been subtracted—are assumed to be "locally" homogeneous and isotropic during the strong motion phase (Harichandran & Vanmarcke 1986; Boissières & Vanmarcke 1995 a, b). But even without "aligning" the motions, one can construct histograms and compute measures of dispersion for ground motion parameters such peak amplitudes, Arias intensity, Fourier amplitude spectra, and strong-motion durations; the width of the histograms depends on magnitude and distance and local geological conditions, as well as, to some extent, on the surface area covered by the accelerographs and the layout of the array.

Stochastic Seismic Analysis of Linear Systems

Based on the simple stochastic representation of the seismic input—sudden exposure, for an interval of s_0 seconds, to stationary Gaussian excitation with given spectral density function $G(\omega)$—random vibration provides approximate closed-form predictions (fractiles of the distribution) of seismic response of one-degree and multi-degree linear systems. In particular, the peak one-degree system response Y_{max} can be expressed as a product of two factors: the response standard deviation σ_Y (evaluated at time $t = s_0$ after the onset of vibration) and the dimensionless peak factor R, which may be thought of as a random variable or its fractile for probability of exceedance p; choosing $p = 0.5$ thus yields the median maximum response. The same basic format can be used to predict peak amplitudes of ground motion, such as the peak ground acceleration: $A_{max} = \sigma_A \times R$, where σ_A is the strong-motion-acceleration standard deviation (the square root of the variance σ_A^2) and R is the peak factor. The methodology also enables prediction of linear-multi-degree-system response, with the form of the expression for the multi-degree-system response variance leading to improved rules for modal combination in the (deterministic) response spectrum approach; it suffices to replace the modal standard deviations by the response spectra coordinates. Vanmarcke (1976) outlined a "stochastic modal superposition" (SMS) procedure that fully accounts for cross-correlation between modal responses, as well as for the transient nature of lightly-damped seismic response and secondary effects due to differences in the peak factors of multi- and single-degree responses. For a historical account of this and other modal combination rules, and references to recent extensions to linear multi-support, multi-input structures, see Heredia & Vanmarcke (1995).

NEW PERSPECTIVES ON THE VARIABILITY OF GROUND MOTION AND RESPONSE

In the methodology just mentioned, maximum amplitudes of ground motion or response are predicted by multiplying a non-random "standard deviation" σ_Y by a chosen fractile of a random peak factor R. We will now make the case that the standard deviation σ_Y, or the corresponding variance σ_Y^2, is itself *inherently* random when the time history is seen as a relatively short "sample function" of a stationary random process with known spectral density function $G_Y(\omega)$. Sample spectral density functions possess inherent variability that translates into inherent variability of excitation and response statistics. This brings to stochastic seimic analysis a heretofore unaccounted-for source of uncertainty. Theoretical backup and analytical tools for analyzing it are provided by statistical theory of (stationary) random functions, while empirical support for the findings comes from processing of accelerograms recorded during a number of seismic events at dense-accelerograph-array sites. We present herein some typical results pertaining two events recorded by the SMART1 array.

"Sample Spectra" as Random Functions of Frequency

The *sample* spectral density function of a stationary random process $X(t)$ may be defined as follows (Parzen 1962):

$$\hat{G}(\omega) = \frac{1}{\pi s_0} \left| \int_0^{s_0} exp(-i\omega t) X(t) dt \right|^2, \quad 0 < \omega < \infty, \tag{1}$$

where s_0 is the length of the sampling interval and, in the application to strong ground motion modeling, the "duration of strong ground shaking". $\hat{G}(\omega)$ is the Fourier transform of the *sample* covariance function $\hat{B}(\tau)$, an *even* function which for positive values of the "lag" τ can be expressed as

$$\hat{B}(\tau) = \frac{1}{s_0}\int_0^{s_0-\tau} [X(t) - \hat{m}]\,[X(t+\tau) - \hat{m}]dt, \quad 0 \le \tau \le s_0\,; \tag{2}$$

also, $\hat{B}(\tau) = 0$ when $\tau \ge s_0$. In the above expression, $\hat{m}$ is the "sample mean",

$$\hat{m} = \frac{1}{s_0}\int_0^{s_0} X(t)dt, \tag{3}$$

which is close to the "true mean", $m = 0$, in case $X(t)$ represents earthquake ground motion or the seismic response of linear systems. The "sample variance" of $X(t)$, denoted by $\hat{\sigma}^2$, can be variously expressed as

$$\hat{\sigma}^2 = \frac{1}{s_0}\int_0^{s_0} [X(t) - \hat{m}]^2 dt = \int_0^{\infty} \hat{G}(\omega)\, d\omega = \hat{B}(0). \tag{4}$$

The appearance of raw sample spectra, characterized by erratic fluctuations, suggests that $\hat{G}(\omega)$ can itself be interpreted as a one-dimensional random function of the continuous parameter ω (Vanmarcke 1983; p. 341). In particular, as Figure 1 indicates, the frequency-dependent random process $\hat{G}(\omega)$ is stationary when $X(t)$ is ideal white noise, and is weakly non-stationary for wide-band spectral density functions typical of earthquake ground acceleration.

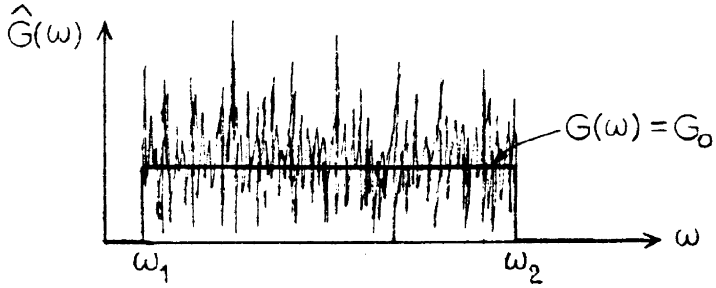

FIGURE 1

Basic Statistics of Sample Spectra. The marginal statistics of $\hat{G}(\omega)$ are given in the literature on spectral estimation (Jenkins 1961). For Gaussian processes $X(t)$ and relatively long sampling intervals s_0, $\hat{G}(\omega)$ is approximately exponentially distributed with mean

$$m_{\hat{G}(\omega)} = E\left[\hat{G}(\omega)\right] = G(\omega), \tag{5}$$

and variance

$$\sigma^2_{\hat{G}(\omega)} = Var\left[\hat{G}(\omega)\right] = G^2(\omega). \tag{6}$$

Hence, the mean and the standard deviation of the sample s.d.f. are equal, so their quotient, the coefficient of variation, is one:

$$V_{\hat{G}(\omega)} = \sigma_{\hat{G}(\omega)}/m_{\hat{G}(\omega)} = 1. \tag{7}$$

The high positive skewness of $\hat{G}(\omega)$ is consistent with the high peaks seen in squared Fourier amplitude spectra of typical ground acceleration records. It is also known that, in the limit when $s_0 \to \infty$, $\hat{G}(\omega_1)$ and $\hat{G}(\omega_2)$ associated with a pair of adjacent frequencies ω_1 and ω_2 are uncorrelated. For sample-function segments with finite duration, however, the scale of fluctuation of $\hat{G}(\omega)$—the "correlation distance" in the frequency domain—varies inversely with s_0, namely (Vanmarcke 1983)

$$\Omega_G = 2\pi/s_0, \tag{8}$$

When $s_0 \to \infty$, $\Omega_G \to 0$ and the frequency-dependent process becomes truly uncorrelated, but for finite record lengths one expects there to be positive correlation between spectral ordinates at frequencies separated by less than Ω_G. For earthquake records, a typical strong-motion duration is $s_0 = 10sec$, giving $\Omega_G = 2\pi/10 \approx 0.628\,cps$ or $0.1Hz$.)

Variability of Sample Spectral Densities at Array Sites

When a dense array of accelerographs—such as SMART1—is triggered during a seismic event, many three-component records are generated, and for a particular component of the ground motion, squared Fourier amplitude spectra $|F_i(\omega)|^2 \propto \hat{G}_i(\omega)$ can be calculated at each instrumental location $i = 1, 2, \ldots, n$ (where $n = 37$ for SMART1). For a given frequency ω, this yields a total n values $|F_i(\omega)|^2$ whose (sample) mean, standard deviation, and coefficient of variation characterize the local spatial field of squared Fourier amplitude spectra across the array site. Figures 2 through 5 illustrate typical results, for the East-West horizontal components during Events 43 and 45. The top part in Figures 2 and 3 shows the mean and the mean-plus-onestandard-deviation of the squared Fourier amplitude spectra, while the bottom part shows their coefficient of variation as a function of frequency ω, which is seen fluctuating near *one* at moderate and high frequencies. The lower c.o.v.'s at lower frequencies reflect the relatively high spatial correlation of low-frequency components of the ground motion, while the spuriously high c.o.v.'s at near-zero frequencies stem from the mean spectra being negligible. At moderate and high frequencies, the spectral ordinates tend to have low spatial correlation and behave as do ordinates of sample spectral densities as predicted by the theory of statistics of stationary and ergodic random processes.

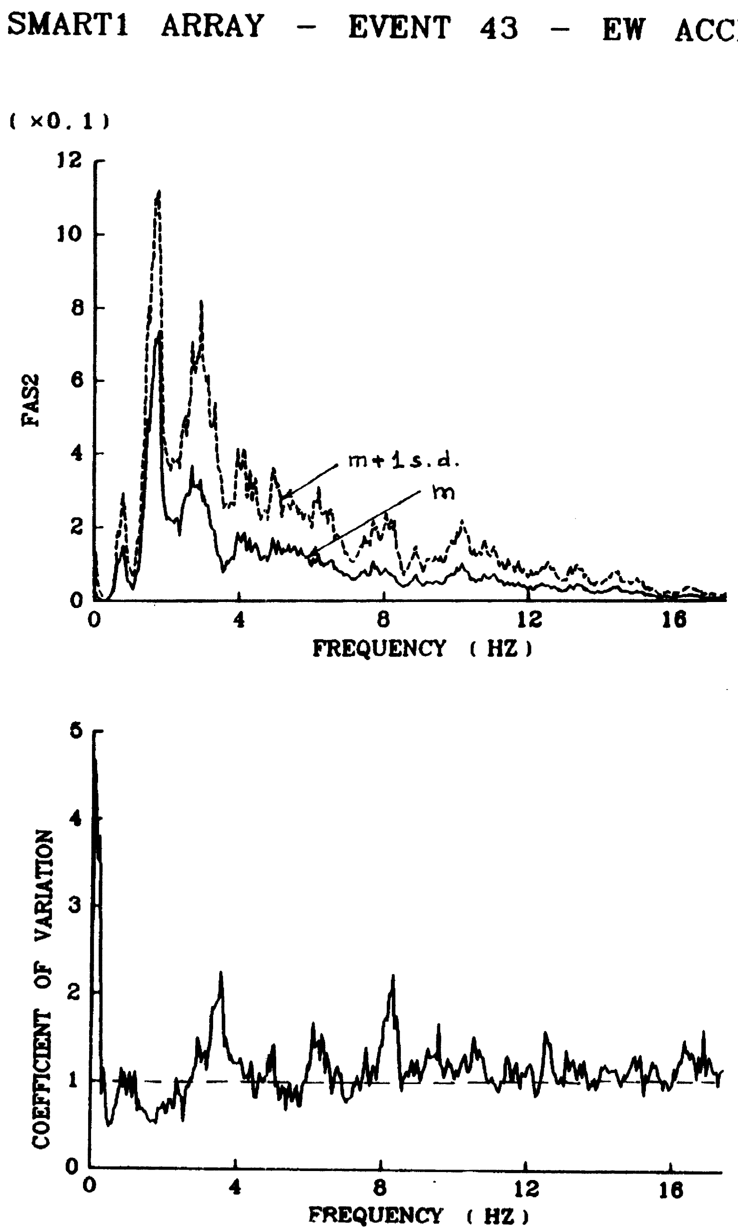

FIGURE 2

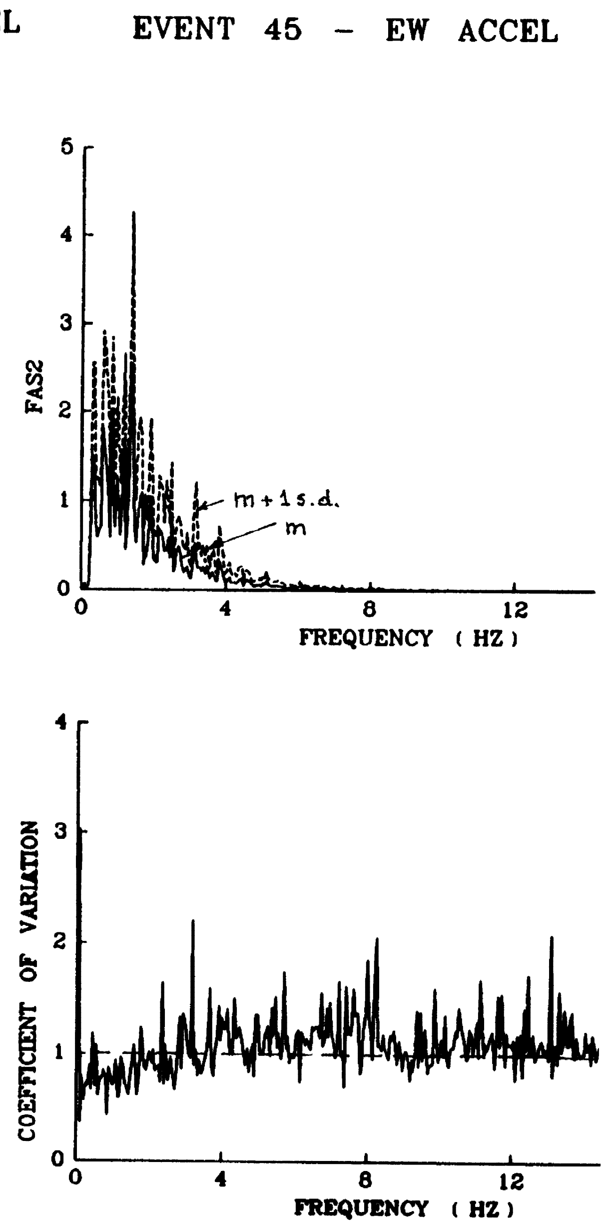

FIGURE 3

Figures 4 and 5 show the histograms of the squared Fourier spectra at selected frequencies, confirming the tendency for the probability distribution to be exponential. It is as if the array stations yield, for each seismic event, a collection of (to first approximation uncorrelated) limited-duration sample functions from a stationary random process (with s.d.f. $G(\omega)$; for an array with size and configuration as SMART1, this interpretation works because the sample functions (the records) do have low correlation at moderate and high frequencies. More generally, the array's instruments "sample" the space-time random field of earthquake ground motion at a discrete set of locations; within the "local field" corresponding to the array site, the spectral content (and strong motion duration) are *nominally* the same but also possess inherent variability stemming from their limited duration, as well as a frequency-dependent spatial correlation structure.

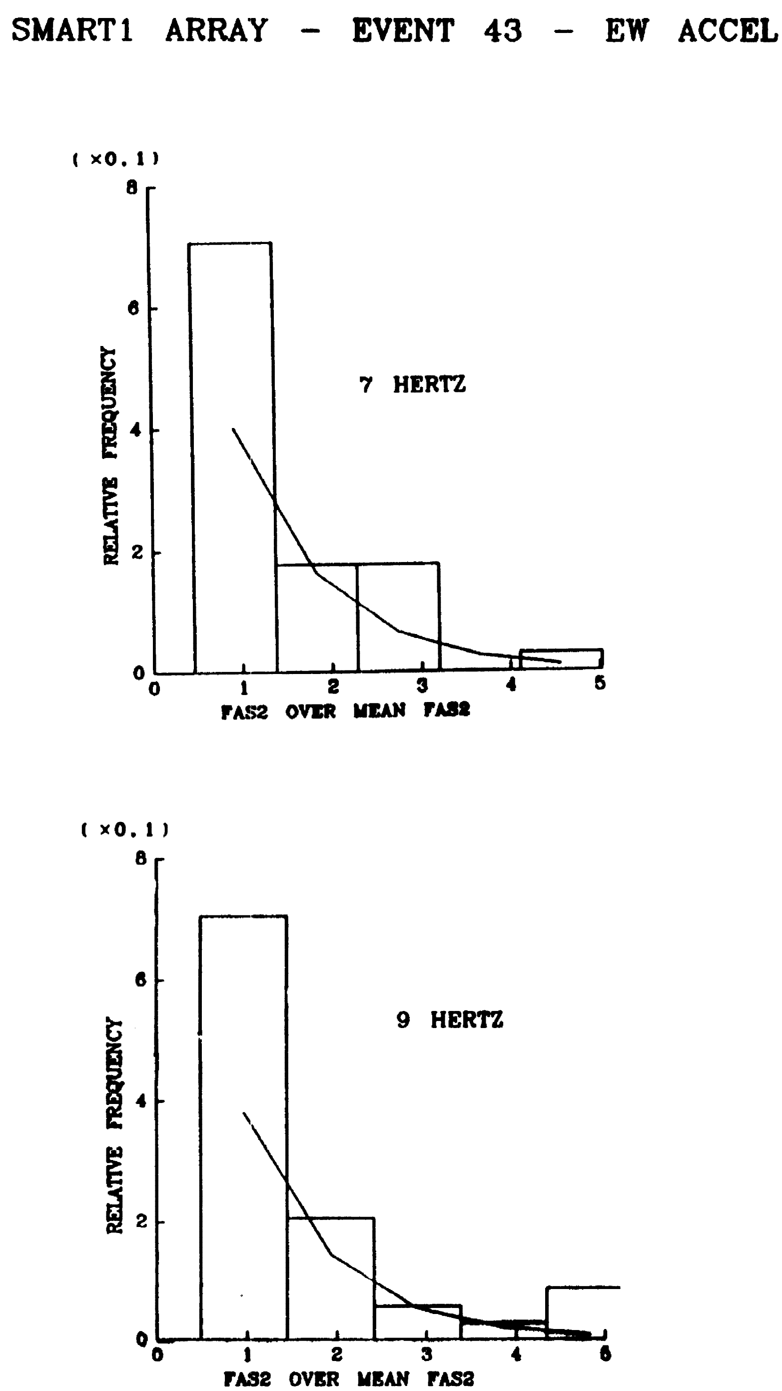

FIGURE 4

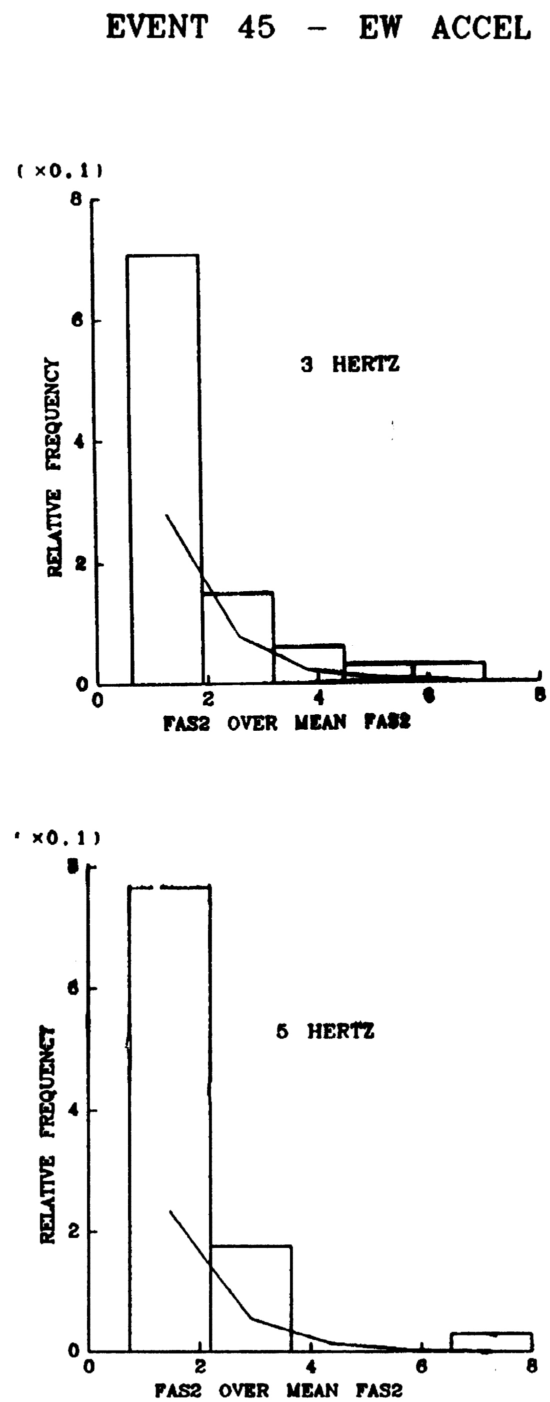

FIGURE 5

Indications are that the statistics of "sample" spectral density functions $\hat{G}(\omega)$ predicted by theory agree quite well with those obtained empirically from recordings at array sites, this despite the limitation that quasi-stationarity is assumed (within the strong-motion segment of accelerograms), along with homogeneity in space within the confines of each "local field". The actual evolution of the frequency content of earthquake ground motions and spatial nonhomogeneity are likely to *add* to the variability, so the theoretical *unit* c.o.v. at high frequencies may be interpreted as a lower bound. In spite of these limitation, the theory (further developed below) provides a basic framework for quantifying a significant and irreduceable component of the variability of earthquake ground motion and response (whose very existence reduces the need and value of making the simple stochastic representation of earthquake ground motion in terms of $G(\omega)$ and s_0 much more complicated.)

Uncertainty about the Ground Acceleration Variance

Consider again a sample of limited duration s_0 of a stationary (and ergodic) random process $X(t)$ with spectral density function $G(\omega)$. The *sample* spectral density function $\hat{G}(\omega)$ tends to fluctuate strongly variable about its mean $G(\omega)$, with standard deviation the same as the mean. The expression for $\hat{\sigma}^2$, the sample variance of $X(t)$, is given by equation (4). When the duration $s_0 \to \infty$, the sample variance $\hat{\sigma}^2$ converges toward the "true" variance $\sigma^2 = B(0)$, which equals the integral of the "true" spectral density function $G(\omega)$. But the sample variance $\hat{\sigma}^2$ is in general a *random variable* with mean

$$E[\hat{\sigma}^2] = \sigma^2 = \int_0^\infty G(\omega) d\omega, \tag{9}$$

and variance

$$Var[\hat{\sigma}^2] = \frac{2\pi}{s_0} \int_0^\infty G^2(\omega) d\omega = \frac{4}{s_0} \int_0^\infty B^2(\tau) d\tau. \tag{10}$$

Derivations of these and other results in this section are presented in Vanmarcke (1983). If the factor $(2/s_0)$ is canceled, the last equality reduces to Parseval's theorem, applicable to any Fourier transform pair (see, e.g., Bracewell 1965), in this case $G(\omega)$ and $B(\tau)$.

Uncertainty of the Ground Acceleration Standard Deviation. In general, the c.o.v. of the sample standard deviation $\hat{\sigma}$ can be estimated based on the lognormal approximation for $\hat{\sigma}^2$ or $\hat{\sigma}$, yielding the following approximation:

$$\frac{Var[\hat{\sigma}]}{\sigma_Y^2} \approx \left[1 + \frac{Var[\hat{\sigma}^2]}{\sigma^4}\right]^{1/4} - 1. \tag{11}$$

The lognormal approximation can also be used to evaluate fractiles of $\hat{\sigma}$, and to estimate, by convolution, the probability density function of the maximum value $X_{max} = R \times \hat{\sigma}$, the product of two random variables (which are, to first approximation, statistically independent). A practical alternative is to assume lognormality for both $\hat{\sigma}$ and R, which makes X_{max} also lognormal.

Example: Band-Limited White Noise Excitation. If the acceleration $X(t)$ is a band-limited white noise with constant spectral density between ω_0 and ω_1, and zero elsewhere, the c.o.v. of $\hat{\sigma}^2$ becomes

$$V_{\hat{\sigma}^2} = \left[\frac{2\pi}{s_0(\omega_1 - \omega_0)}\right]^{1/2}. \tag{12}$$

When this simple model represents the s.d.f. of ground acceleration on bedrock, the two frequency limits depend, respectively, on earthquake magnitude and site-to-source distance; ω_0 tends to decrease as the magnitude increases, while ω_1 decreases as the distance from the source grows. The strong-motion duration also grows, in general, with both magnitude and distance. So the measure of the inherent uncertainty of the acceleration variance, expressed by equation (12), depends in analytically tractable ways on magnitude and distance.

Uncertainty of Linear-System-Response Variance

Many new possibilities are created by the insight that sample spectral density functions $\hat{G}(\omega)$ are random functions of frequency with mean and standard deviation $G(\omega)$ and scale of fluctuation $\Omega_G = 2\pi/s_0$. All aspects of the theory of random processes can be applied to the process $\hat{G}(\omega)$, including information about the heights of local peaks, level crossings and extreme values. More important, the variances and covariances of linear transformations of $\hat{G}(\omega)$ can be evaluated by adapting linear random vibration methodology. Consider, in particular, the second-order stochastic input-output relation,

$$\hat{G}_Y(\omega) = |H(\omega)|^2 \hat{G}_X(\omega), \tag{13}$$

where $H(\omega)$ is the complex transfer function of a linear, time-invariant system. The transfer function may relate ground acceleration or ground velocity to ground displacement, in which case $H(\omega) = i\omega$ or $H(\omega) = -\omega^2$, or it may be the transfer function of the response of a simple oscillator to random excitation. The *sample* spectral density function of the response is the (nonstationary) frequency-dependent random process $\hat{G}_Y(\omega)$, with mean and standard deviation $G_Y(\omega) = |H(\omega)|^2 G_X(\omega)$. The "sample variance" of the response $Y(t)$ is itself a linear transformation of $\hat{G}_X(\omega)$:

$$\hat{\sigma}_Y^2 = \int_0^\infty \hat{G}_Y(\omega)d\omega = \int_0^\infty |H(\omega)|^2 \hat{G}_X(\omega)d\omega. \tag{14}$$

Example: Low-Pass Filtering of Ideal White Noise. Consider a linear system characterized by the "low-pass-filter" transfer function

$$|H(\omega)|^2 = \frac{\omega_0^4}{(\omega_0^2 + \omega^2)^2}, \tag{15}$$

subjected to ideal-white-noise excitation, and let s_0 denote the time interval of stationary excitation and response, i.e., the sample function's "duration". The excitation has a sample s.d.f. $\hat{G}_X(\omega)$ with constant mean and standard deviation G_0. The response has the sample s.d.f. $\hat{G}_Y(\omega) = |H(\omega)|^2 \hat{G}_X(\omega)$. Its sample variance $\hat{\sigma}_Y^2$ is as expressed by equation (4), having mean value

$$E[\hat{\sigma}_Y^2] = \sigma_Y^2 = \int_0^\infty \frac{\omega_0^4 G_0}{(\omega_0^2 + \omega^2)^2} d\omega = \frac{\pi}{4}\omega_0 G_0, \tag{16}$$

variance

$$Var[\hat{\sigma}_Y^2] = \frac{2\pi}{s_0} \int_0^\infty \frac{\omega_0^8 G_0^2}{(\omega_0^2 + \omega^2)^4} d\omega = \frac{5\pi^2}{16} \frac{1}{s_0} \omega_0 G_0^2, \tag{17}$$

and coefficient of variation (c.o.v.)

$$V_{\hat{\sigma}_Y^2} = \left[\frac{5}{\omega_0 s_0}\right]^{1/2} \tag{18}$$

For instance, if $s_0 = 50/\omega_0$, the coefficient of variation equals $10^{-1/2}$ or about 30 percent. The c.o.v. of the sample standard deviation, computed using equation (11), is $\{[1 + (1/10)]^{1/4} - 1\}^{1/2} \approx 0.155$.

Predicting Fluctuations of Response Spectra. The technique just outlined can be used to quantify the variability of "sample response spectra". In this case, a given fractile (say, $p = 0.5$) of the "peak factor" R depends weakly on the oscillator's natural frequency ω_n and damping ratio ζ and on the strong-motion duration s_0. The variability of R is due to the random phasing of the sinusoidal components, while that of the one-degree-system response standard deviation $\hat{\sigma}_Y$ is related to the observed fluctuations of response spectra (plotted, as is common, against natural frequency for a fixed damping ratio). Based on the standard second-order stochastic theory, we can evaluate the coefficient of correlation between the "samples variances" $\hat{\sigma}^2_{Y_1}$ and $\hat{\sigma}^2_{Y_2}$ of the response-spectral ordinates of two oscillators "1" and "2", for instance, two modes of a multi-degree system with a common damping ratio. Also, by assuming that the logarithms of $\hat{\sigma}^2_{Y_1}$ and $\hat{\sigma}^2_{Y_2}$ follow a joint normal distribution, one can express the coefficient of correlation between the samples standard deviations $\hat{\sigma}_{Y_1}$ and $\hat{\sigma}_{Y_2}$ of the response-spectral ordinates of two oscillators or two modes of a multi-degree-of-freedom system.

Inherent Variability of Ground Motion Amplitudes. As indicated, the theory applies to the peak ground motion amplitudes (acceleration A_{max}, velocity V_{max} and displacement D_{max}), or more specifically to the sample standard deviations ($\hat{\sigma}_A$, $\hat{\sigma}_V$, and $\hat{\sigma}_D$). These can also be calculated for records at many stations of a dense accelerograph array, enabling one to compare theory-based and empirical statistics. The sample standard deviations of, say, the ground velocity at two locations within a "local field", $\hat{\sigma}_V^{(1)}$ and $\hat{\sigma}_V^{(2)}$, are generally correlated, their coefficient of correlation dependent on the separation distance, so the theory points new ways of exploration of the local spatial correlation structure of earthquake ground motion. Finally, since the spectral density function of bedrock ground motion can be expressed as a function of magnitude M and focal distance R (using geophysical models), and modified to account for local geological conditions, the theory implies a way to quantify the variability of ground motion amplitudes as predicted by attenuation laws, enabling empirically observed effects (for instance, how the variability of peak amplitudes within a "local field" depends on magnitude) to be systematically quantified within a broad theoretical framework.

CONCLUSIONS

Subject to limitations (to be further investigated) owing to the assumed quasi-stationarity and local spatial homogeneity of ground motions, we outlined a theoretical framework for quantifying the inherent uncertainty of ground motion and structural response stemming from their limited duration. The variability of ordinates of Fourier amplitude spectra and the fluctuations of response spectra of individual earthquake records generally match the corresponding measures of spatial variation at dense array sites, as exemplified by some data from the SMART1 accelerograph array. The analytical tractability of this aspect of the randomness of "local fields" of earthquake ground motion, and the value it adds to information implicit in individual accelerograms, should lead to: (a) improved understanding and predictability of (spatial) patterns of peak ground motion amplitudes within a "local field"; (b) a clearer interpretation of a local field's "effective peak acceleration" and "probable-local-maximum peak acceleration (to which design response spectra might be anchored); (c) quantification, from first principles, of the variability of ground-motion-amplitude attenuation laws; (d) improved random-vibration-based predictions of the response of both linear and nonlinear systems, singly- or multiply-supported; (e) and instrumentally-based approach to predicting patterns of damage within a local field, complementing (and for many purposes replacing) measures such as Modified Mercalli Intensity.

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